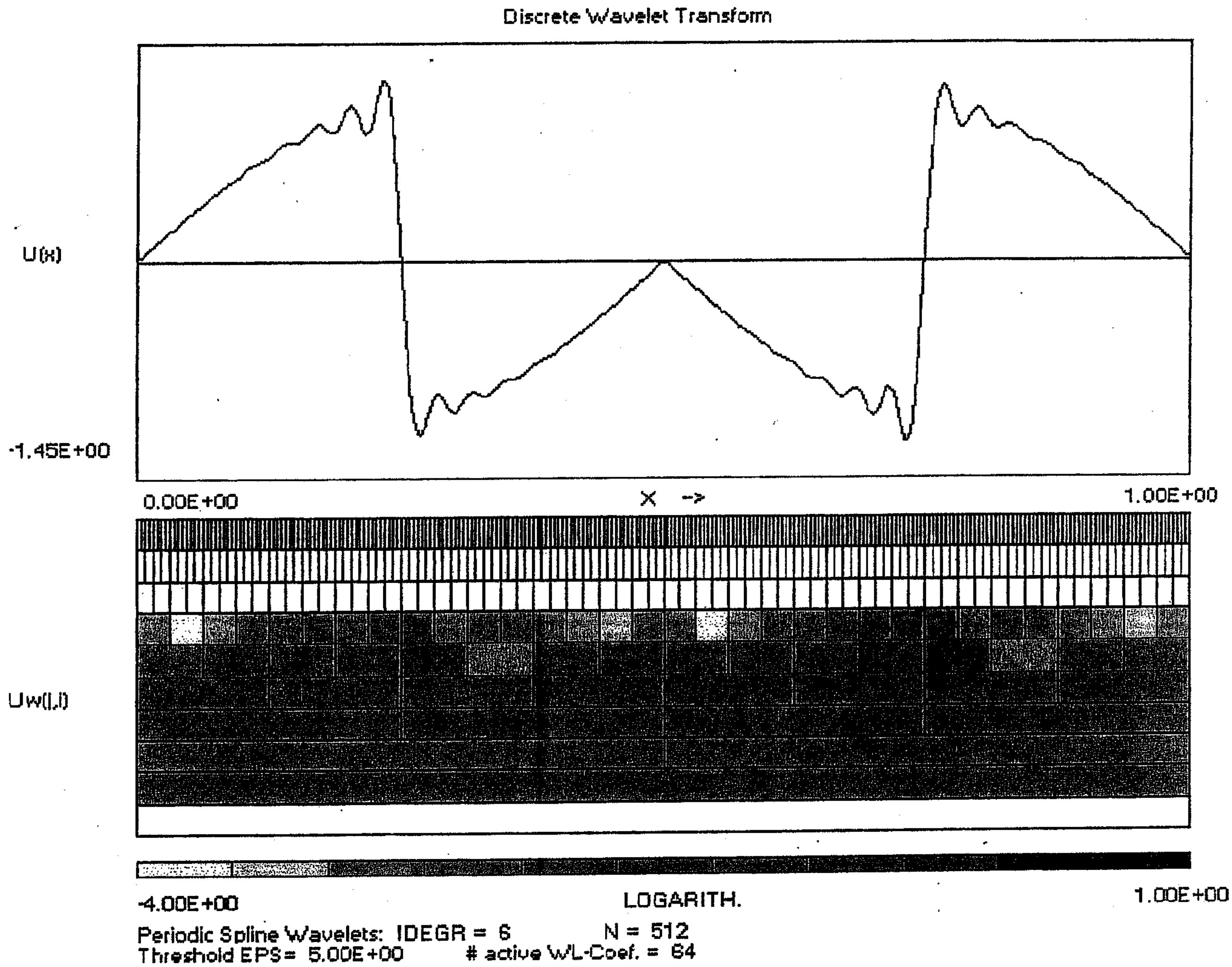
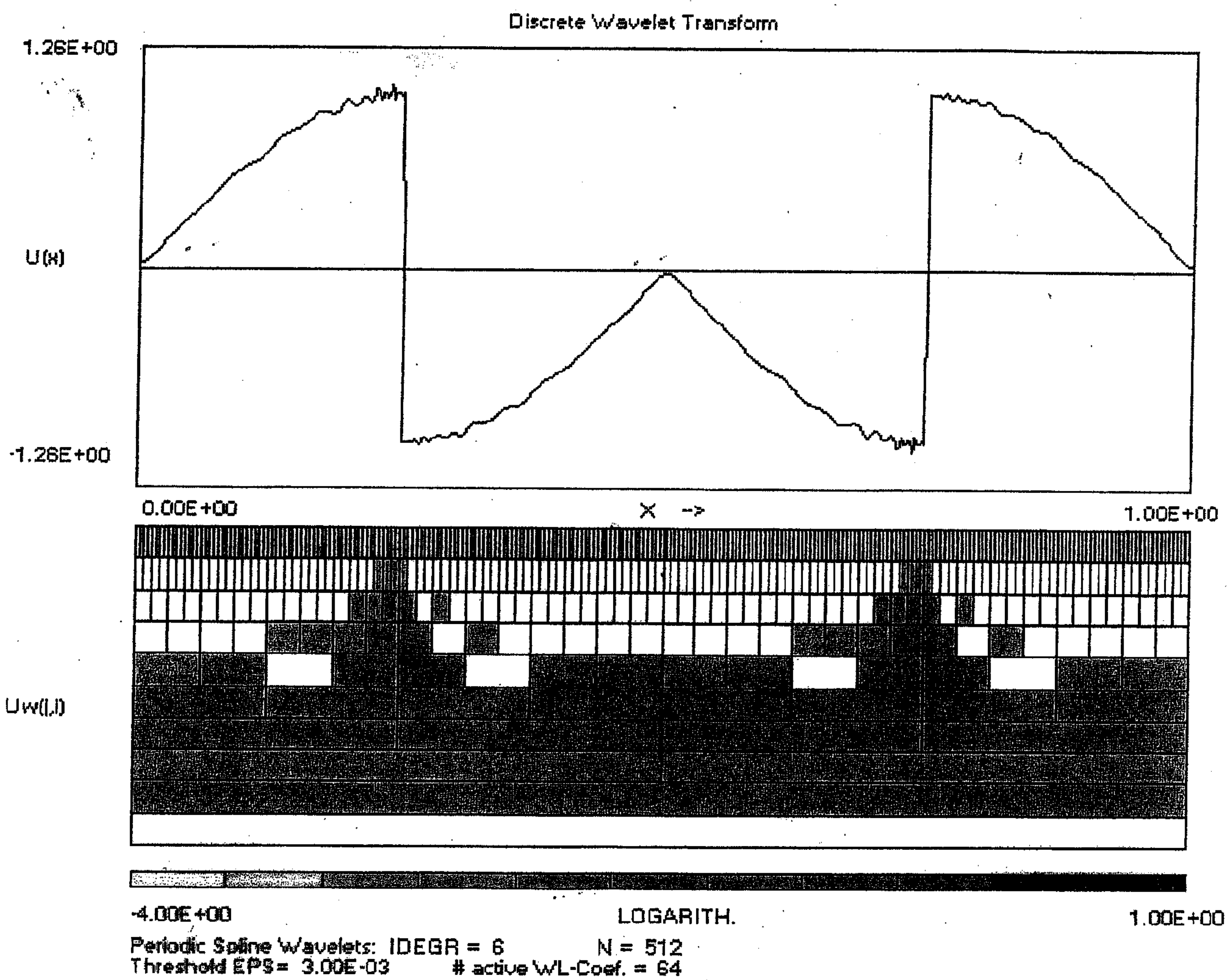
Linear Approximation

