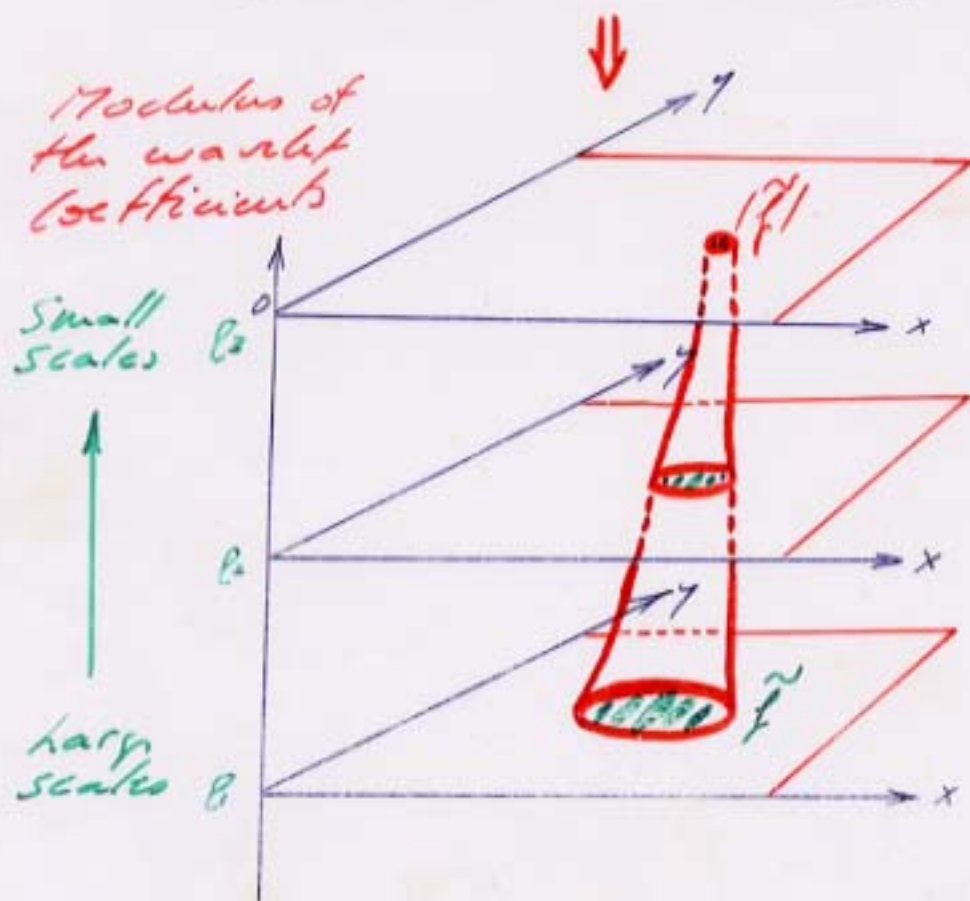
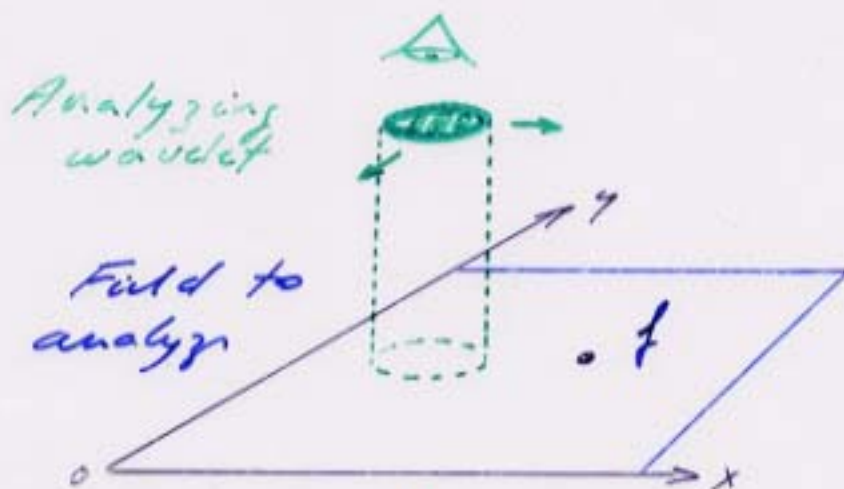


# TWO-DIMENSIONAL WAVELET ANALYSIS OF TURBULENT FIELDS



Logarithm  
of the scale  
 $b$

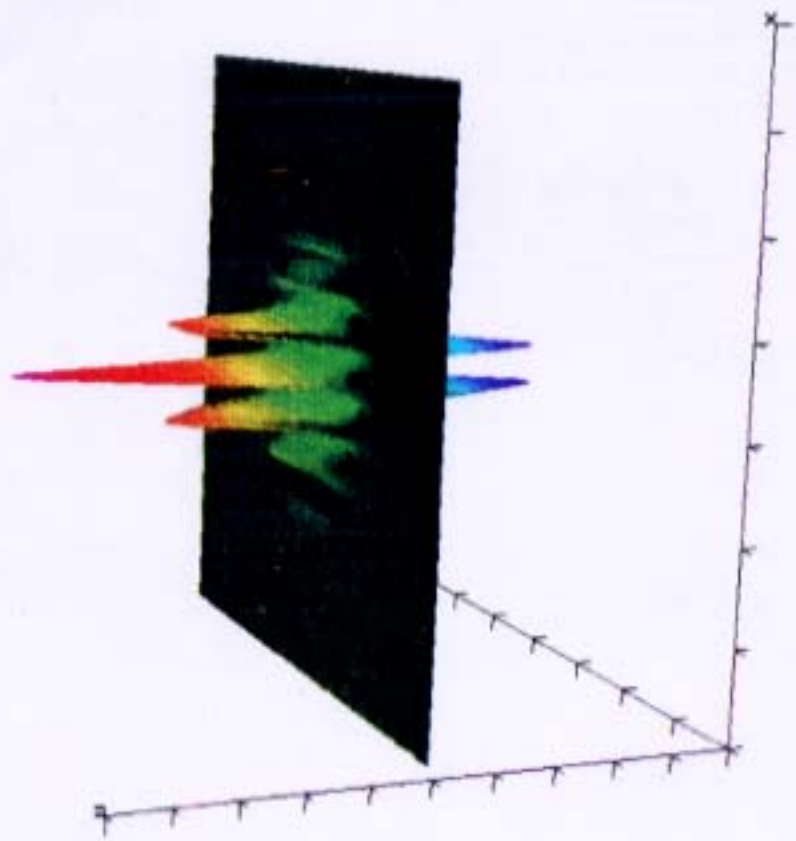
```

0.00x10**0 Nu
0.00x10**0 Ut.au
0.00x10**0 R2
0.00x10**0 Time
64x64 GRID

