Wavelets and Wavelet Packets to Analyse, Filter and Compress Two-dimensional Turbulent Flows

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1. Introduction

In a signal most of the time the useful information is carried by both its frequency content and its time evolution. If one considers only the time representation, one does not know the spectrum, while the Fourier spectral representation does not give information on the instant of emission of each frequencies. A more appropriate representation should combine these two complementary descriptions. This is true in particular for turbulent signals, especially the very intermittent ones presenting bursts or some quasi-singular behaviours. Indeed, there is no perfect representation due to the limitation resulting from the uncertainty principle, which forbids us to perfectly analyse the signal from both sides of the Fourier transform at the same time due to the limitation $\Delta t.\Delta\nu \geq 1$ (normalized information cell). Therefore there is always a compromise to be made in order to have, either a good time resolution Δt but loosing the spectral resolution $\Delta\nu$, which is the case when we sample a signal by convolving it with a Dirac comb (**Figure 1.1**), or a good spectral resolution $\Delta\nu$ but loosing the time resolution Δt, which is the case with the Fourier transform (**Figure 1.2**). These two transforms are the most commonly used in practice because they allow to construct orthogonal bases onto which one projects the signal to be analysed and eventually computed.

In order to try to recover some time locality while using the Fourier transform, Gabor /1946/ has proposed the windowed Fourier transform, which consists of convolving the signal with a set of Fourier modes localized in a Gaussian envelop of constant width a_0 (**Figure 1.3**). This transform allows then a time-frequency decomposition of the signal at a given scale a_0, which is kept fixed. But unfortunately, as shown by Balian /1981/, the bases constructed with such windowed Fourier modes cannot be orthogonal. More recently, Grossmann and Morlet /1984, 1985/ have devised a new transform, so called the wavelet transform, which consists of convolving the signal with a set of affine functions presenting all the same frequency ν_0; the family of analysing wavelets $\psi_{a,b}$ is obtained by dilation and translation of a given function ψ presenting at least one oscillation. The wavelet transform allows therefore a time-scale decomposition of the signal at a given frequency ν_0, which is kept fixed. Actually the wavelet transform realizes the best compromise in view of the uncertainty principle, because it adapts the time-frequency resolution $\Delta t.\Delta\nu$ to each scale a. In fact it gives a good spectral resolution $\Delta\nu$ with a limited time resolution Δt in the large scales, and on the contrary it gives a good time localization Δt with a limited spectral resolution $\Delta\nu$ in the small scales (**Figure 1.4**). The continuous wavelet transform has been extended to n dimensions by Murenzi /1989/.

In 1985 Meyer, while trying to prove the same kind of impossibility to build orthogonal bases as done by Balian in the case of the windowed Fourier transform, has been quite surprised to discover an orthogonal wavelet basis built with spline functions, now called the Meyer-Lemarié wavelets /Lemarié and Meyer 1986/. In fact the Haar orthogonal basis, which has been proposed in 1909, is now recognized as the first orthogonal wavelet basis known, but the functions it uses are not regular, which drastically limits its application. In practice one likes to build orthogonal wavelet bases using functions having a prescribed regularity, as good as one needs enough spectral decay depending on the application. In particular, following Meyer's work, Daubechies /1988/ has proposed new orthogonal wavelet bases built with compactly supported functions of prescribed regularity defined by discrete Quadratic Mirror Filters of different lengths, the longer the filter, the more regular the associated functions. Mallat /1989/ has devised a fast algorithm to compute the orthogonal wavelet transform using wavelets defined by QMF; it has been used in particular to devise more efficient technics for numerical analysis /Beylkin, Coifman and Rokhlin 1992/. Then, more recently, Malvar /1990/, Coifman and Meyer /1991/ has found a new kind of windows of variable width which allows the construction of orthogonal adaptative local cosine bases. The elementary functions of such bases are then parametrized by their position b, their scale a (width of the window) and their wavenumber k (proportional to the number of oscillations inside each window). In the same spirit, Coifman, Meyer, Quake and Wickerhauser /1990/, Wickerhauser /1990/ and Coifman, Meyer and Wickerhauser /1992/ have proposed the so called wavelet packets which, similarly to compactly supported wavelets, are wavepackets of prescribed regularity defined by discrete Quadratic Mirror Filters, from which one can construct orthogonal bases. A review of the different types of wavelet transforms and their applications to analyse or compute turbulent flows in 2D and 3D is given in Farge 1992a and 1992b.