$$\langle f, \psi_{j,k} \rangle = 0 \text{ for } j > J_{MAX}$$

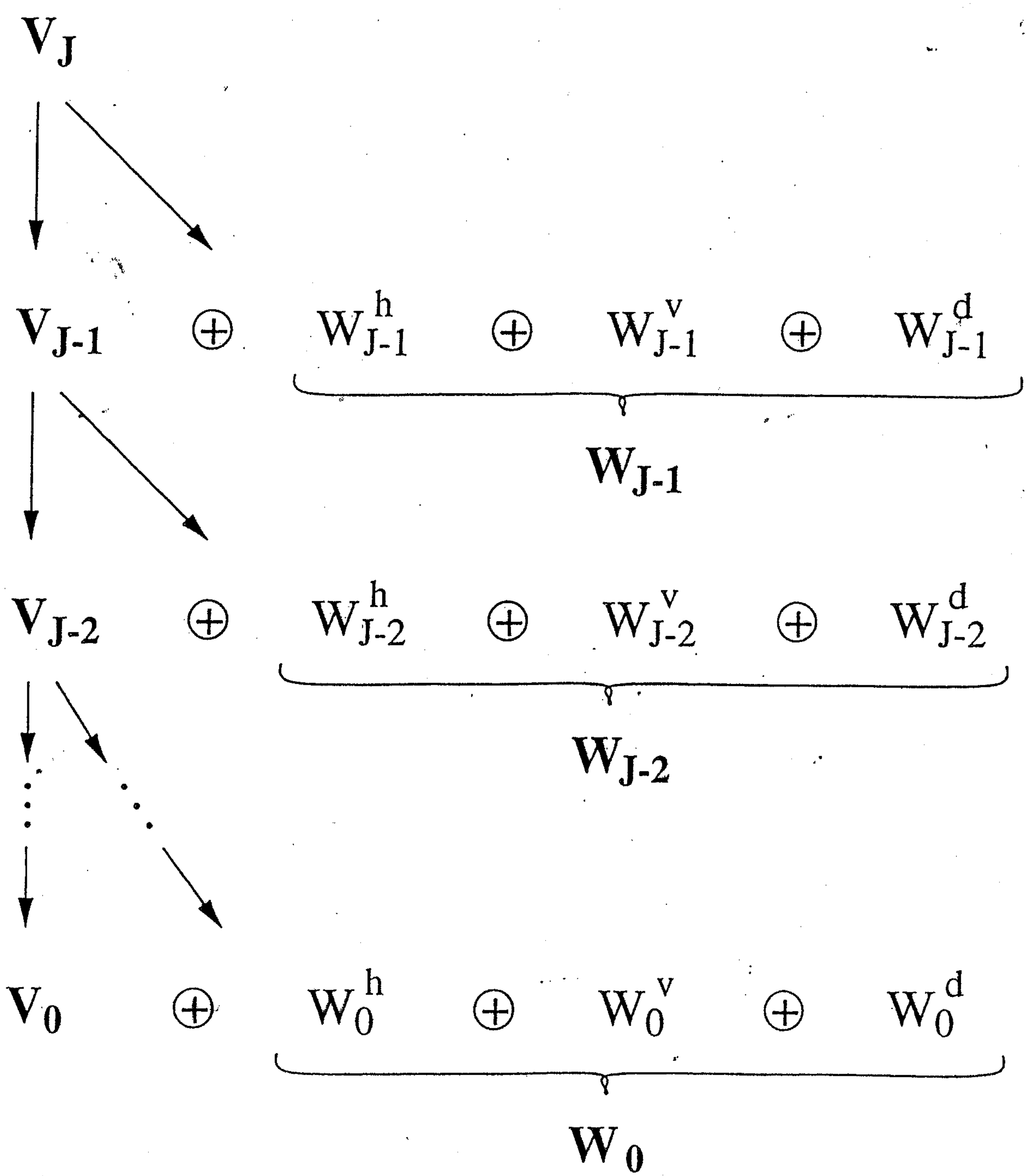


Nonlinear Approximation

