COHERENT STRUCTURE EDUCTION IN WAVELET-FORCED TWO-DIMENSIONAL TURBULENT FLOWS

MARIE FARGE
LMD-CNRS, Ecole Normale Supérieure
24 rue Lhomond, 75231 Paris cedex 05, France

KAI SCHNEIDER
Centre de Physique Théorique, CNRS - Luminy -
Case 907, 13288 Marseille cedex 09, France
and ICT, Universität Karlsruhe (TH)
Kaiserstraße 12, 76128 Karlsruhe, Germany

AND

NICHOLAS K.-R. KEVLAHAN
LMD-CNRS, Ecole Normale Supérieure
24 rue Lhomond, 75231 Paris cedex 05, France

Abstract. We analyze vorticity fields obtained from direct numerical simulations (DNS) of statistically stationary two-dimensional turbulence where the forcing is done in wavelet space. We introduce a new eduction method for extracting coherent structures from two-dimensional turbulent flows. Using a nonlinear wavelet technique based on an objective universal threshold we separate the vorticity field into coherent structures and background flow. Both components are multi-scale with different scaling laws, and therefore cannot be separated by Fourier filtering. We find that the coherent structures have non-Gaussian statistics while the background flow is Gaussian, and we discuss the implications of this result for turbulence modelling.

1. Introduction

Our aim is the numerical simulation of large Reynolds number flows using present computers. In this context the key question is: which quantities should be computed and which quantities can be discarded or modelled? The usual choice, based on the statistical theory of homogeneous and isotropic turbulence (in three dimensions (Kolmogorov, 1941) and two dimensions (Kraichnan, 1967)), is to compute the low-wavenumber modes, considered to be active, and to model the high-wavenumber modes, considered to be controlled by the active ones, using an *ad hoc* subgrid scale parameterization, for example an eddy-diffusivity. This wavenumber separation is assumed by most of the numerical methods presently used to compute turbulent flows, from Reynolds Averaging to Large Eddy Simulation (LES) and Nonlinear Galerkin methods. It is important to understand that this choice of a wavenumber separation is done *a priori*, supposing an hypothetical spectral gap, which has not been observed, and without reference to the first principles stated in the Navier–Stokes equations.

Statistical theories of homogeneous and isotropic turbulence rely on the hypothesis that energy (or enstrophy in two dimensions) is injected at low wavenumbers and is dissipated at high wavenumbers. This separation of scales, between energy (or enstrophy) production and dissipation, allows for the existence of an intermediate range of wavenumbers, called the inertial range, where the nonlinear dynamics is supposed to be conservative. As far as we know, the existence of such an inertial range has never been demonstrated from first principles. Moreover, according to Kolmogorov's theory, in three-dimensional turbulence energy cascades from low to high wavenumbers. Unfortunately this behaviour is only defined for ensemble averages, and in a single realization of a turbulent flow significant amounts of energy are transferred in the opposite direction, from high to low wavenumbers (Domaradzki, 1992), (Meneveau, 1991). This energy 'back-scatter' invalidates the assumption that the high wavenumber modes are passive. The back-scatter is due to the presence of organized structures, such as vortex tubes (often called 'vortex filaments' although they have nothing to do with the vortex filaments of two-dimensional turbulent flows) or horseshoe vortices, which interact and transfer energy to larger scales. Because these instabilities are local and intermittent, the Fourier representation is not able to separate them from the rest of the flow. This back-scattering problem is even worse in two-dimensional turbulence, for which Kraichnan (Kraichnan, 1967) has predicted that even ensemble averages (and not only individual realization) present an inverse energy cascade, due to the conservation of both energy and enstrophy in the inertial range.

We believe that the problem of turbulence modelling and subgrid-scale parameterization has to be reconsidered, for both two-dimensional and three-dimensional turbulent flows, and we propose that the classical division between large and small eddies be replaced by a new separation between coherent structures and background flow.

Since the pioneering paper by McWilliams (McWilliams, 1984) the emergence of coherent structures out of initially random vorticity fields has been recognized as a generic feature of two-dimensional turbulent flows. The definition of coherent structures is still rather subjective. Usually one thinks of coherent structures as localized accumulations of vorticity which concentrate most of the enstrophy, $Z = 1/2 < \omega^2 >$ (ω being the vorticity and $<>$ the space average) in a small fraction of the spatial domain and which survive on time-scales much longer than the average eddy turn-over time $\tau = \sqrt{1/Z}$. A more precise characterization of coherent structures was proposed in 1981 by Weiss (Weiss, 1992) as elliptical flow regions where rotation dominates deformation. This definition assumes that the stress tensor varies slowly, in space and time, compared to vorticity gradients.