2. The Continuous Wavelet Transform

The only condition a function $\psi(x) \in L^2(\mathbb{R})$, real or complex-valued, should satisfy to be called a wavelet is the admissibility condition:

(1)

(2) with

If ψ is integrable, this condition implies that the wavelet has a zero mean :

(3)

In practice one also wishes the wavelet to be as localised as possible on both sides of Fourier, namely that:

(4)

(5) and

k_0 being the frequency of the wavelet
and n as large as possible.

Figure 2 shows examples of the most commonly used wavelets: the Marr wavelet (**Figure 2.1**), also called the Mexican hat, real-valued function which is used for the isotropic continuous wavelet transform, the Morlet wavelet (**Figure 2.2**), complex-valued function which is used for the non-isotropic continuous wavelet transform, the Meyer-Lemarié wavelet (**Figure 2.3**) and the Daubechies wavelet, (**Figures 2.4 and 2.5**) which are real-valued functions used to build orthogonal bases.

For several applications, in particular to study fractals, one also wishes the wavelet to have a good regularity, namely that decays rapidly near 0, or equivalently that the wavelet has enough cancellations such as:

(6)

with n as large as possible.

Then, after having chosen the so-called 'mother wavelet' ψ, ones generates the family of wavelets $\Psi_{b,a}(x)$, by continuously translating (parameter b) all along the signal and continuously dilating (parameter a) to all accessible scales the 'mother wavelet' ψ, which gives:

(7)

with N(a) a normalization coefficient equal, either to $a^{1/2}$ if one wishes the squarred modulus of the wavelet coefficients to correspond to an energy density (L^2 norm), or to a if one uses the wavelet coefficients to analyse the local regularity of the signal (L^1 norm).

The continuous wavelet analysis of the function $f(x) \in L^2(\mathbb{R})$ is then the inner product between f(x) and the set of all translated and dilated wavelets $\Psi_{b,a}(x)$, such as:

(8)

where * indicates the complex conjugate.

The wavelet transform therefore projects the $L^2(R)$ space of finite energy functions into the $L^2(RXR^+)$ wavelet coefficients having a measure $dadb/a^2$, which is the Haar measure associated to the affine group. Figure 3 shows five examples of wavelet analysis of academic signals: a Dirac spike (**Figure 3.1**), the superposition of two cosine functions having different frequencies (**Figure 3.2**), the superposition of two cosine functions of very different amplitudes (**Figure 3.3**), a tchirp (**Figure 3.4**), a Gaussian white noise (**Figure 3.5**), and finally a tchirp in presence of a strong noise (**Figure 3.6**).

From the wavelet coefficients , one is able to reconstruct the function f(x) using the inverse wavelet transform, defined as:

(9)

with

finite valued coefficient given by the admissibility condition (1).

One verifies that the wavelet transform conserves energy (as the Plancherel identity we have for the Fourier transform), namely that:

(10)

If the function f(x) belongs to the functional space $L^2(R)$, and if the wavelet is regular enough and therefore well localised in Fourier (5), the wavelet analysis may be interpretated as a pass-band filter with dk/k constant (**Figure 1.4.d**) :

(11)

The extension of the continuous wavelet transform to analyse signals in n dimensions has been done by Murenzi /1989/, considering in this case the Euclidean group with dilations. The generation of the wavelet family $\Psi_{a,r,\mathbf{b}}(x)$ is obtained by translation (vector **b**), dilation (parameter a) and rotation (corresponding to the operator r defined in $\mathbb{R}^n$), such as:

(12)

For R^2, r is the rotation matrix:

(13)

with θ, the rotation angle.

In n dimensions the admissibility condition becomes:

(14)

The analysis and synthesis are then:

(15)

(16)

The energy conservation is still verified:

(17)

Holschneider / 1988/ has shown that one can reconstruct the function f(x) from its wavelet coefficients (b,a) by using any other function $\phi(x)$, which verifies a modified admissibility condition such as:

(18)

This for instance allows to reconstruct f(x) by a simple summation of all wavelet coefficients along the verticals b = constant, which in fact corresponds to using a Dirac function as function $\phi(x)$ to reconstruct the signal, which gives:

(19)

with

3. Properties of the Continuous Wavelet Transform

31. Covariance by Translation and Dilation

One of the property of the continuous wavelet transform, which is lost in the case of the orthogonal wavelet transform, is its covariance, by both translation, i.e. shift by x_0:

(20)

with W the continuous wavelet transform operator,

and dilation, i.e. under scale changes by a factor λ:

(21)

32. Linearity

The continuous wavelet transform is a linear transform and therefore we have the following superposition principle:

(22)

with α and β two arbitrary constants.