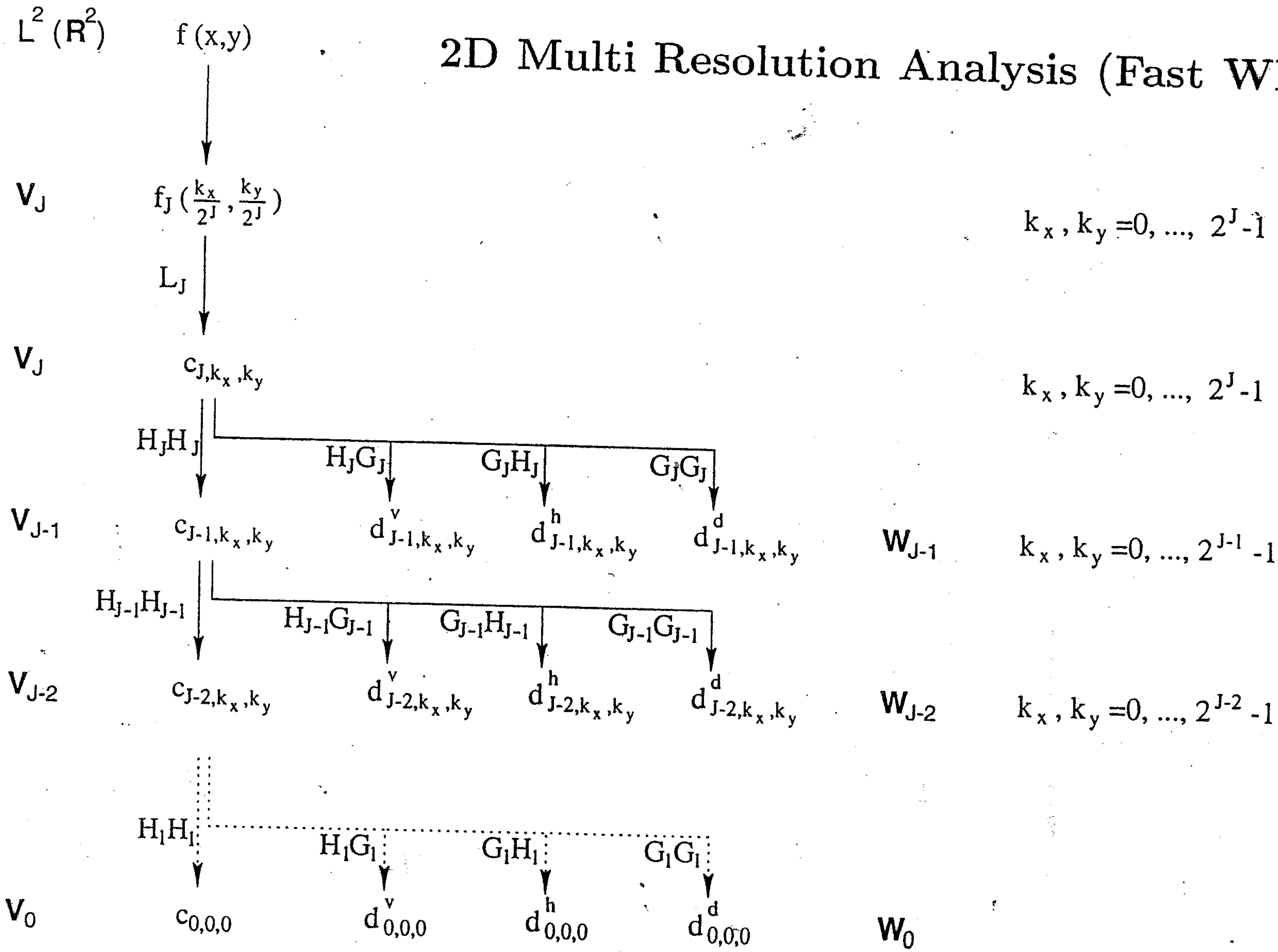
$$\langle f, \psi_{j,k} \rangle = 0 \text{ for } |\langle f, \psi_{j,k} \rangle| \le \epsilon$$

