Properties of Wavelets

Norm equivalences:

$$||f||_X = ||2^{j\alpha}\langle f, \psi_{j,k}\rangle||_x$$

for Hölder, Lebesgue, Sobolev and Besov spaces.

Nonlinear approximation:

$$||f - f_N|| \leq CN^{-s}$$

where f_N is the best N-term approximation, i.e. one retains N coefficients such that

$$|\langle f, \psi_{j,k}\rangle| > \epsilon.$$

Compression and preconditioning of operators

$$Lf = \int K(x, y,)f(y)\, dy$$

The matrix $\langle L\psi_{j,k}, \psi_{j',k'}\rangle$ is sparse. Diagonal precoditioning yields a uniformly bounded condition number.

Fast Wavelet Transform: $O(N)$ complexity

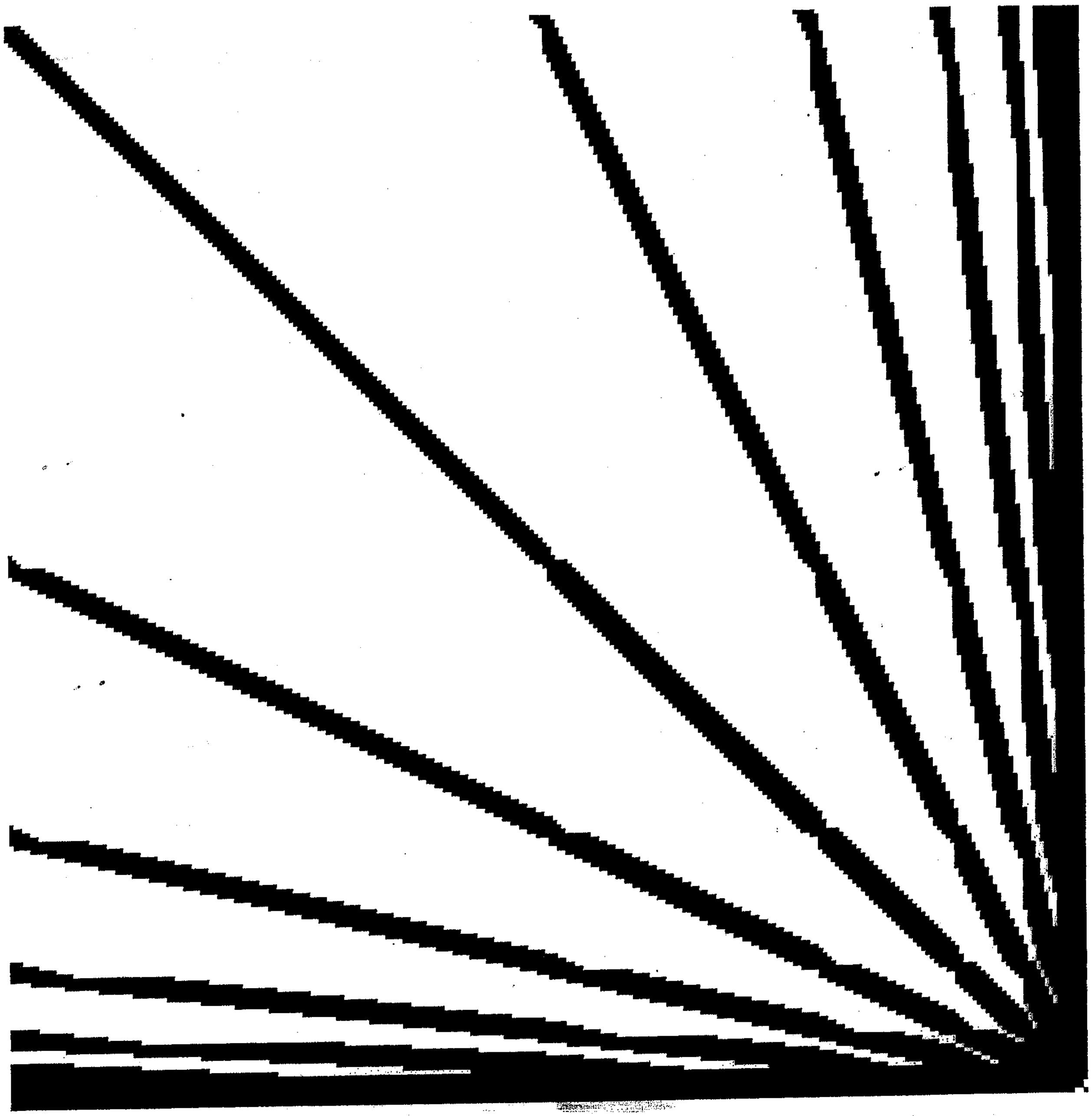
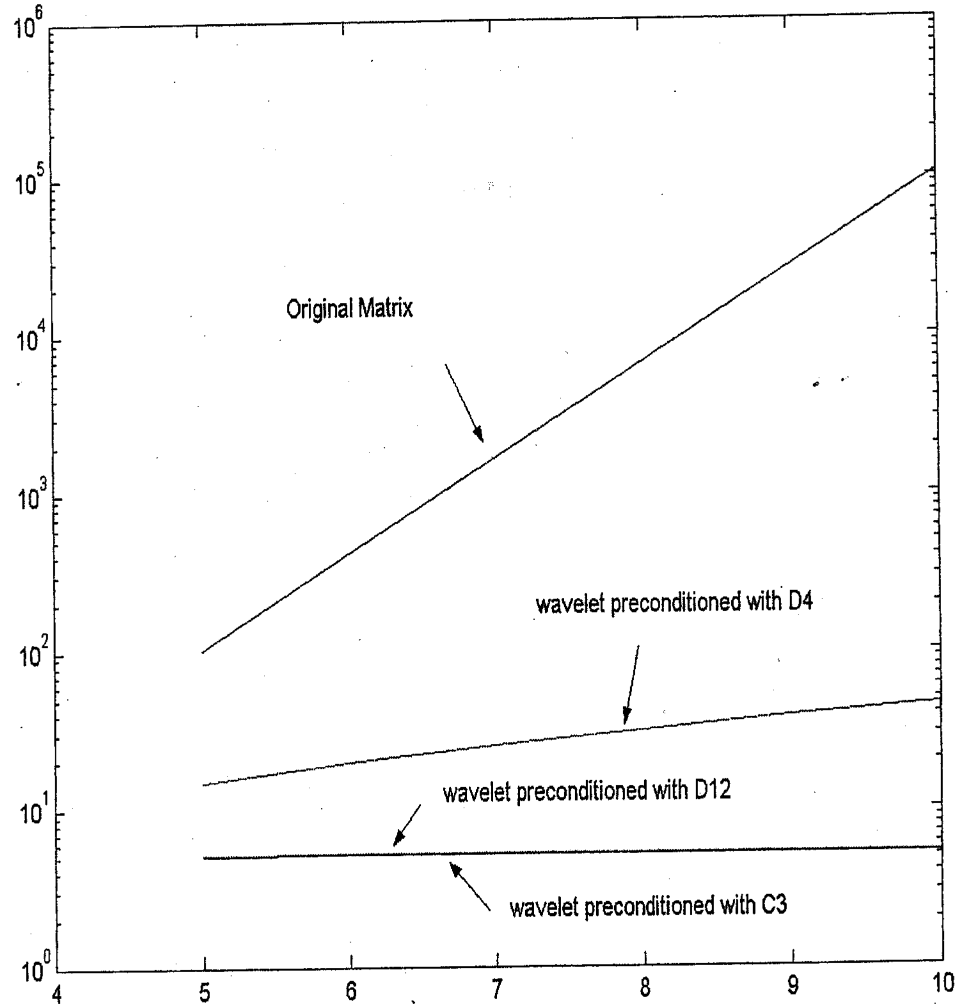