```

CONTOUR LEVELS

- .850000
- .800000
- .750000
- .700000
- .650000
- .600000
- .550000
- .500000
- .450000
- .400000
- .350000
- .300000
- .250000
- .200000
- .150000
- .100000
- .050000
- 0.000000
- 0.050000
- 0.100000
- 0.150000
- 0.200000
- 0.250000
- 0.300000
- 0.350000
- 0.400000
- 0.450000
- 0.500000
- 0.550000
- 0.600000
- 0.650000
- 0.700000
- 0.750000
- 0.800000



# N-DIMENSIONAL CONTINUOUS WAVELET TRANSFORM

Choice of the 'mother wavelet'

$$C_{\psi} = (2\pi)^N \int \dots \int_{-\infty}^{+\infty} |\hat{\psi}(\vec{k})|^2 \frac{d^N \vec{k}}{|\vec{k}|^{2N}} < \infty$$

Generation of the 'wavelet family'

Euclidean Group with dilatation  $\left\{ \begin{array}{l} \text{by translation (vector } \vec{s}) \\ \text{dilatation (parameter } a > 0) \\ \text{and rotation (operator } r): \end{array} \right.$

$$\Psi_{a,r,\vec{s}}(\vec{x}) = a^{-N/2} \psi(a^{-1} r^{-1}(\vec{x} - \vec{s}))$$

Analysis

$$\tilde{f}(a,r,\vec{s}) = \int \Psi_{a,r,\vec{s}}(\vec{x}) f(\vec{x}) d^N \vec{x}$$

Synthesis

$$f(\vec{x}) = \frac{1}{C_{\psi}} \iiint \Psi_{a,r,\vec{s}}(\vec{x}) \tilde{f}(a,r,\vec{s}) \underbrace{\frac{da dr d\vec{s}}{a^{N+1}}}_{\text{Haar measure}}$$

Parseval

$$\int \|f(\vec{x})\|^2 d^N \vec{x} = \frac{1}{C_{\psi}} \iiint \|\tilde{f}(a,r,\vec{s})\|^2 \frac{da dr d\vec{s}}{a^{N+1}}$$

## 2D WAVELETS

### Real-valued wavelets

Isotropic Mexican hat:

$$\Psi(\vec{x}) = (2 - |\vec{x}|^2) e^{-\frac{|\vec{x}|^2}{2}}$$

$$\hat{\Psi}(\vec{k}) = |\vec{k}|^2 e^{-\frac{|\vec{k}|^2}{2}}$$

Anisotropic Mexican hat:

$$\Psi(\vec{x}) = [2 - (\vec{x} \cdot A \vec{x})] e^{-\frac{\vec{x} \cdot A \vec{x}}{2}}$$

$$\hat{\Psi}(\vec{k}) = |\vec{k}|^2 e^{-\frac{(A^{-1} \vec{k})^2}{2}}$$

with  $A$  deformation matrix

### Complex-valued wavelets

Morlet wavelet:

$$\Psi(\vec{x}) = e^{i(\vec{k}_0 \cdot \vec{x})} e^{-\frac{|\vec{x}|^2}{2}} + \text{c.T.}$$

$$\hat{\Psi}(\vec{k}) = e^{-\frac{(\vec{k} - \vec{k}_0)^2}{2}} + \text{c.T.}$$

Stretched Morlet wavelet

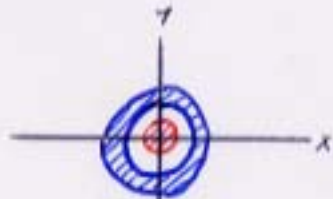
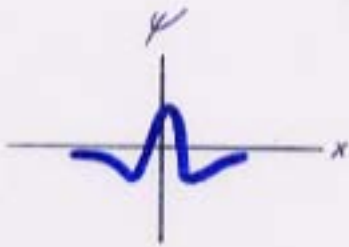
$$\Psi(\vec{x}) = e^{i(\vec{k}_0 \cdot \vec{x})} e^{-\frac{1}{2} (\vec{x} \cdot A \vec{x})} + \text{c.T.}$$

$$\hat{\Psi}(\vec{k}) = e^{-\frac{1}{2} [(\vec{k} - \vec{k}_0) \cdot A^{-1} (\vec{k} - \vec{k}_0)]} + \text{c.T.}$$

with  $A$  deformation matrix

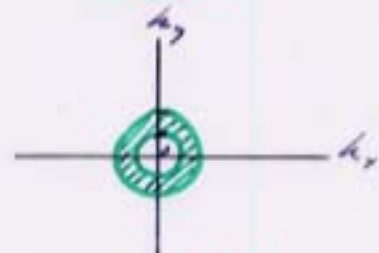
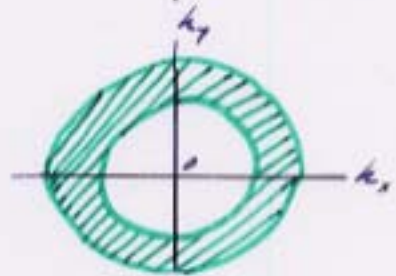
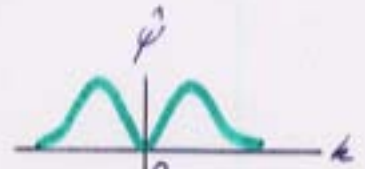
# COMMONLY USED WAVELETS

Real wavelet:  
Mexican hat



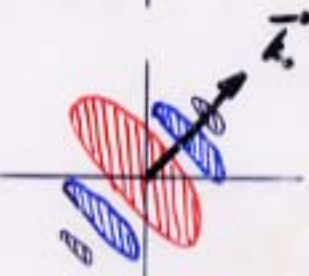
$$\psi = (\vec{x}^2 - 1) e^{-\frac{\vec{x}^2}{2}}$$