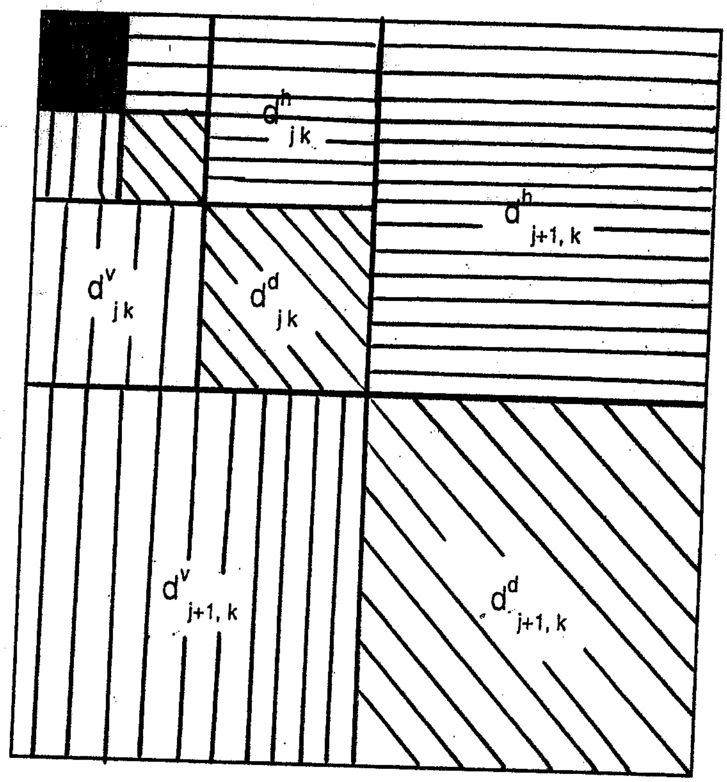


2D Multi Resolution Analysis



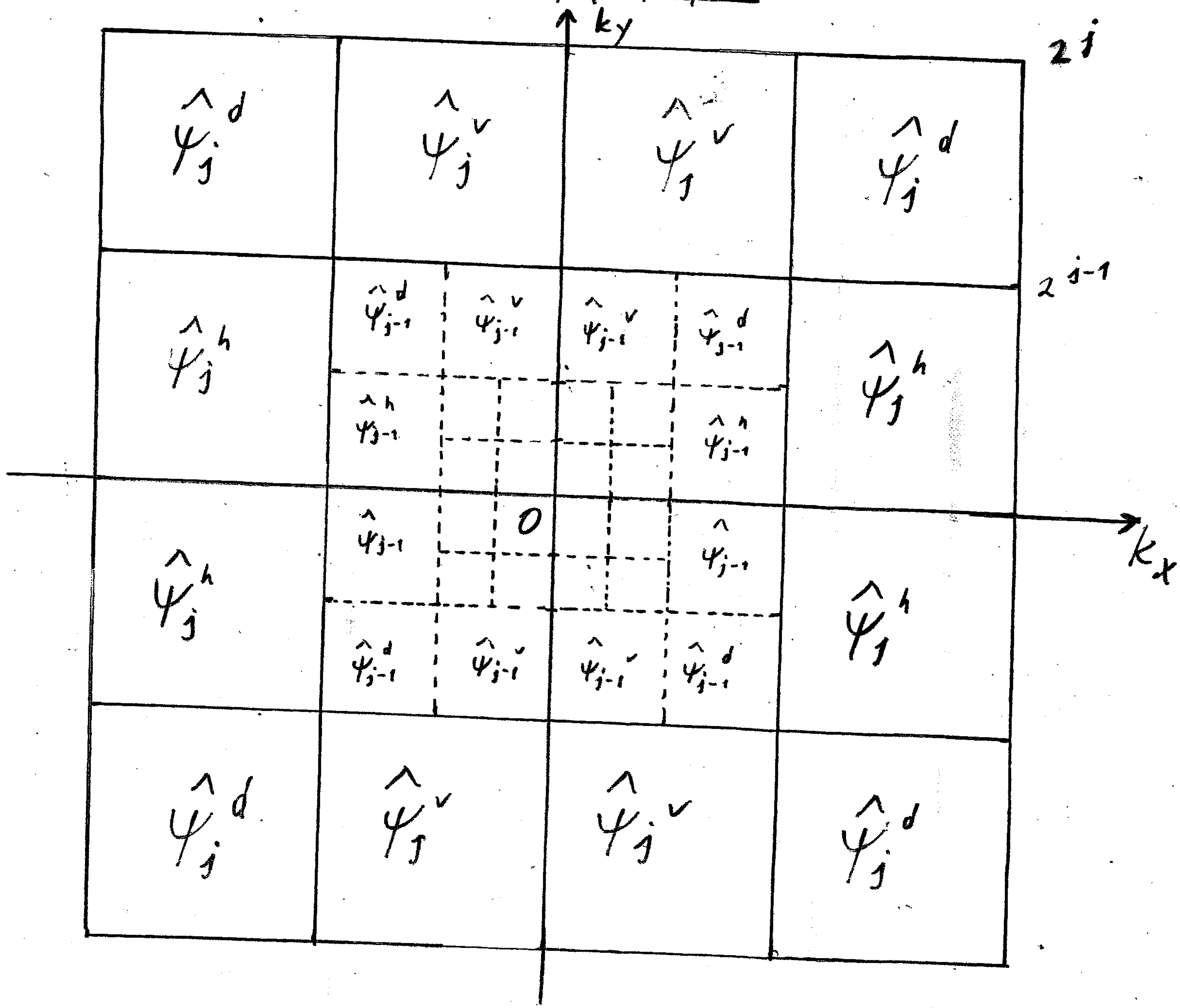
2D Multi Resolution Analysis (Fast WLT)