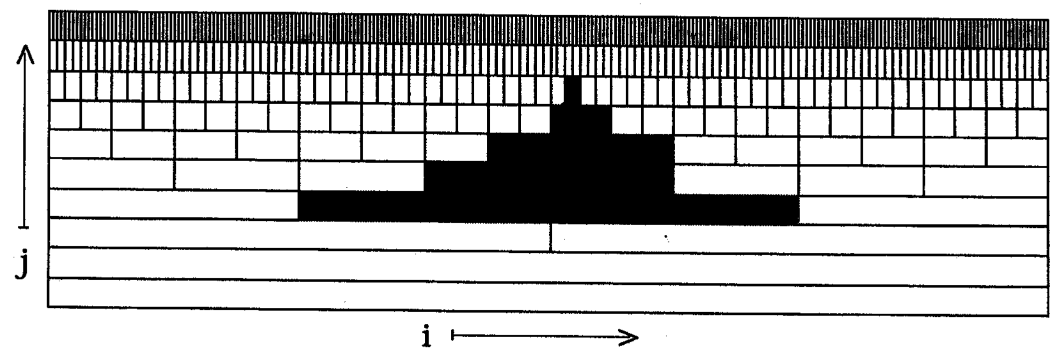
Figure 2: Matrix $\mathbf{B}$ (in the case $\mathbf{A} = \mathbf{D}$) of size 255×255 in the system of coordinates associated with the basis of Daubechies' wavelets with 3 vanishing moments. Entries with the absolute value greater than 10^{-14} are shown black.