Local  
Filter



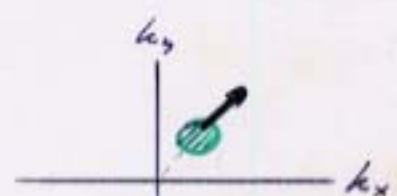
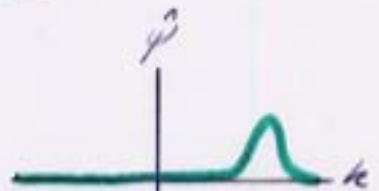
$$\hat{\psi} = \vec{k}^2 e^{-\frac{\vec{k}^2}{2}}$$

Complex wavelet:  
Morlet



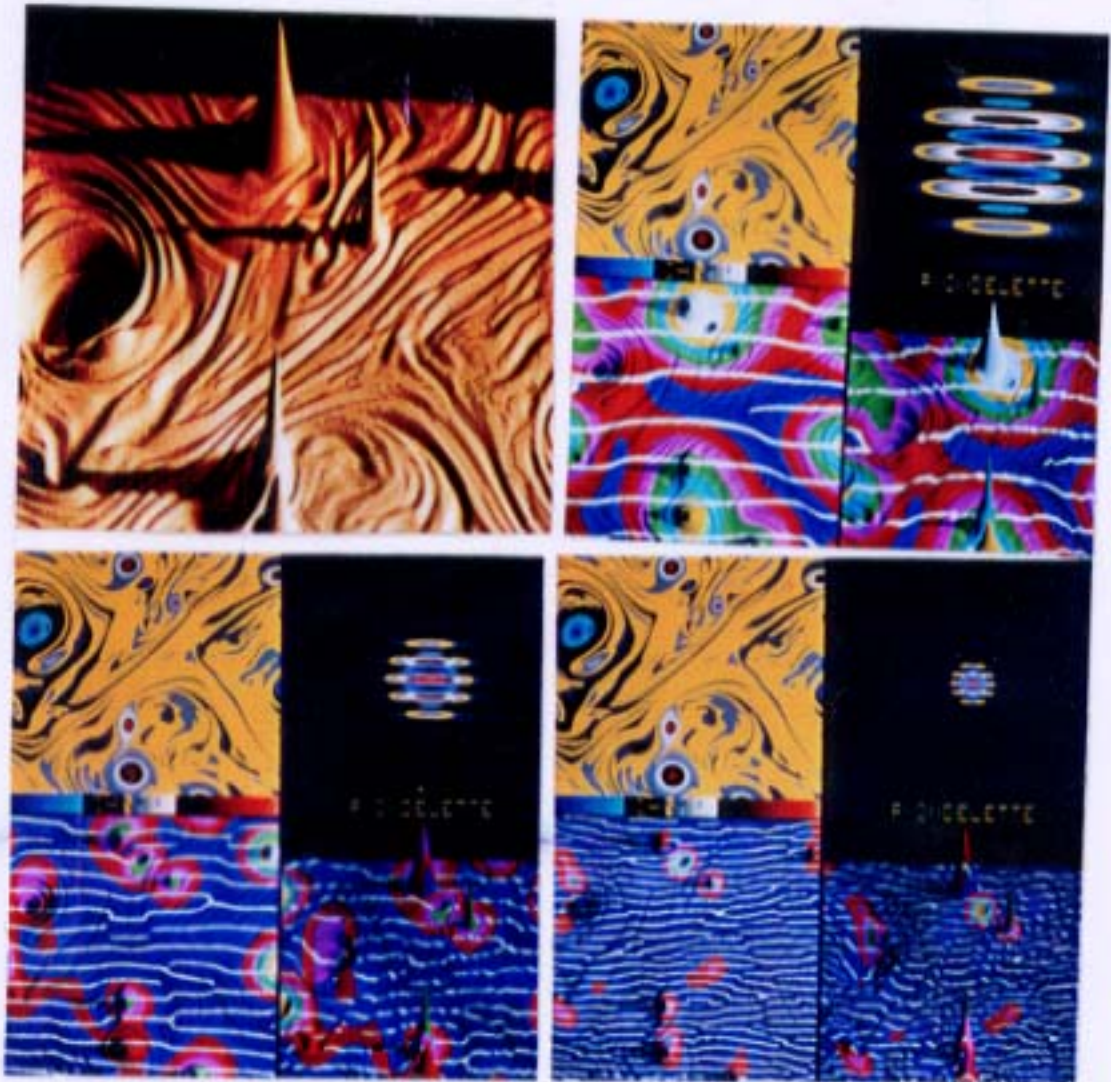
$$\psi = e^{i k_0 \vec{x}} e^{-\frac{\vec{k}^2}{2}}$$