Scheme of the wavelet coefficients for a two–dimensional MRA.

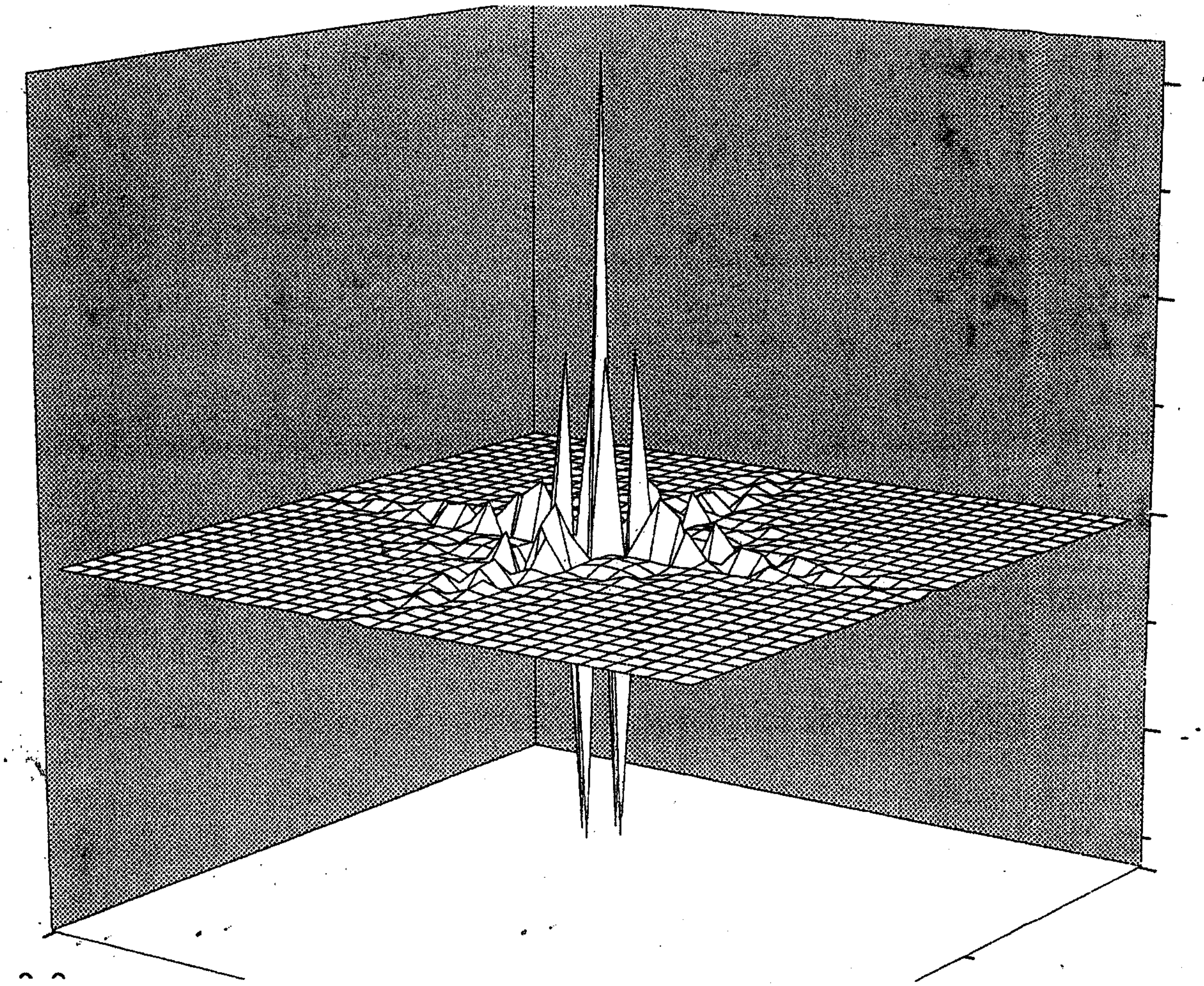
Schematic localization in Fourier space of orth. Wavelets, ψ^h, ψ^v, ψ^d