33. Locality in both Space and Scale

The double localization of wavelets in both positions b and scales a allows to read both informations from the wavelet coefficients. This is not the case with the Fourier coefficients because the basis functions are non local: a given Fourier coefficient therefore depends on the behaviour of the whole signal. On the contrary a given wavelet coefficient (b_o,a_o) does not depend on the value of the signal outside the so called 'influence cone' localized in , with Δb depending on the support of the wavelet (**Figure 4.a**). Likewise the wavelet coefficients at a given scale a_o depends only on the spectral behaviour of the signal in the bandwidth $[k_{min}/a_o, k_{max}/a_o]$ with k_{min} and k_{max} given by the support of (**Figure 4.b**). The support of is defined as the region where ψ is larger than a given value, because wavelet ψ has at least an exponential decay.

34. Characterisation of the Local Regularity of a Function

One of the most useful property of the wavelet transform to analyse turbulent flows is the fact that the local scaling of the wavelet coefficients computed in L^1 norm, i.e. with the normalization N(a)=a in (7), allows to characterize the regularity of the signal /Holschneider 1988/ and /Jaffard 1989/.

Thus, if exists, i.e. if f is m times continuously differentiable in x_o, then:

(23)

when a tends to 0.

If , the space of Lipschitz functions having exponent $-1<\alpha<1$, which are continuous functions non differentiable in x_o , such that:

(24)

with C, constant >O,

then

(25)

when a tends to 0.

Thus the behaviour of the wavelet coefficients (x_o,a) at x_o in the limit $a \rightarrow o$ measures the local regularity of the function f in x_o, which is given by the slope of the modulus of (x_o,a) represented in Log-Log coordinates. For instance the wavelet coefficients computed in norm L^1 of a function presenting a Lipschitz singularity α in x_o will diverge in the very small scale limit (**Figure 5.a**), while those of a function which is regular in x_o will tend to zero in the same limit (**Figure 5.b**).

4. Analysis of Two-dimensional Turbulent Flows

'In the last decade we have experienced a conceptual shift in our view of turbulence. For flows with strong velocity shear... or other organizing characteristics, many now feel that the spectral description has inhibited fundamental progress. The next "El Dorado" lies in the mathematical understanding of coherent structures in weakly

dissipative fluids: the formation, evolution and interaction of metastable vortex-like solutions of nonlinear partial differential equations...' Norman Zabusky /1984/.

As Norman Zabusky stated it in this quotation, it is essential before modelling turbulent flows to understand the dynamical role of coherent structures and analyse their contribution to the different nonlinear interactions. With the Fourier transform we cannot separate the coherent structures from the rest of the flow, because the Fourier modes contain a non local information, and we are therefore unable to discriminate the role of coherent structures. On the contrary this local spectral analysis becomes possible when using the wavelet transform and we can with it devise new types of diagnostics. After defining them, we will apply them to analyse some vorticity fields corresponding to long time evolution of a forced two-dimensional flow, computed with a resolution 128^2.

41. The wavelet coefficients

If we denote the position as b, the scale as a and the angle as θ, the wavelet coefficients computed in L^p norm are:

(26)

(27) with

If p=2, the wavelet coefficients are in L^2 norm and the squarred wavelet coefficients correspond to the local energy density of the signal at location b, scale a and direction θ. If p=1, the wavelet coefficients are in L^1 norm and in this case the local scaling of the wavelet coefficients gives information on the local regularity, or the Lipschitz exponent in the case of discontinuities, of the signal at location b, scale a and direction θ.

On figure 6 we show the 1D continuous wavelet analysis of a one-dimensional cut done in a two-dimensional turbulent vorticity field. The wavelet coefficients are computed, either in L^2 norm (**Figure 6.1**), or in L^1 norm (**Figure 6.2**), using the Morlet wavelet with k_0=5.

