University of California, Santa Barbara — Winter 2004
Department of Mechanical and Environmental Engineering
ME 252 B, Computational Fluid Dynamics:
Wavelet transforms and their applications to turbulence
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Exercise sheet 1: Fourier transforms and sampling

Exercise 1:

Given a one–dimensional signal $s(t) = \exp(-(t - t_0)^2/\sigma^2)$.

- Compute its Fourier transform

$$\widehat{S}(f) = \int_{-\infty}^{+\infty} s(t) \exp(-i2\pi f t) dt$$

- Discuss the role of t_0 and σ in Fourier space.

Exercise 2:

Given a discrete signal $s_n = s(t_n)$ with $t_n = n/N$ for $n = 0, ..., N - 1$ and with $t_0 = 0.5$, $\sigma^2 = 1/500$ and $N = 2^{10} = 1024$.

- Plot the signal (e.g. using matlab plot(s)).
- Compute its discrete Fourier transform

$$\widehat{s}_k = \frac{1}{N} \sum_{n=0}^{N-1} s_n \exp(-i2\pi kn/N) \quad , \quad k = 0, ..., N - 1$$

  (e.g. using matlab fft(s))
- Plot the spectrum (modulus of the Fourier coefficients), $|\widehat{s}_k|$, $k = 0, ..., N - 1$ and compare the result with the computation of exercise 1.

Exercice 3:

Given a discrete Dirac pulse

$$d_n = \begin{cases} 1 & \text{for } n = mN/f_0 \; , \quad m \in \mathbb{N} \\ 0 & \text{else} \end{cases}$$

for $n = 0, ..., N - 1$ with $N = 2^{10} = 1024$, and frequency $f_0 = 2^6 = 64$.

- Plot the Dirac pulse.
- Compute its discrete Fourier transform.
- Plot the spectrum.

Exercise 4:

Multiply the signal s with the Dirac pulse and plot the resulting sampled signal $r_n = s_n d_n$ for $n = 0, ..., N-1$.

- Plot the spectrum of r.
- What do you observe?

Exercise 5:

Reconstruct the signal s^{rec} from the sampled signal r by applying an ideal low pass filter in Fourier space with cut off frequency $k_c = f_0/2 = 32$, i.e.

$$\widehat{s}_k^{rec} = \begin{cases} \widehat{r}_k & \text{for } |k| \leq k_c \\ 0 & \text{else} \end{cases}$$

for $k = 0, ..., N-1$.

Apply the inverse discrete Fourier transform (e.g. using matlab ifft(s^{rec})) to get

$$s_n^{rec} = \sum_{k=0}^{N-1} \widehat{s}_k^{rec} \exp(i2\pi kn/N) \quad , \quad n = 0, ..., N-1$$

and plot s^{rec}.

Exercise 6:

Repeat exercises 2-5 with different values $f_0 > 64$ and $f_0 < 64$.
What do you observe?