A key question, which remains open, is the following: do coherent structures have a generic shape? The answer to this question strongly influences our analysis of vorticity fields, and in particular our interpretation in terms of scale, which is intrinsically linked to the generic shape we assume for the vortices. This hypothetical shape is not clearly defined, the notion of scale, as well as the notions of vortex size and circulation, are meaningless. This point has been a source of misunderstanding for years in the study of turbulence due to the identification of scale with the inverse of wavenumber. In fact, this traditional definition of scale makes sense only if one analyzes an homogeneous and isotropic velocity or vorticity field, e.g. a wave field, or an ensemble average of velocity or vorticity fields which is homogeneous and isotropic due to the translational and rotational invariance of the coherent structure motion. However, this definition does not make sense if one analyzes a given realization containing isolated coherent structures, which is necessarily inhomogeneous, intermittent and highly phase-correlated. In this case, to define the notion of scale one should look for the generic shape of the coherent structures observed in the vorticity field. The vorticity field should be preferred to the velocity field for dynamical reasons, because vorticity is Galilean invariant and volume preserving in the inviscid limit. It should be noted that in statistical analysis *a prioris* are as essential as hypotheses are in modelling: we should state them clearly, otherwise our results will be nonsensical. Once the reference shape has been defined we can compute the correlation between the vorticity field and this shape, which is dilated and translated in order to obtain the scale content of the vorticity field.

In this paper we analyze two-dimensional forced turbulent vorticity fields using an orthogonal wavelet basis. The mother wavelet is chosen as the generic shape for coherent structures. Such localized functions are better suited to analyze turbulent flows than the trigonometric functions used as basis elements for Fourier decomposition. This analysis allows us to perform a local spectral analysis of the flow, which is not possible with the classical Fourier transform. In selecting the analyzing functions we are limited by the Heisenberg uncertainty principle, which means that we cannot have perfect localization in both space and wavenumber. To analyze two-dimensional turbulent fields the best solution is to use functions whose shape correspond to our *a priori* model of an isolated coherent structure, namely a localized and isotropic distribution of vorticity. We can then use a self-affine set of such functions in order to generate a multi-scale analysis of the vorticity field. In order to perform a quantitative analysis, we also require isometry between the enstrophy computed in physical space and the enstrophy computed from the inner-product of the vorticity field with the analyzing functions. All these reasons have led us to use wavelets, which are well-localized in both physical and Fourier spaces, self-affine (since they are obtained by dilation from one another), and allow an isometric transform which conserves energy.

2. Wavelet–forced two–dimensional turbulence

The enstrophy dissipated by the dissipative term has to be re-injected in order to simulate a statistically stationary flow, i.e. non-decaying fully-developed turbulence. The usual technique is to inject enstrophy and energy locally in Fourier space. In this article, however, we employ a new wavelet forcing technique introduced in (Schneider and Farge, 1997). The forcing is defined in wavelet space in order to control the smoothness of the vortices thus excited. Using this method the injection of energy and enstrophy is as local as possible in both physical and spectral space. For details we refer the reader to (Schneider and Farge, 1997).

2.1. GOVERNING EQUATIONS

We consider the two-dimensional Navier–Stokes equations written in velocity–vorticity form with a forcing term $F = \nabla \times \mathbf{f}$. Furthermore we include an artificial frictional term $\lambda\Psi$, which acts as an infrared energy sink to remove the energy that accumulates at large scales due to the inverse energy cascade. The equations we solve are:

$$\partial_t \omega + \mathbf{v} \cdot \nabla \omega = \nu \nabla^2 \omega + \lambda \Psi + F \quad , \qquad \nabla \cdot \mathbf{v} = 0, \tag{1}$$

where the velocity vector $\mathbf{v} = (u, v)$, the vorticity $\omega = \nabla \times \mathbf{v}$ and the stream function $\Psi = \nabla^{-2} \omega$. Further parameters are the strength of the friction term λ and the kinematic viscosity ν.

We assume periodic boundary conditions in both directions, i.e. our domain is the two-dimensional flat torus $\mathbb{T}^2$ with $\mathbb{T} = 2\pi \mathbb{R}/\mathbb{Z}$. These boundary conditions have been chosen in order to simulate turbulent flows far from walls, and thus avoid the treatment of boundary layers.