On figure 7 we show the 2D continuous wavelet analysis of a two-dimensional turbulent vorticity field. The wavelet coefficients are computed in L^2 norm at three different scales, namely 32 pixels (**Figure 7.1**), 16 pixels (**Figure 7.2**), and 2 pixels (**Figure 7.3**), using the isotropic Marr wavelet (there is no angular dependence of the wavelet coefficients in this case due to the wavelet isotropy).

42. The intermittency factor

The intermittency factor is given by the wavelet coefficients renormalized by the space averaged energy at each scale, such that:

(28)

It gives information on the space variance of the energy spectrum, namely if I(a,b)=1 the field is homogeneous and there is no space variance of the energy at scale a. If I(a,b) is large the field on the contrary is very intermittent, namely all the energy contribution at scale a comes from few very excited regions, while the rest of the field has little energy at this scale.

Figure 8 shows the intermittency factor computed at three different scales, namely 32 pixels (**Figure 8.1**), 8 pixels (**Figure 8.**), and 2 pixels (**Figure 8.**) using the isotropic Marr wavelet (there is no angular dependence of the wavelet coefficients in this case due to the wavelet isotropy).

43. The local energy spectrum

It is defined from the wavelet coefficients, such that:

(29)

Figure 9 shows the local energy spectra (**Figure 9.4**) computed by integrating in space the Marr wavelet coefficients after segmenting the vorticity field (**Figure 9.1**) into three different regions using the Weiss criterium (Weiss 1981): the elliptical region corresponding to the cores of the coherent structures (**Figure 9.2**), the parabolic region corresponding to the shear layers at the periphery of the coherent structures (**Figure 9.3**) and the hyperbolic region corresponding to the vorticity filaments of the incoherent background flow. We observe that the elliptic region scales around k^{-6}, the parabolic region around k^{-4}, while the hyperbolic region scales around k^{-3} . Therefore more coherent the region is, steeper its spectrum, while the incoherent region, such as the background flow, which is much more homogeneous, has a flater spectrum similarly to a noise.

5. Filtering of Two-dimensional Turbulent Flows using Continuous Wavelets

The wavelet transform being invertible it is always possible to select a subset of the coefficients and reconstruct a filtered version of the field from them. We propose several filtering technics to extract coherent structures from the background vorticity in two-dimensional turbulent flows. The first one consists of discarding all wavelet coefficients outside the influence cones (**Figure 4.a**) attached to the local maxima of the vorticity field which correspond to the coherent structures cores. The second one consists of discarding all wavelet coefficients which are smaller than a given threshold, which depends on the quantity of enstrophy we want to retain in the filtered vorticity field.

Figure 10 shows the extraction of one coherent structure, done by filtering all wavelet coefficients which are outside the influence cone attached to the center of this coherent structure, before computing the inverse wavelet transform. We display the complete vorticity field (**Figure 10.1**), the coherent structure alone (**Figure 10.2**), the vorticity field without the coherent structure (**Figure 10.3**) and the energy spectra of the three previous fields (**Figure 10.4**).

Figure 11 shows the extraction of the 40 most excited coherent structures, done by filtering all wavelet coefficients which are outside the influence cones attached to the center of these coherent structures, before computing the inverse wavelet transform. We display the complete vorticity field (**Figure 11.1**), the 40 coherent structures alone (**Figure 11.2**), the vorticity field without the coherent structures (**Figure 11.3**) and the energy spectra of the three previous fields (**Figure 11.4**).

Figure 12 shows the extraction of all excited coherent structures, done by filtering all wavelet coefficients which are smaller than a given threshold and then computing the inverse wavelet transform. We display the complete vorticity field (**Figure 12.1**), the coherent structures alone (**Figure 12.2**), the vorticity field without the coherent structures (**Figure 12.3**) and the energy spectra of the three previous fields (**Figure 12.4**).

As seen with the local energy spectra,these filtering techniques show again that the spectral behaviour depends on the region of the flow, with a tendency to scale as k^{-6} near the cores of the coherent structures, as k^{-4} or k^{-5} at their periphery and as k^{-3} in the background.