Local  
filter and  
polarizer



$$\hat{\psi} = e^{-\frac{(\vec{k} - \vec{k}_0)^2}{2}}$$

## 2D Wavelet Analysis of Vorticity



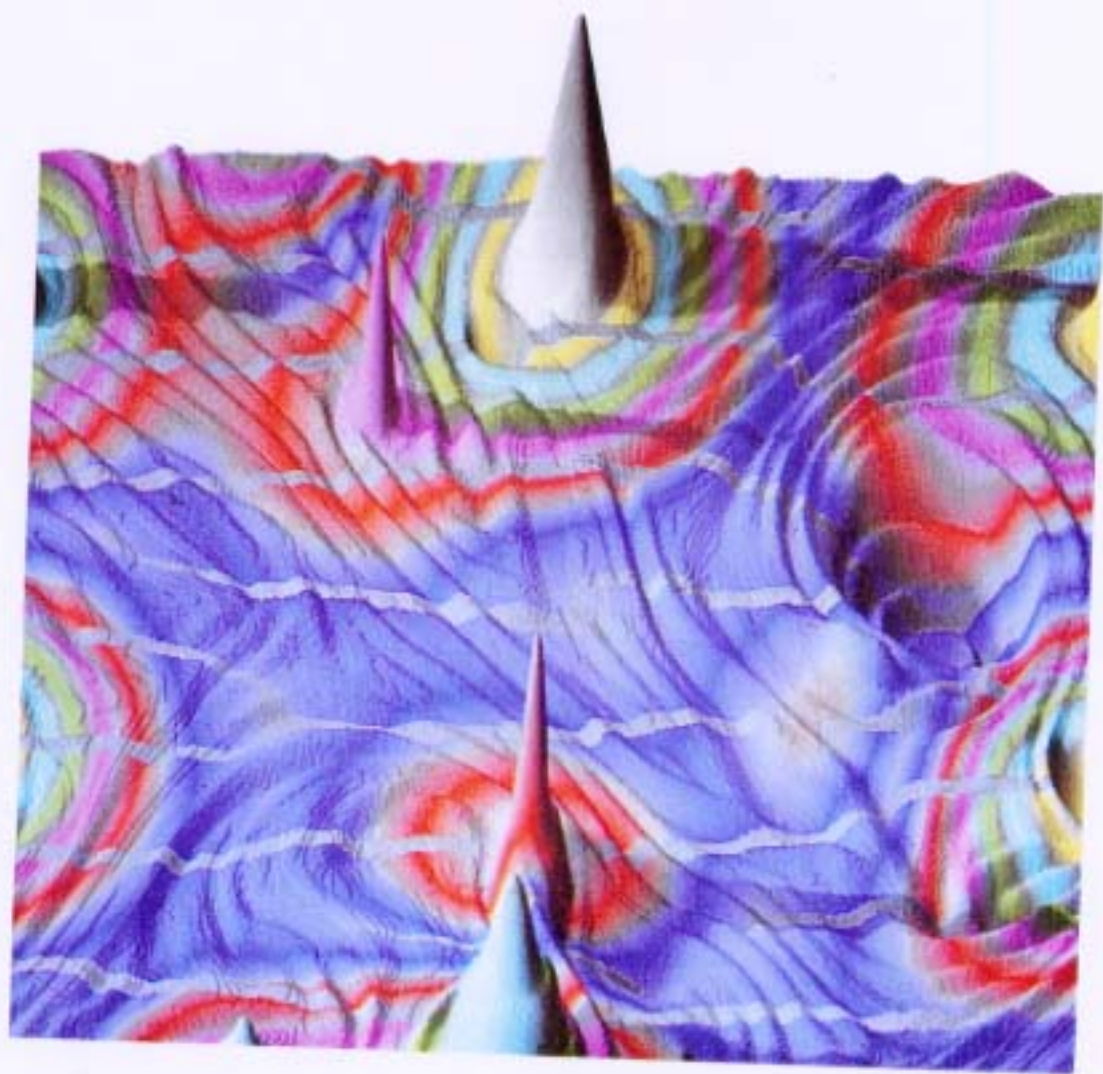
Strong wavelet coefficients  
correspond to coherent structures  
Weak wavelet coefficients  
correspond to background flow

### Analyses en ondelettes de champs turbulents

La transformée en ondelettes est une nouvelle technique d'analyse de signal, et nous avons été les premiers à l'utiliser pour étudier la turbulence. Nous avons écrit le premier code de transformée en ondelettes bidimensionnelles et nous l'avons utilisé pour analyser les structures cohérentes dans les écoulements turbulents, à deux et trois dimensions, résultats qui ont fait l'objet d'une dizaine de publications.

Sur la figure est représentée l'analyse en ondelettes de Morlet bidimensionnelles (montrées en haut à droite de chaque vue) d'un champ turbulent (montré en haut à gauche) à trois échelles différentes. Pour cela nous avons superposé le champ (en vue cavalière), le module des coefficients d'ondelettes (en couleur) et leur phase (l'isoligne blanche correspondant à la phase zéro).

Coherent structures condensate  
enstrophy ( $\langle \omega^2 \rangle$ ) on a wide  
range of scales



20 Vorticity field  $w(x,y)$   
(surface plot)

Modulus of the wavelet coefficients  
for large scale and  $y$  direction  
(Colors encode the magnitude)

Phase  $\phi=0$  of the wavelet coefficients  
for large scale and  $y$  direction  
(grey isolines)

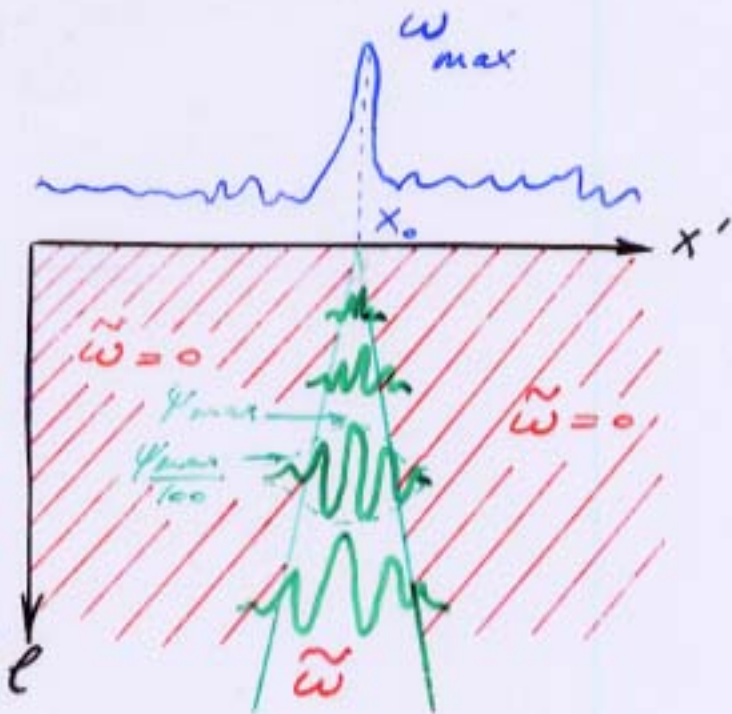
# COHERENT STRUCTURES FILTERING & ANALYSIS

1. By wavelet influence coeffs

$$\psi(\vec{x}) = e^{-1} \psi[\sigma^{-1}(\theta) \frac{\vec{x} - \vec{x}_0}{\ell}]$$

$$\tilde{\omega}(\ell, \vec{x}; \theta) = \langle \psi_{\ell, \vec{x}_0} | \omega \rangle$$

$$\omega_2(\vec{x}) = \frac{1}{\psi_0} \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{\omega}(\ell, \vec{x}; \theta) \psi(\vec{x}) \frac{d\ell d^2x d\mu(\theta)}{\ell^3}$$