2.2. NUMERICAL METHOD

For the numerical solution of the system (1) we employ a classical Fourier pseudo–spectral method with a semi–implicit time discretization using finite differences (Euler–backwards for the viscous term and Adams–Bashforth extrapolation for the nonlinear term, both of second order). The vorticity field and the other variables are represented as Fourier series: $\omega(\mathbf{x}) = \sum_{|\mathbf{k}|\leq N/2} \widehat{\omega}(\mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}}$.

2.3. WAVELET FORCING

Furthermore at time step n we develop ω^n as an orthonormal wavelet series from the largest scale $l = 2^0$ to the smallest scale $l = 2^J$ using a two–dimensional multi-resolution analysis (MRA) (Daubechies, 1992) (Farge, 1992):

$$\omega^n(x,y) \;=\; c^n_{0,0,0}\, \phi_{0,0,0}(x,y) \;+\; \sum_{j=0}^{J-1} \sum_{i_x=0}^{2^j-1} \sum_{i_y=0}^{2^j-1} \sum_{\mu=1}^{3} d^{\mu,n}_{j,i_x,i_y}\; \psi^{\mu}_{j,i_x,i_y}(x,y) \quad , \tag{2}$$

with $\phi_{j,i_x,i_y}(x,y) \;=\; \phi_{j,i_x}(x)\; \phi_{j,i_y}(y)$, and

$$\psi^{\mu}_{j,i_x,i_y}(x,y) \;=\; \begin{cases} \psi_{j,i_x}(x)\; \phi_{j,i_y}(y) & ;\ \mu = 1 \quad , \\ \phi_{j,i_x}(x)\; \psi_{j,i_y}(y) & ;\ \mu = 2 \quad , \\ \psi_{j,i_x}(x)\; \psi_{j,i_y}(y) & ;\ \mu = 3 \quad , \end{cases} \tag{3}$$

where $\phi_{j,i}$ and $\psi_{j,i}$ are the 2π–periodic one–dimensional scaling function and the corresponding wavelet, respectively. Due to the orthogonality the coefficients are given by $c^n_{0,0,0} = \langle \omega^n\,,\, \phi_{0,0,0} \rangle$ and $d^{\mu,n}_{j,i_x,i_y} = \langle \omega^n\,,\, \psi^{\mu}_{j,i_x,i_y} \rangle$ where $\langle \cdot, \cdot \rangle$ denotes the inner product.

The forcing term F at time step $n+1$ is then defined as a function of ω at time step n:

$$F^{n+1}(x,y) \;=\; C \sum_{J_0<j<J_1} \sum_{k_x=0}^{2^j-1} \sum_{k_y=0}^{2^j-1} \sum_{\mu=1,2,3} \langle \omega^n\,,\, \psi^{\mu}_{j,k_x,k_y} \rangle\, \psi^{\mu}_{j,k_x,k_y}(x,y) \quad , \tag{4}$$

with $0 \leq J_0 \leq J_1 \leq J$, where J denotes the finest scale in the simulation, $C > 0$ and $|\langle \omega^n(x,y)\,,\, \psi^{\mu}_{j,k_x,k_y} \rangle| > \epsilon$. The scale parameters J_0 and J_1 define the scale range of the forcing. The restriction to wavelet coefficients above a given threshold implies that only the dynamically active part of the flow, i.e. the coherent structures (Farge, 1992), (Farge *et al.* , 1996), are forced. The constants C and λ, responsible for the strength of the forcing and the Rayleigh friction respectively, are adjusted in such a way that we obtain a statistically stationary state. For further details on the numerical simulation we refer the reader to (Schneider and Farge, 1997).

3. Coherent structure eduction

The term 'eduction' was introduced by (Hussain, 1986) to describe the extraction of coherent structures out of three-dimensional laboratory turbulent flows. We will keep the same terminology to characterize the technique we propose to separate the coherent structures from the background flow in two-dimensional turbulent flows obtained by DNS.