6. Compression of Two-dimensional Turbulent Flows using Wavelet Packets

Wavelet packets represent a family of orthogonal bases which unifies wavelets with Dirac, Fourier and wavepacket functions, affording increased flexibility in tiling the information plane, because now each element of the basis is parametrized independently in position b, scale a and wavenumber k. For a given signal sampled on N points the wavelet packet algorithm generate 2^N possible orthogonal bases and then selects the one which minimizes the number of coefficients having significant contributions to the total signal. In this sense, the wavelet packet algorithm defines the most efficient basis, so called the Best Basis, upon which to expand a given signal. If the flow is dominated by point vortices, then it is optimally represented using the Dirac grid point basis, and the output of the wavelet packet algorithm will reflect this. On the contrary, if the flow is dominated by waves, then it is optimally represented using the Fourier basis, and the output of the wavelet packet algorithm will again reflect this. If the flow behaviour is in between these two extreme situations, other bases will be more appropriate and the wavelet packet algorithm will give us the Best Basis in which the vorticity field will be represented with the smallest number of significant coefficients. The computation of the Best Basis for a signal sampled on N points is done in $N.\log_2 N$ operations, while the reconstruction of the signal from its projection onto the Best Basis is done in N operations.

Figure 13 shows the compression of a two-dimensional vorticity field using its wavelet packet coefficients with three different compression ratios. For a compression by 2 (**Figure 13.1**) we split the field into the 50 % strongest wavelet packet coefficients and the 50 % weakest wavelet packet coefficients, then for a compression by 20 (**Figure 13.2**) we split the field into the 5 % strongest wavelet packet coefficients and the 95 % weakest wavelet packet coefficients and for a compression by 200 (**Figure 13.3**) we split the field into the 0.5 % strongest wavelet packet coefficients and the 99.5 % weakest wavelet packet coefficients. For the three compression ratios we display the uncompressed fields with their energy spectrum, the compressed fields with their energy spectrum and the discarded fields with their energy spectrum. These results have been obtained in collaboration with Meyer, Pascal and Wickerhauser and are extensively discussed in Farge et al. /1992/.

With these compression techniques we find as before that the spectral behaviour depends on the region of the flow we analyse, with a tendency to scale as k^{-6} near the cores of the coherent structures, as k^{-4} at their periphery and as k^{-3} in the background.

7. Conclusion

Nowadays turbulence is commonly viewed from one of two alternative perspectives, depending upon which side of the Fourier transform one looks from. In physical space, we observe coherent vortices and wonder if there is universality in their structure and interactions. In Fourier space, we see transfers of energy and enstrophy between different scales of motion and ask, for example, if the slope of the energy spectrum is universal. The selection of bases in which turbulence may be examined must be extended if these perspectives are to be effectively reconciled. Through the use of wavelets and wavelet packets, we have constructed a class of bases, which includes grid point and Fourier representations as special cases, from which we select the basis which is optimal for a given flow, namely the one which compress the most the information while keeping track of the behaviour of the flow in both space and scale.

With such a wavelet or wavelet packet representation we can compute a local energy spectrum. Using the continuous wavelet transform, we have shown that different regions of the flow present different slopes for the local energy spectrum. Clearly the Fourier transform is unable to detect these different spectral behaviours which vary in space, while the wavelet transform is here the appropriate tool. Typically we have observed that the cores of the coherent structures, which correspond to the elliptic regions, scale around k^{-6}, the shear layers around the coherent structures, which correspond to the parabolic regions, scale around k^{-4}, while the vorticity filaments in the background, which correspond to the hyperbolic regions, scale around k^{-3}. From this constatation we infer that the variation of the Fourier spectral slope we commonly observe for two-dimensional flows may be related to the variation of the density of coherent structures which varies depending on the initial conditions and on the forcing. If it is true we may hope that the local scaling of the different regions may be universal enough in order to be able to model their behaviour, each regions having then its own parametrization.

Using the orthogonal wavelet packet transform, we have shown that the significant coefficients correspond to the coherent structures, while the weak coefficients correspond to the vorticity filaments which are only passively advected by the coherent structures. One possible application of the wavelet packet algorithm is to apply it from time to time during a numerical simulation, in order to separate regions with highly active small scales, which need a better grid resolution, from regions with inactive small scales, which do not contribute much to the dynamics and can either be discarded or modelled. Indeed the wavelet packet Best Basis seems to distinguish the low-dimensional dynamically active part of the flow from the high-dimensional passive components. It gives us some hope of drastically reducing the number of degrees of freedom necessary to the computation of two-dimensional turbulent flows.