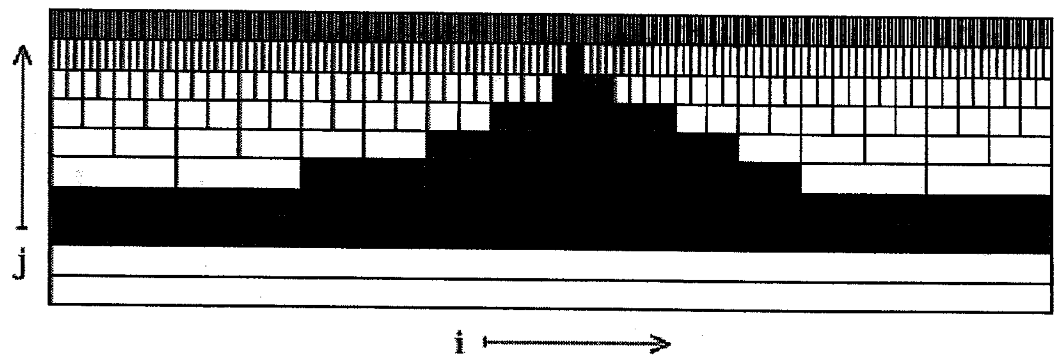
10^6
10^5
10^4
10^3
10^2
10^1
10^0
4
5
6
7
8
9
10
Original Matrix
wavelet preconditioned with D4
wavelet preconditioned with D12
wavelet preconditioned with C3

Adaption Strategy

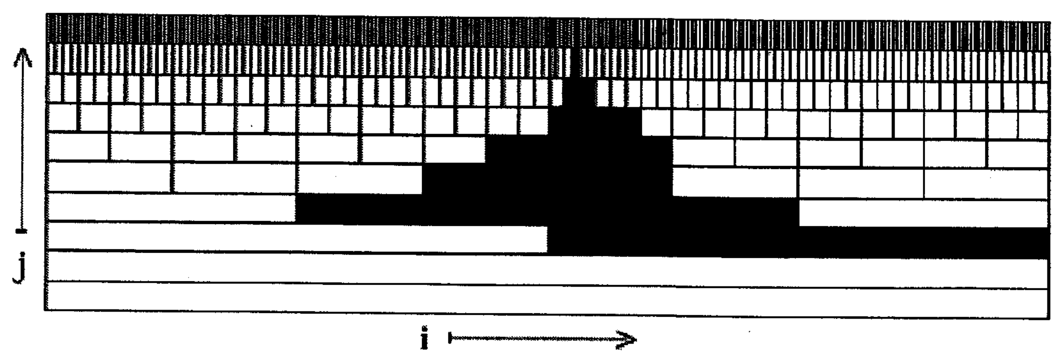
Time t^n



Time $t^{n+1/2}$



Time t^{n+1}



Scale (j) space (i) representation

COHERENT VORTEX SIMULATION (CVS)

Wavelet–filtered 2d Navier–Stokes equation

$$\partial_t \omega_> - \nabla \cdot (\omega \vec{v})_> - \nu \nabla^2 \omega_> = \nabla \times \vec{F}_> \qquad , \qquad \nabla \cdot \vec{v} = 0$$

with $\omega = \omega_> + \omega_<$ we decompose the nonlinear term into

$$(\omega \vec{v})_> = \omega_> \vec{v}_> + L + C + R$$

time discretization: semi-implicit scheme of 2nd order (EB2/AB2)
spatial discretization: Petrov-Galerkin scheme

Trial functions: wavelets $\omega^n(x,y) = \sum_\lambda d_\lambda^n \psi_\lambda(x,y)$

Testfunctions: vaguelettes (Liandrat & Tchamitchian 1990)

$$\theta_\lambda = (Id - \nu \Delta t \nabla^2)^{-1} \psi_\lambda$$

Solution: change of basis

$$d_\lambda^{n+1} = < \omega^n - \mathbf{v}^n \cdot \nabla \omega^n, \theta_\lambda >$$

adaptive vaguelette decomposition (Fröhlich & S., JCP 130, 1997)