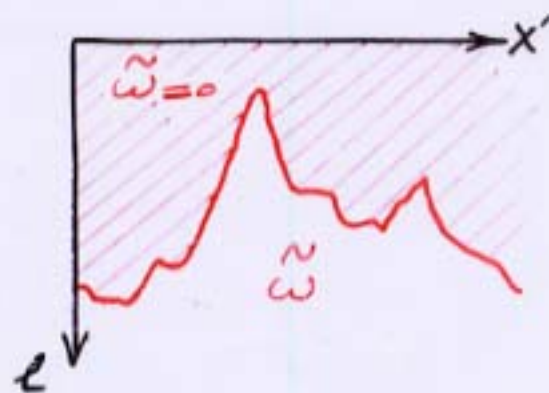


2. By thresholding in  $L^2$  wavelet coefficients space

keep  $\tilde{\omega}^2(\ell, \vec{x}; \theta) > \epsilon$

discard  $= 0$  if  $\leq \epsilon$

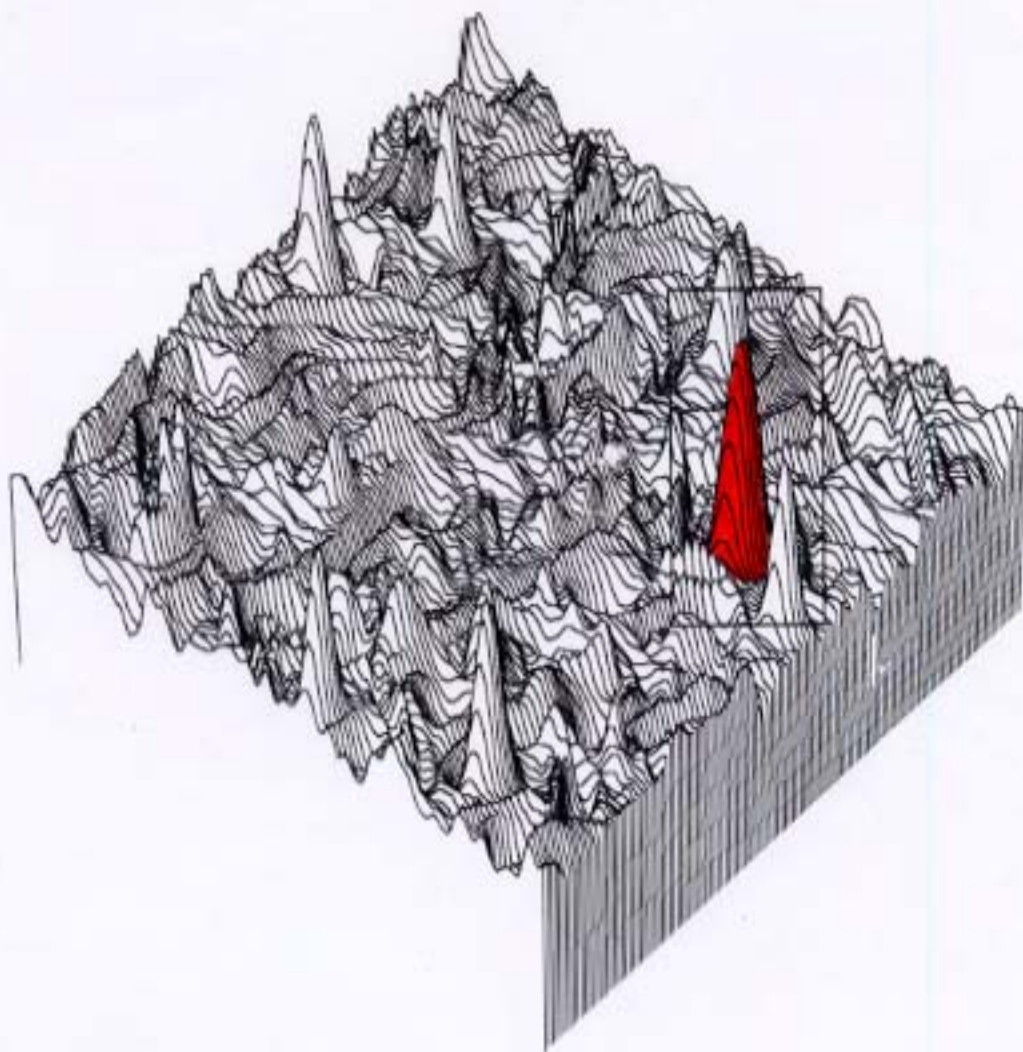
Then reconstruct  $\omega_2(\vec{x})$





*Filtrage d'une structure  
par cône d'influence*

le tourbillon à isoler



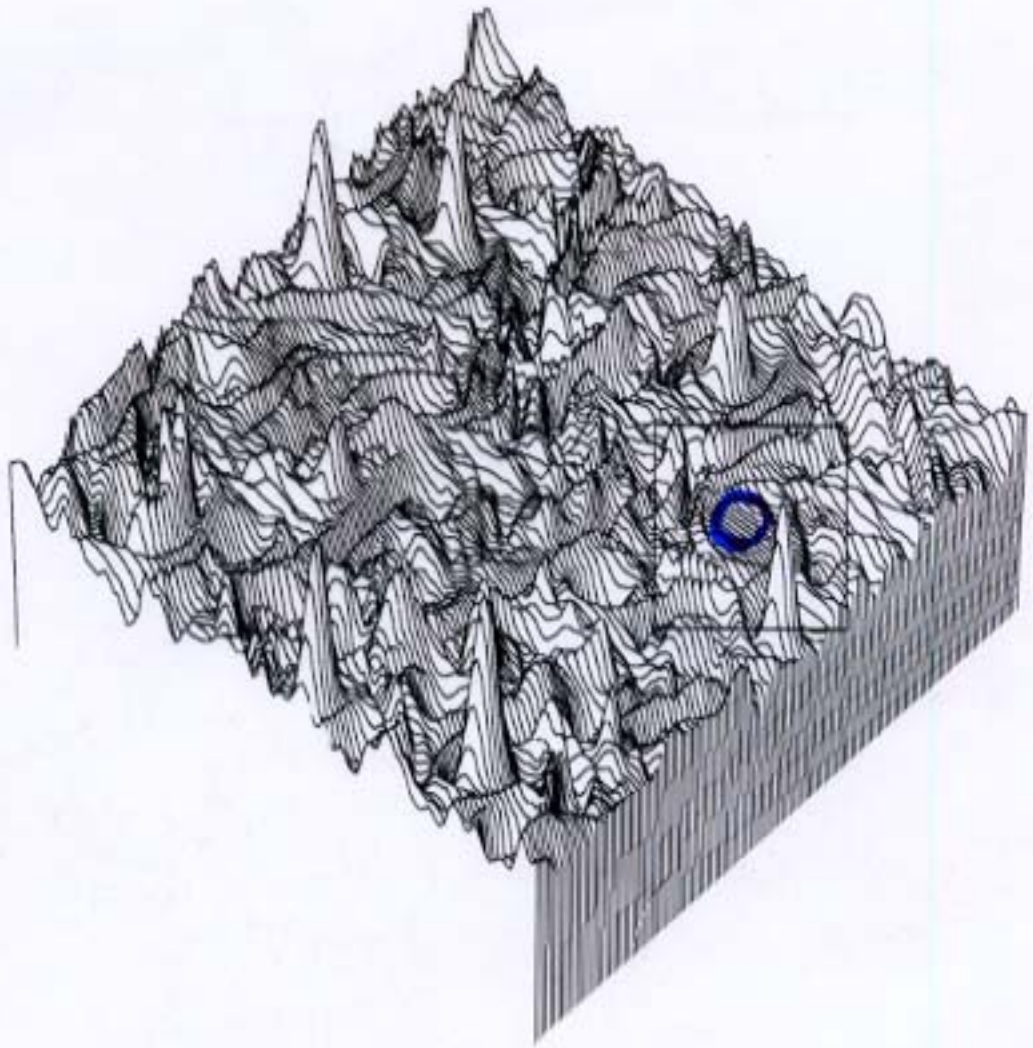
*Filtrage d'une structure  
par cône d'influence*