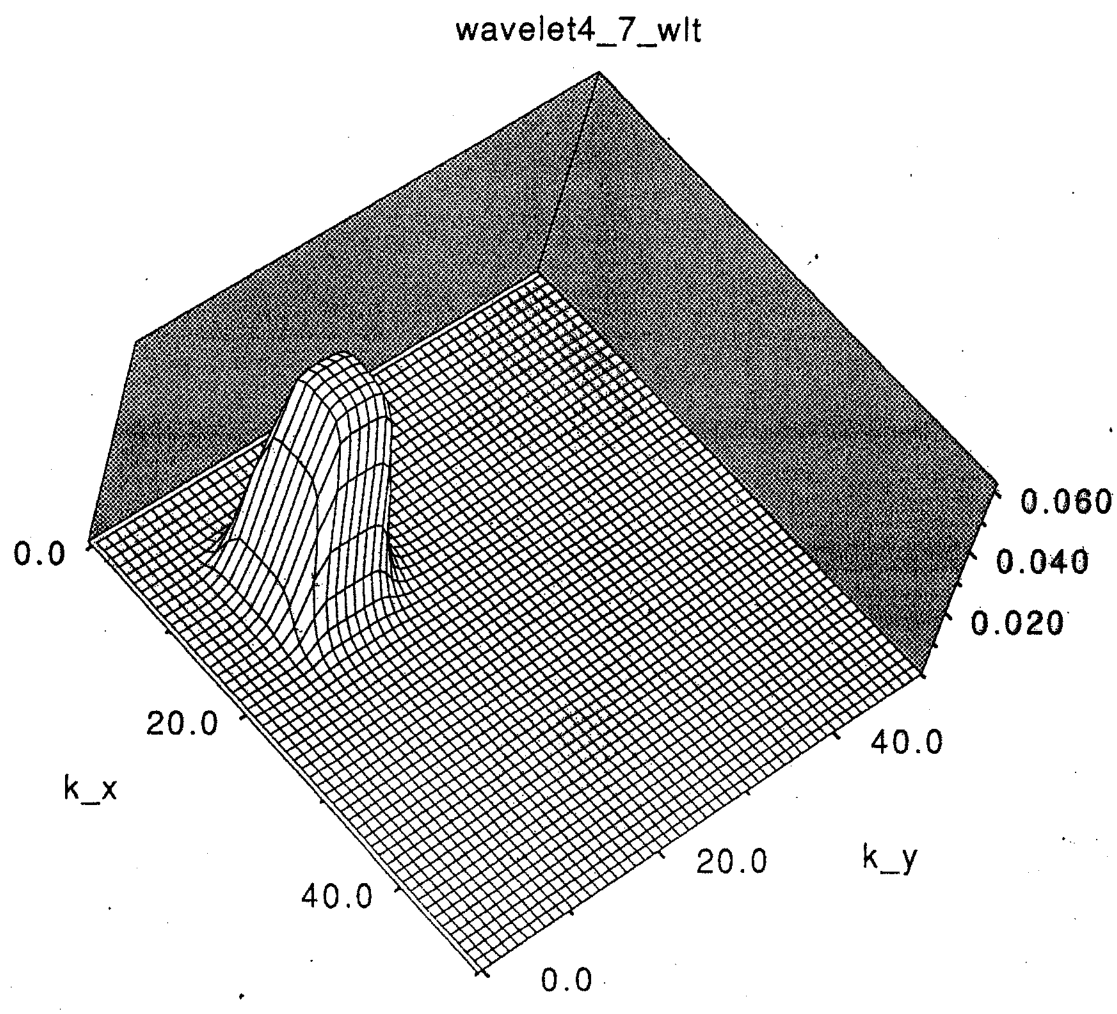


Conclusions DWT

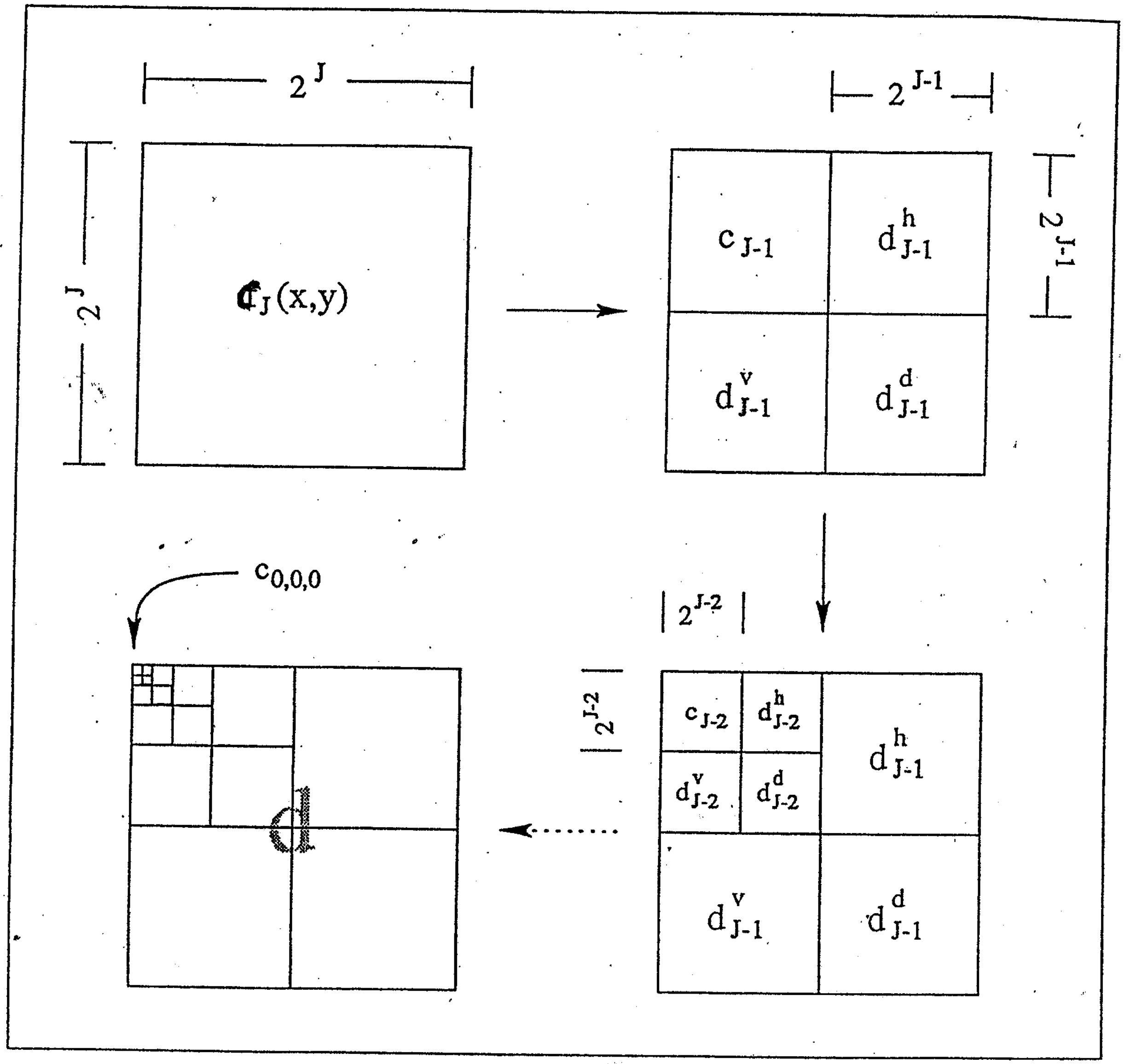
- Fast Wavelet Transform $O(N)$ algorithm
- Compression property of signals/images/ vorticity fields
- Transforms available for:
  - the real line
  - periodic signals
  - intervals (with boundary conditions)
- orthogonal, biorthogonal constructions
- higher dimensions → tensor product construction
- divergence free wavelets ($\nabla \cdot \vec{\psi} = 0$) not yet used in CFD (Lemarié, Federbush, Urban)

- no constructions for complex geometries (domain dec.)
- no translation invariance