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Legends

Figure 1

Comparison between different types of transforms: ***a.*** *the analysing function in physical space,* ***b.*** *the analysing function in Fourier space,* ***c.*** *the corresponding information cells*

1.1 *Sampling by a Dirac comb*
1.2 *Fourier transform*
1.3 *Windowed Fourier transform*
1.4 *Wavelet transform*

Figure 2

Most commonly used wavelets: ***a.*** *the function and* ***b.*** *its Fourier transform*

2.1 *Marr wavelet (Mexican hat)*
2.2 *Morlet wavelet*
2.3 *Meyer-Lemarié wavelet*
2.4 *Daubechies wavelet (n=2)*
2.5 *Daubechies wavelet (n=7)*

Figure 3

Wavelet transform of several academic signals using Morlet wavelet: ***a.*** *the signal,* ***b.*** *the wavelet coefficient modulus,* ***c****. the wavelet coefficient phase.*
We have used the code TecLet 1D (copyright Science & Tec).

3.1 *a Dirac spike*
3.2 *the superposition of two cosine functions having different frequencies*
3.3 *the superposition of two cosine functions of very different amplitudes*
3.4 *a tchirp*
3.5 *a Gaussian white noise*
3.6 *a tchirp in presence of a strong noise*

Figure 4

Locality in wavelet coefficient space: ***a.*** *the influence cone attached to* x_o, ***b.*** *the spectral band attached to wavenumber* k_o

Figure 5

Analysis of the local regularity of a function f in x_o (given by the slope of the modulus of (x_o,a) represented in Log-Log coordinates): ***a****. f is a function presenting a Lipschitz singularity α in $x_{o,}$* **b.** *f is a function which is regular in x_o.*

Figure 6

Continuous wavelet analysis, using Morlet wavelet with k_0=5, of a one-dimensional cut done in a two-dimensional turbulent vorticity field: ***a.*** *normalization $a^{-1/2}$,* ***b.*** *normalization en a^{-1}.*

Figure 7

The wavelet coefficients in L^2 norm computed using the Marr wavelet:
a. *Large scale, 32 pixels,****b.*** *Medium scale, 8 pixels,* ***c.*** *Small scale, 2 pixels*

Figure 8

The intermittency factor computed using the Marr wavelet:
a. *Large scale, 32 pixels,****b.*** *Medium scale, 8 pixels,* ***c.*** *Small scale, 2 pixels*

Figure 9

Local energy spectra computed from the wavelet coefficients after segmenting the vorticity field into three different regions.

- ***9.1*** *The complete vorticity field*
- ***9.2*** *The elliptical region corresponding to the coherent structures*
- ***9.3*** *The parabolic region corresponding to the shear layers at the periphery of the coherent structures*
- ***9.4*** *The hyperbolic region corresponding to the vorticity filaments of the incoherent background flow*
- ***9.5*** *The corresponding energy spectra*

Figure 10

Extraction of one coherent structure, done by filtering all wavelet coefficients which are outside the influence cone attached to the center of this coherent structure, before computing the inverse wavelet transform: ***a.*** *the complete vorticity field,* ***b.*** *the coherent structure alone,* ***c.*** *the vorticity field without the coherent structure,* ***d.*** *the energy spectra of the three previous fields.*

Figure 11

Extraction of the 40 most excited coherent structures, done by filtering all wavelet coefficients which are outside the influence cones attached to the centers of these coherent structures, before computing the inverse wavelet transform: ***a.*** *the complete* *vorticity field,* ***b.*** *the 40 coherent structures alone,* ***c.*** *the vorticity field without the coherent structures,* ***d.*** *the energy spectra of the three previous fields.*

Figure 12

Extraction of all excited coherent structures, done by filtering all wavelet coefficients which are smaller than a given threshold and then computing the inverse wavelet transform: ***a.*** *the complete vorticity field,* ***b.*** *the coherent structures alone,* ***c.*** *the vorticity field without the coherent structures,* ***d.*** *the energy spectra of the three previous fields.*

Figure 13

Compression of a two-dimensional vorticity field using its wavelet packet coefficients: ***a.*** *the uncompressed field and its energy spectrum,* ***b.*** *the compressed field and its energy spectrum,* ***c.*** *the discarded field and its energy spectrum.*
The visualisation has been done in collaboration with Jean-Francois Colonna.

13.1 *Compression by a factor 2*
13.2 *Compression by a factor 20*
13.3 *Compression by a factor 200*