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Vortex excitation in turbulent flows using an adaptive wavelet scheme

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Abstract

Two-dimensional turbulent flows, which are dominated by coherent structures, have a sparse representation in wavelet bases. The coherent structures (e.g. vortices) correspond to very few wavelet modes, the strongest modes of the wavelet transform, while the weaker modes represent the unorganized background flow [4]. These observations suggest that wavelets could be an efficient basis for two-dimensional turbulent flows since the dynamics of such flows is largely controlled by their coherent vortices. Recently we have developed a numerical scheme to solve the Navier-Stokes equations directly in an orthogonal wavelet bases [6]. We demonstrated the possibility to benefit from the compression property of this representation [5, 8]. This approach leads to a reduced number of degrees of freedom in the numerical simulation of 2D turbulence, although the flow exhibits a large number of spatial scales.

In [7] we developed a new method to force turbulent flows by local vortex excitation. The forcing term depends nonlinearly on the wavelet coefficients of the vorticity field. It injects energy and enstrophy as locally as possible in both physical and spectral spaces in an inhomogeneous way. This new forcing is defined in wavelet space in order to control the smoothness of the vortices thus excited and to model the local production of vortices by instabilities observed in turbulent flows. For the simulation of statistically stationary turbulent flows in an adaptive wavelet basis we now couple the wavelet based forcing method with the fully adaptive wavelet code of [6] using partial collocation of the nonlinear convective term. In Fig.1 we depicted a statistically stationary state of the wavelet forced flow, the vorticity field together with the necessary wavelet coefficents (dark markers), the spectra and the probability density function (pdf) which clearly exhibits a non–Gaussian behaviour. One motivation for the simulation of statistically stationary turbulent flows is the study of turbulent mixing which is of 'primordial' importance in many applications, like oceanography, propagation of pollutants or turbulent combustion modelling. Different approaches are currently used to simulate turbulent flow fields with many spatial scales, e.g. direct numerical simulation [1] or Monte Carlo methods using Gaussian stochastic processes [2] also in conjunction with wavelets [3]. Here we propose the use of adaptive wavelet methods in order to simulate Navier–Stokes turbulence with many spatial scales and non–Gaussian behaviour to study the mixing of passive tracers or scalars in fully developed turbulent flows. As an example we show in Fig.2 the high level of complexity of the mixing of a scalar field transported in a turbulent flow field. The initially simple circular concentration distribution is deformated through the extreme mixing and intricate shapes are produced.

In the conclusion we discuss some perspectives for turbulence modelling in wavelet-coefficient space and their impact on turbulent mixing.

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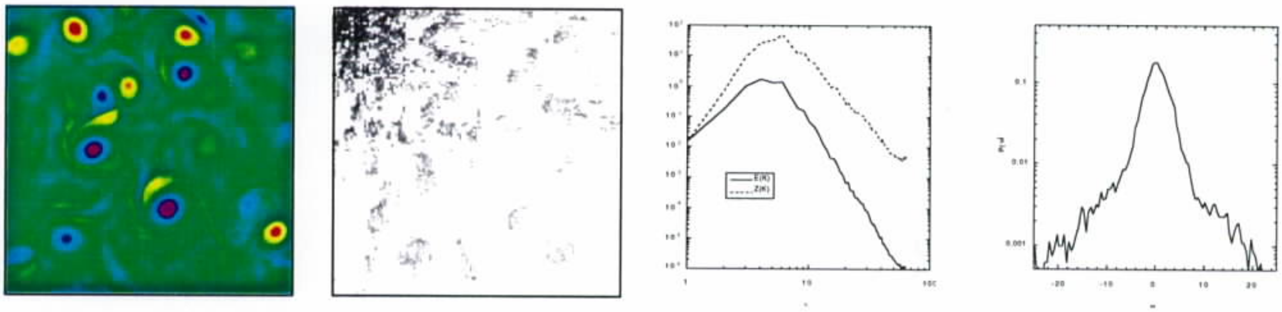

Figure 1: Wavelet forced 2D turbulent flow: Vorticity field, corresponding wavelet coeffcients, energy/enstrophy spectra and pdf of vorticity.

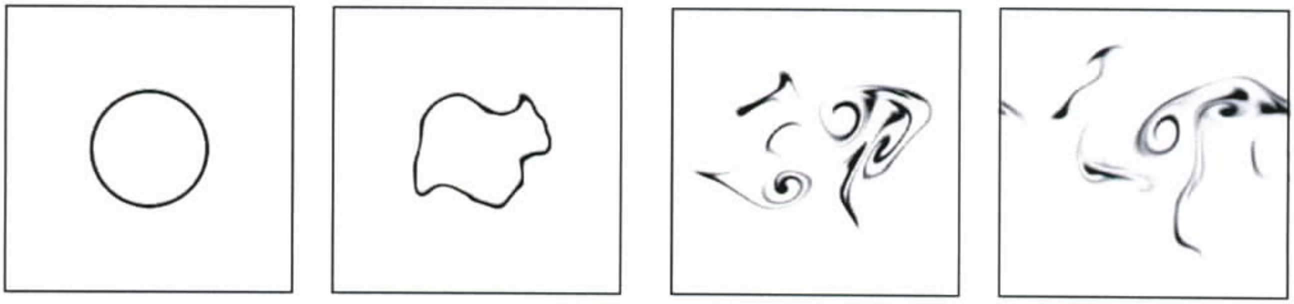

Figure 2: Evolution of concentration in a statistically stationary 2D turbulent flow.

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