Nonlinear term: partial collocation in physical space

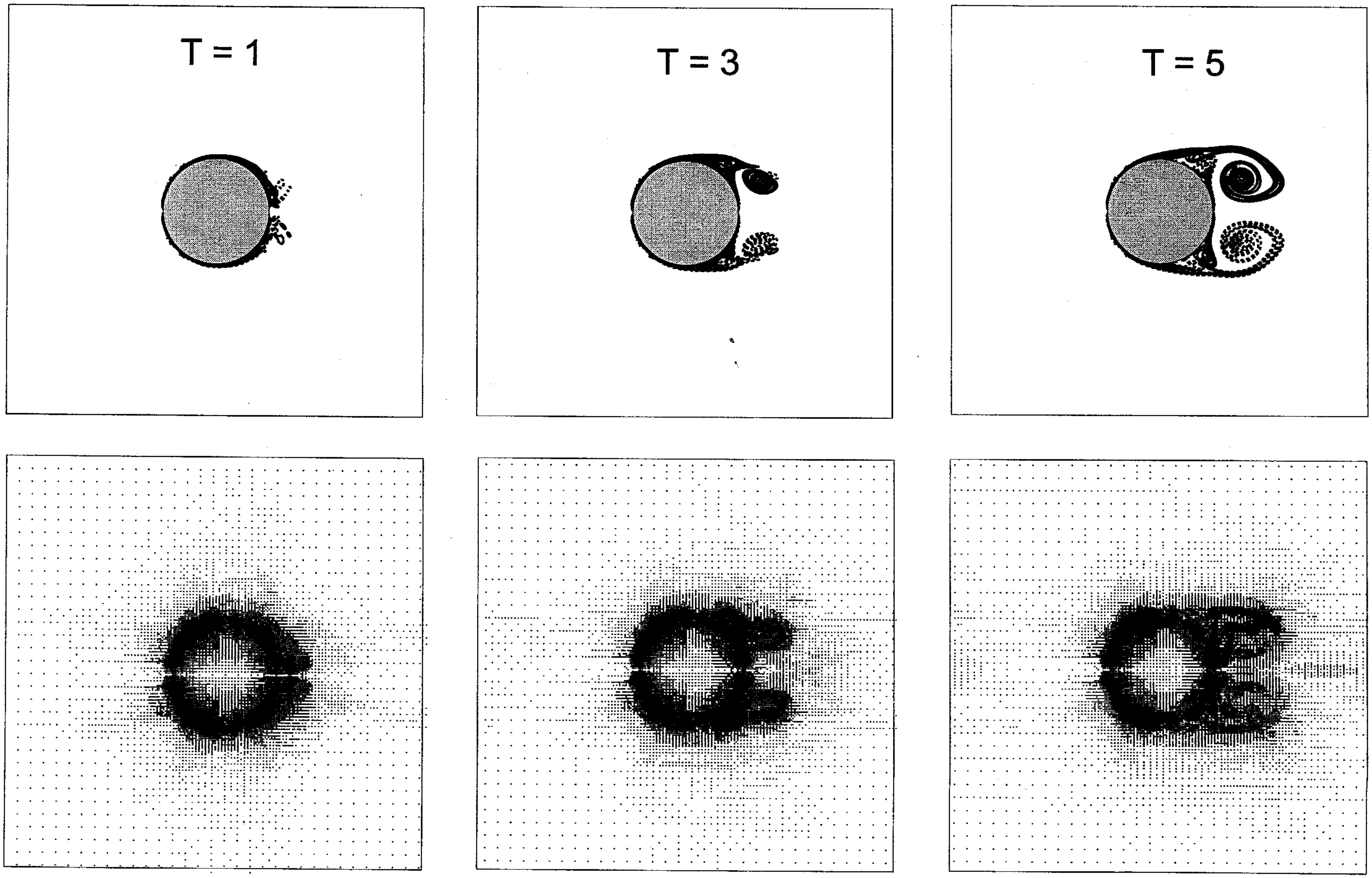
Summary: $O(N)$ algorithm $\qquad N =$ number of d.o.f.

CVS of an impulsively started cylinder at Re = 3000

Schneider& Farge, ACHA, vol. 12 (2002)

Isolines of vorticity

Adaptive grid



$N = 512^2$ — 7.2 % N — 7.7 % N — 7.9 % N