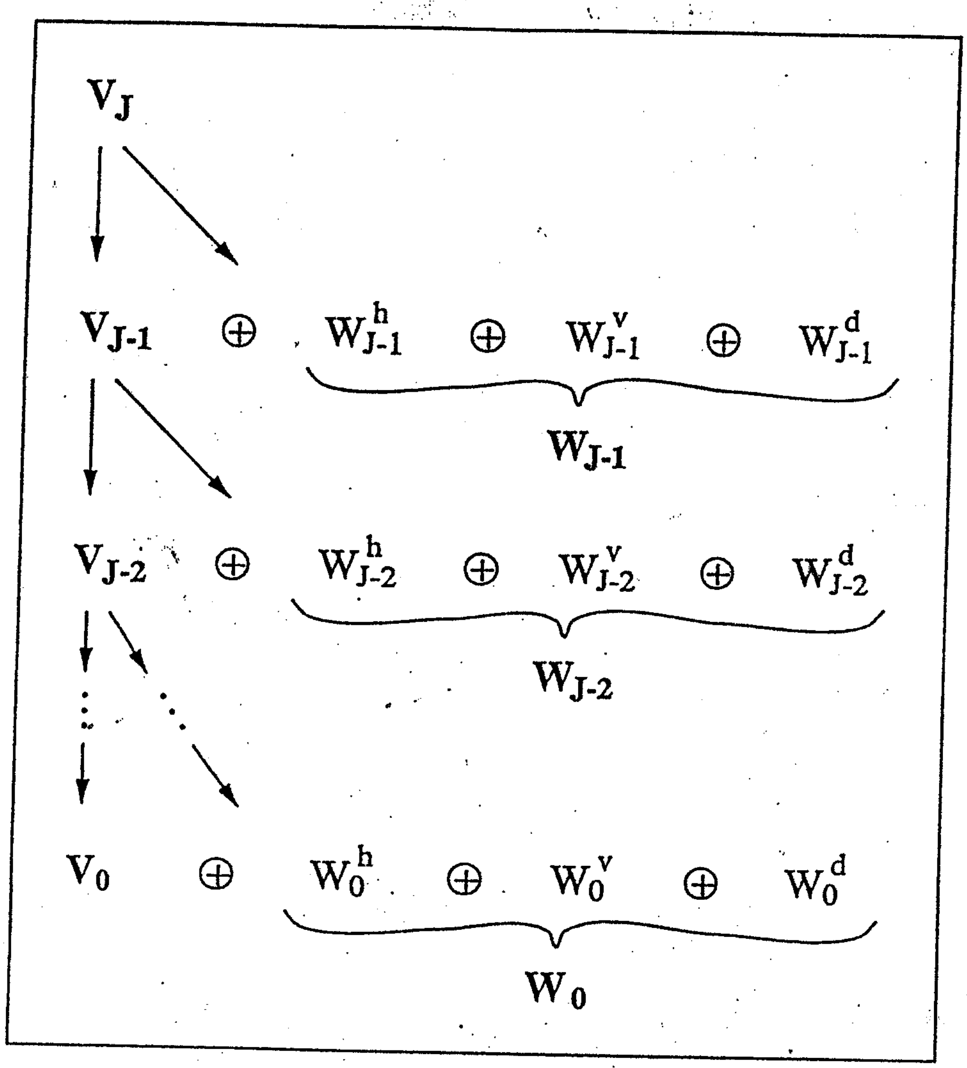




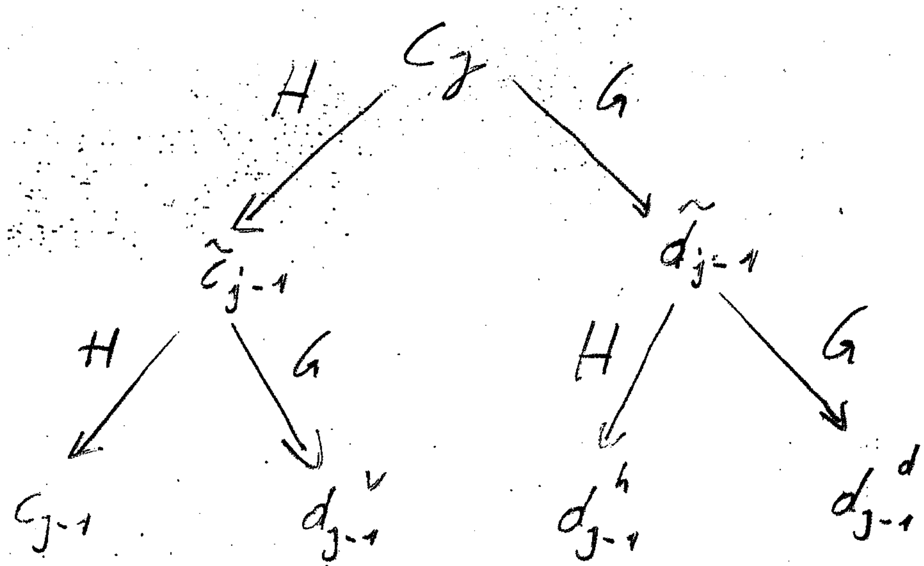
wavelet4_7_wlt
0.060
0.040
0.020
0.0
20.0
k_x
40.0
40.0
20.0
k_y
0.0



2 J
c J (x,y)
2 J-1
c J-1
d h J-1
d v J-1
d d J-1
c 0,0,0
2 J-2
c J-2
d h J-2
d v J-2
d d J-2
d


V_J
$V_{J-1} \oplus W_{J-1}^h \oplus W_{J-1}^v \oplus W_{J-1}^d$
W_{J-1}
$V_{J-2} \oplus W_{J-2}^h \oplus W_{J-2}^v \oplus W_{J-2}^d$
W_{J-2}
$V_0 \oplus W_0^h \oplus W_0^v \oplus W_0^d$
W_0

2d FWT


c_j
H
G
$\tilde{c}_{j-1}$
$\tilde{d}_{j-1}$
H
G
H
G
c_{j-1}
d_{j-1}^v
d_{j-1}^h
d_{j-1}^d

b) two dim. case

Tensor product of two one-dimensional MRA

$$\mathbb{V}_j = V_j \otimes V_j = \overline{\text{span}}\{\phi_{j k_x k_y}(x,y) = \varphi_{j k_x}(x)\,\varphi_{j k_y}(y)\}$$

Orthogonal complement space

$$\mathbb{W}_j \quad \text{in} \quad \mathbb{V}_{j+1} \quad \text{of} \quad \mathbb{V}_j$$

$$\mathbb{V}_{j+1} = V_{j+1} \otimes V_{j+1} = (V_j \oplus W_j) \otimes (V_j \oplus W_j)$$

$$= \underbrace{V_j \otimes V_j}_{\mathbb{V}_j} \oplus \underbrace{[(W_j \otimes V_j) \oplus (V_j \otimes W_j) \oplus (W_j \otimes W_j)]}_{}$$

$$= \mathbb{V}_j \oplus \mathbb{W}_j$$

It follows that $\mathbb{W}_j$ consists of three pieces

$$\psi^{v}(x,y) = \varphi(x)\,\psi(y)$$

$$\psi^{h}(x,y) = \psi(x)\cdot\varphi(y)$$

$$\psi^{d}(x,y) = \psi(x)\,\psi(y)$$

with horizontal, vertical and diagonal wavelets.

$\Rightarrow$

$$\rho(x,y) = \sum c_{0k}\,\phi_{0}(x,y) + \sum\sum\sum d^{\mu}_{jk}\,\psi^{\mu}_{jk}(x,y)$$

Compression of functions

$$f(x) = \sum_{j,k} \langle f, \psi_{j,k} \rangle \, \psi_{j,k}(x)$$

with

$$\langle f, \psi_{j,k} \rangle = \int f(x) \, \psi_{j,k}(x) \, dx$$