Wavelet and Wavelet Packet Transforms: Theory and Applications

Marie FARGE
LMD - CNRS
Ecole Normale Supérieure
Paris

1. Continuous Wavelet Transform
2. Analysis of Turbulent Flows
3. Frames and Orthogonal Wavelets
4. Wavelet Packets and Local Cosines
5. Compression of Turbulent Flows
6. Matching pursuit
7. Resolution of PDEs

Theory

1. Fourier transform

Fourier series, Fourier integrals
Parseval's identity
Convolution theorem
Paley-Wiener theorem
Uncertainty relations
Examples

2. Continuous wavelet transform

Definition, invertibility
Parseval's identity
Properties
Examples
Reproducing kernel, frames

3. Orthogonal wavelet transform

Definition, dyadic grid
Multiresolution analysis
quadratic mirror filters
Properties, examples

4. Wavelet packet transform

5. Windowed Fourier and Malvar wavelet transforms

Applications

1. Fourier

   Analytical solution of the heat equation

   Numerical solution of Navier Stokes equation, FFT

   Pseudo-spectral methods

   Fourier spectrum of turbulent flows

   Analysis of stochastic signals

   Fourier filtering

2. Continuous wavelet transform

   Analysis of sound, algorithm

   Analysis of 2D turbulent flows

   Analysis of 3D turbulent flows

   Satellite image analysis

3. Orthogonal wavelet transform

   Image processing, algorithm

   Image filtering and compression

   Numerical resolution of 1D PDEs

4 Wavelet packet transform

   Fingerprint data compression

   2D turbulent flow compression

   Numerical resolution of 1D PDEs

5. Windowed Fourier and Malvar wavelet transforms

   2D turbulent flow compression

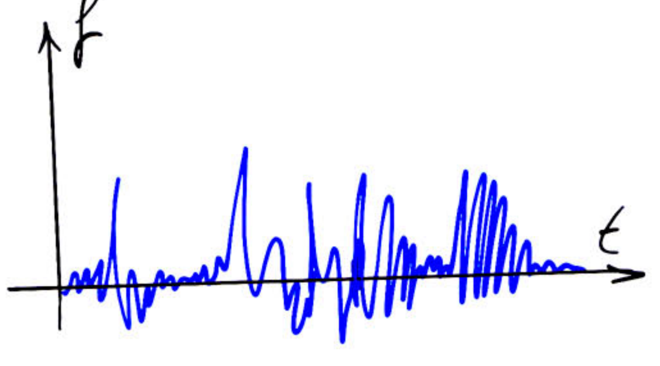
TIME-FREQUENCY REPRESENTATION

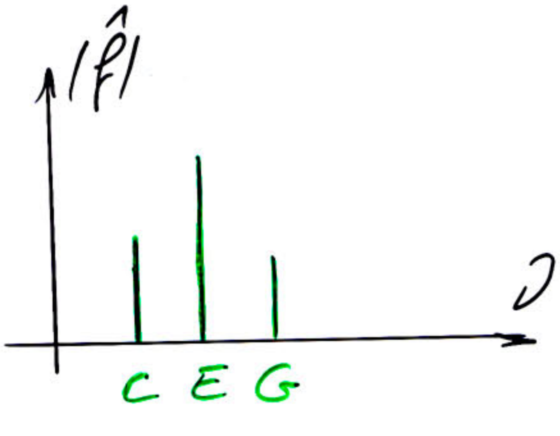
Roger BALIAN

'Un principe d'incertitude fort en théorie du signal'

C.R.A.S., 292, II (1981)

'It could be interesting, in communication theory, to represent an oscillating signal by a superposition of elementary wavelets, each of them having both a frequency and a time localization quite well defined. The useful information is indeed often carried by both the emitted frequencies and by the time structure of the signal (music is a characteristic example for that). The representation of a signal as function of time cannot exhibit the frequency content, which in contrast its Fourier analysis hides the time of emission and the duration of each elements of the signal. An adequate representation should combine the advantages of these two complementary descriptions, while providing a discrete character appropriate to the theory of communication.





NEED FOR A TIME-FREQUENCY REPRESENTATION

Jean VILLE, 1948
'Théorie et application de la notion de signal analytique',
Câbles et transmissions.

'If we consider a musical piece and that a note, for instance an 'A', appears at least once in it, the harmonic analysis will represent the corresponding frequency with a certain amplitude and a certain phase, but without localizing the 'A' in time. However, it is obvious that during this musical piece there are instants for which we do not hear the 'A' note.

Although the Fourier representation is mathematically correct, because the phases of nearby notes are organized in such a way that they destroy by interference the 'A' when we do not hear it or reinforce it, also by interference when we hear it; but, if there is in this conception a skill which honours mathematical analysis, we should not hide the fact that there is a deformation of reality: indeed, when we do not hear the 'A', the genuine reason is that it has not been emitted.

To analyze a signal we have to look for a mixed definition, comparable to the one proposed.

by Gabor: at each instant we have a certain number of frequencies, which give the pitch and the timbre of the sound emitted such as we hear it; to each frequency is associated a certain time distribution which corresponds to the instants when the note is emitted.

We are then led to define an instantaneous spectrum, function of time, which gives the structure of the signal at a given instant; the signal spectrum, in the usual Fourier sense, which gives the frequency structure of the signal for its entire duration, is then obtained by adding all the instantaneous spectra'.

BRIEF HISTORY

Theory:

1984 Continuous wavelet transform (CWT),
Alex Grossmann & Jean Morlet

1986 orthogonal wavelet transform,
Yves Meyer & Pierre-Gilles Lemarié

1987 Compactly supported wavelets,
Ingrid Daubechies

1988 Fast wavelet transform,
Stéphane Mallat

1989 CWT in n dimensions,
Romain Murenzi

Applications:

1984 Geophysics, Morlet
1986 Music, Risset & Kronland-Martinet
1986 quantum mechanics, Paul & Daubechies
1987 Fractals, Holschneider & Jaffard
1988 Image processing, Mallat
1988 Turbulence, Farge
1990 PDE's, Liandrat & Tchamitchian
1991 Numerical analysis, Coifman & Beylkin
1996 Navier-Stokes, Schneider, Perrier

Reference book
Barbara Burke
The world according to wavelets
A.K. Peters, Wellesley, 1996