tourbillon isolé

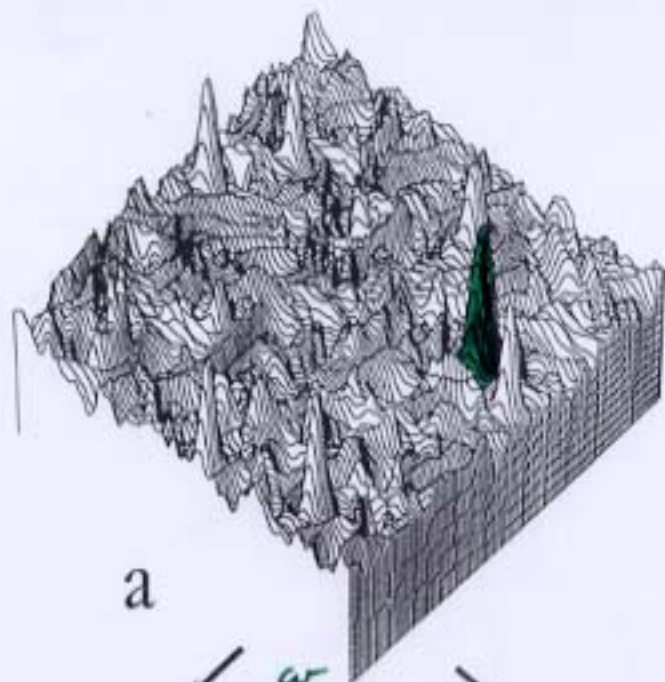


*Filtrage d'1 structure  
par côm d'influence*

champ sans le tourbillon

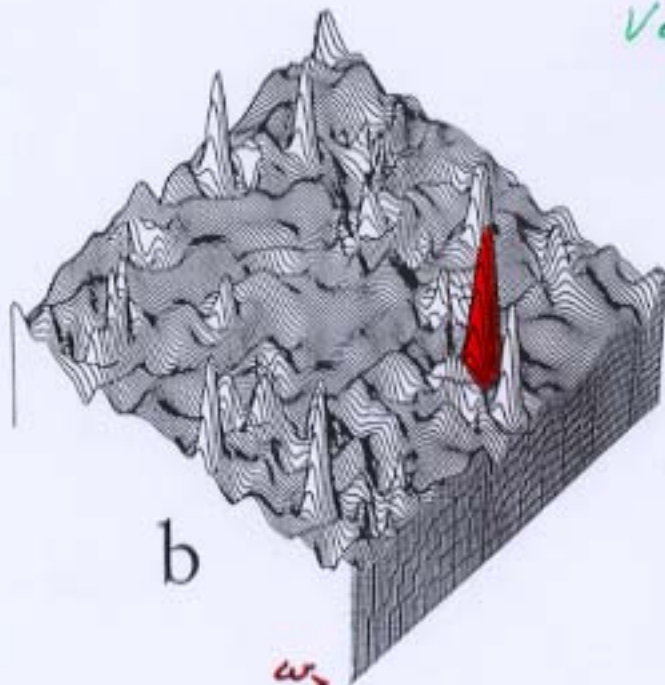


Splitting  
using  
continuous  
wavelets



a

$\omega$   
Vorticity field  
 $\nabla \times \vec{v}$

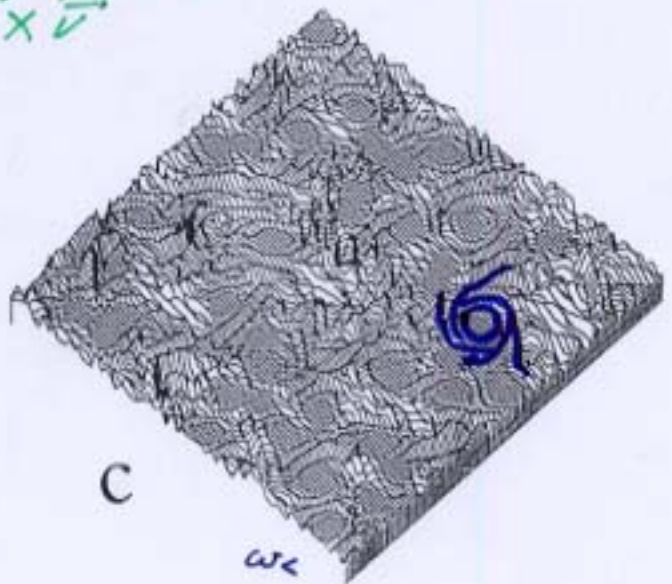


b

$\omega_1$

Coherent  
flow  
of vortices

+

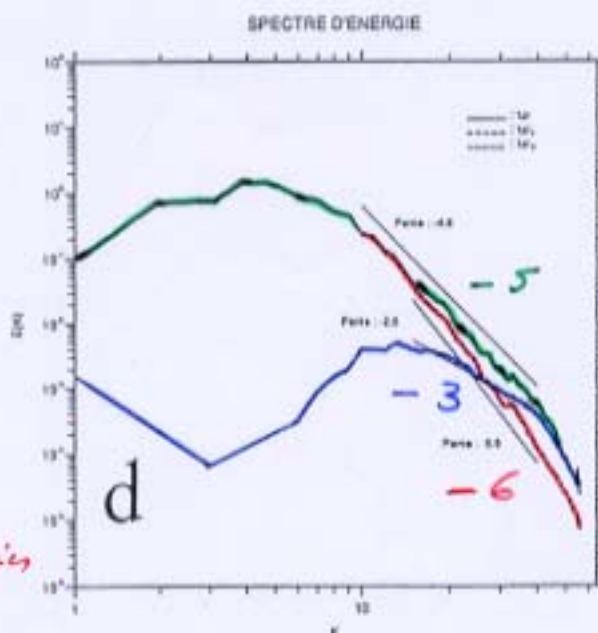


c

$\omega_c$

Incoherent  
