Properties of Wavelets

Norm equivalences: \[ \|f\|_X = \|2^{j\alpha} \langle f, \psi_{j,k} \rangle \|_X \]
for Hölder, Lebesgue, Sobolev and Besov spaces.

Nonlinear approximation:
\[ \|f - f_N\| \leq CN^{-s} \]
where \( f_N \) is the best \( N \)-term approximation, i.e. one retains \( N \) coefficients such that
\[ |\langle f, \psi_{j,k} \rangle| > \epsilon. \]

Compression and preconditioning of operators
\[ Lf = \int K(x, y) f(y) dy \]
The matrix \( \langle L \psi_{j,k}, \psi_{j',k'} \rangle \) is sparse. Diagonal preconditioning yields a uniformly bounded condition number.

Fast Wavelet Transform: \( O(N) \) complexity
Figure 2: Matrix $B$ (in the case $A = D$) of size $255 \times 255$ in the system of coordinates associated with the basis of Daubechies' wavelets with 3 vanishing moments. Entries with the absolute value greater than $10^{-14}$ are shown black.
Adaption Strategy

Time $t^n$

Time $t^{n+1/2}$

Time $t^{n+1}$

Scale (j) space (i) representation
COHERENT VORTEX SIMULATION (CVS)

Wavelet-filtered 2d Navier-Stokes equation
\[
\partial_t \omega_\sigma - \nabla \cdot (\omega \vec{v})_\sigma - \nu \nabla^2 \omega_\sigma = \nabla \times \vec{F}_\sigma, \quad \nabla \cdot \vec{v} = 0
\]

with \( \omega = \omega_\sigma + \omega_< \) we decompose the nonlinear term into
\[
(\omega \vec{v})_\sigma = \omega_\sigma \vec{v}_\sigma + L + C + R
\]

time discretization: semi-implicit scheme of 2nd order (EB2/AB2)
spatial discretization: Petrov-Galerkin scheme

Trial functions: wavelets
\[
\omega^n(x, y) = \sum_\lambda d^n_\lambda \psi_\lambda(x, y)
\]

Testfunctions: vaguelettes (Liandrat & Tchamitchian 1990)
\[
\theta_\lambda = (Id - \nu \Delta t \nabla^2)^{-1} \psi_\lambda
\]

Solution: change of basis
\[
d^{n+1}_\lambda = \langle \omega^n - \nu^n \cdot \nabla \omega^n, \theta_\lambda \rangle
\]

adaptive vaguelette decomposition (Fröhlich & S., JCP 130, 1997)

Nonlinear term: partial collocation in physical space

Summary: \( O(N) \) algorithm \( N = \) number of d.o.f.
CVS of an impulsively started cylinder at $Re = 3000$


Isolines of vorticity

$T = 1$

$T = 3$

$T = 5$

Adaptive grid

$N = 512^2$

7.2% $N$

7.7% $N$

7.9% $N$