background  
flow

Dynamically  
'active'  
elliptic  
regions:  
rotation  $\gg$  strain



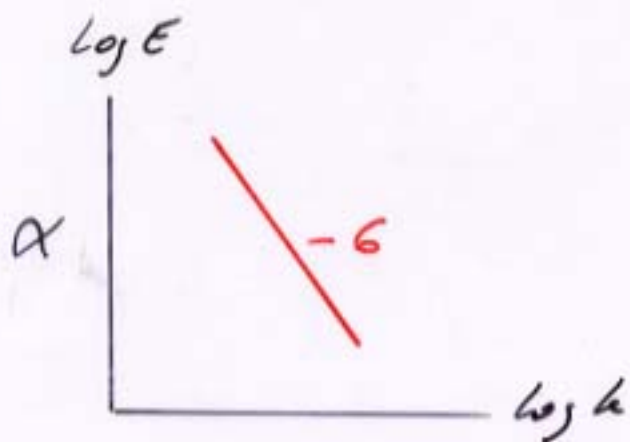
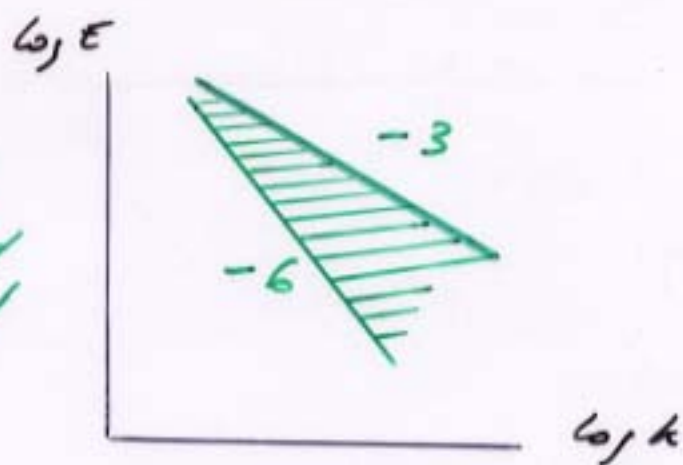
d

Passively  
advectiond  
hyperbolic  
regions:  
strain  $\gg$  rotation

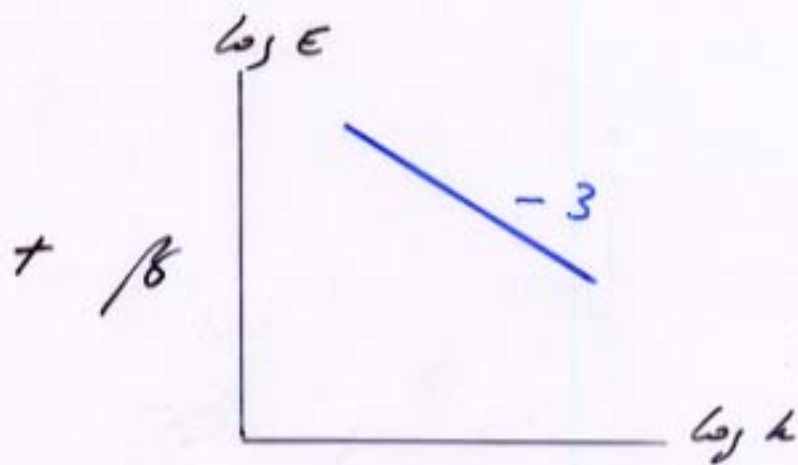
Figure 1

## CONJECTURE

Non universality of the spectral laws observed in numerical simulations



Possible universal law for the vortices scaling?



Universality law ('à la Kraichnan') for the incoherent flow

with  $\alpha$  and  $\beta$  depending on the probability density of vortices

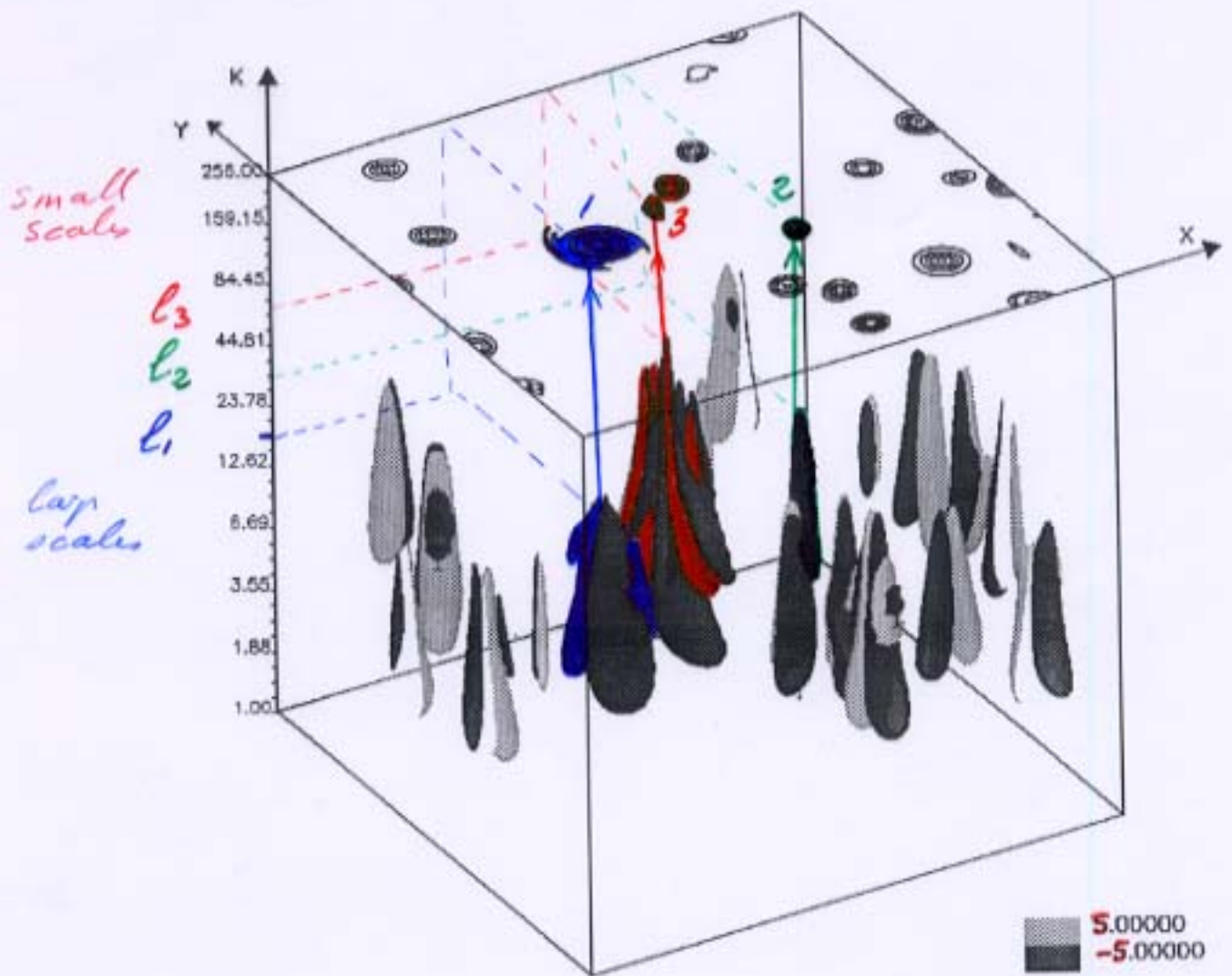


FIG. 4.94 - Champ:R512d-38.(h<sub>4</sub>). Transfert local d'enstrophie. Isosurface:±5.

Local transfers  
of  
enstrophy

$$\overline{\omega \frac{\partial \omega}{\partial t}}$$

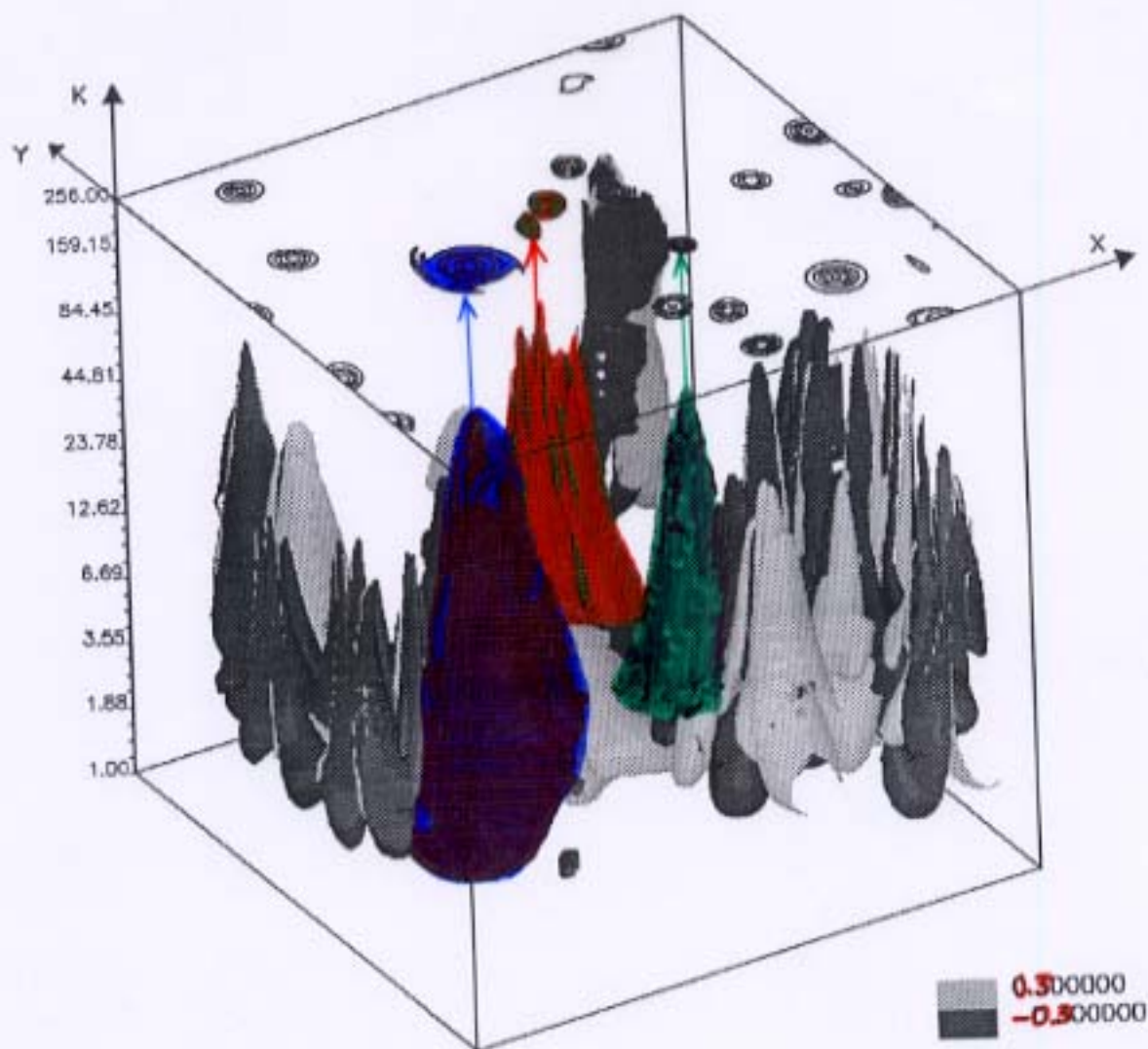


FIG. 4.88 - Champ:R512d-38.(h<sub>4</sub>). Transfert local d'énergie. Isosurface:±0.3 .

local transfer  
of  
energy  
 $\omega \frac{\partial \omega}{\partial t}$