As we have already noted, there is at present no consensus on the precise definition of coherent structures. The only definition of a coherent structure which seems objective is that of a locally meta-stable state, such that, in the reference frame associated with the coherent structure, the nonlinearity of Navier Stokes equations becomes negligible. In consequence, a coherent structure can be characterized by a functional relation between the vorticity ω and the streamfunction ψ in the form $\omega = F(\psi)$, where F is called the coherence function (Farge and Holschneider, 1990). One possible coherent structure eduction technique would be to plot the diagram $\omega = F(\psi)$ and extract the branches which can be fitted by a function F; the points belonging to these branches would correspond to locations where the vorticity field $\omega_>$ is coherent, while the scattered points which do not belong to any branch would correspond to locations in the incoherent background flow $\omega_<$.

Other possible eduction techniques are less objective than the one described above, because they depend on a threshold value which has to be defined *a priori*. The simplest method, already proposed by several authors (McWilliams, 1984) (Babiano *et al.* , 1987), is to choose a vorticity threshold, for instance $\omega_T = \sqrt{Z}$, and retain as coherent the regions where $|\omega| > \omega_T$, while the background flow corresponds to the regions where $|\omega| \leq \omega_T$. The drawback of this method is that it does not preserve the smoothness of ω, and both fields, $\omega_>$ and $\omega_<$, will have spurious discontinuities which will affect their Fourier energy spectra. To avoid this problem we propose to replace the grid-point representation by a wavelet representation, which, on the contrary, does not introduce discontinuities and therefore conserves the spectral properties of ω.

Since the wavelet transform is invertible, it is always possible to select a subset of the coefficients and reconstruct a filtered version of the field from them. Using this property we have proposed several coherent structure eduction techniques (Farge and Philipovitch, 1993), (Farge *et al.* , 1992). The first consists of discarding all wavelet coefficients outside the influence cones, namely the spatial support of the wavelets, attached to the local maxima of the vorticity field which correspond to the centers of coherent structures (Farge and Philipovitch, 1993).

The second technique, introduced here, consists of discarding all wavelet coefficients which are smaller than a given threshold which depends only on the variance of the field $< \omega^2 >$ and on the number of samples N, such that the threshold value is: $\widetilde{\omega_T} = (2 < \omega^2 > \log_{10} N)^{1/2}$. This technique is based on the wavelet shrinkage method of Donoho (Donoho, 1992). Donoho has shown that this is the optimal denoising technique for a signal containing a Gaussian white noise of a given variance. However, we are not sure that our signal actually contains a Gaussian noise, and anyway we do not know its variance, therefore we will consider instead the variance of the total vorticity field. This is an overestimate which leads to a higher treshold value and a stronger compression rate. Anyway we can always *a posteriori* check that the discarded components correspond to a Gaussian noise (characterized by a Gaussian PDF, skewness zero and kurtosis three).

Applying the second nonlinear thresholding technique to the wavelet packet coefficients of the vorticity field, we have extracted the coherent structures from the background flow and shown that both components are multi-scale (Farge *et al.* , 1992), however they exhibit different scaling laws, the background having an energy spectra scaling in k^{-3} compatible with Kraichnan's prediction while the coherent structures scale in k^{-6} (Farge *et al.* , 1992).

We have also tried (Wickerhauser *et al.* , 1994) to use adaptive local cosines instead of wavelet packets to separate coherent structures from background flow. We showed that the local cosine representation does not compress the enstrophy as well as wavelets or wavelet packets. First, it smoothes the coherent structures and therefore loses enstrophy, and secondly it introduces spurious oscillations in the background, due to the loss of the phase information attached to the weak coefficients. These drawbacks are common to any Fourier or windowed Fourier representation, because each Fourier component contains non-local information and we need the phase information of all Fourier components to reconstruct precisely a given region of the field. Therefore no Fourier technique can properly educe coherent structures, because as the vorticity field is compressed the coherent structures disappear and become increasingly mixed up with the background flow (Wickerhauser *et al.* , 1994). In this paper we will focus on the second (variance based) nonlinear wavelet compression as the optimal solution for coherent structure eduction. We will also work with the wavelet representation rather than the wavelet packet representation.

4. Results

The wavelet forced two-dimensional Navier–Stokes equation were solved using a pseudo-spectral scheme with resolution $N = 128^2$. The dissipation is modelled by a Laplacian operator with a kinematic viscosity $\nu = 2 \times 10^{-3} m^2/s$. We have reached a statistically stationary solution (Schneider and Farge, 1997) that we have maintained for 16 000 time steps ($\Delta t = 10^{-3} s$), i.e. for 60 eddy-turn over times. We now analyze the vorticity field obtained at the final time step which corresponds to $t = 16s$.

We project the vorticity field ω onto an orthogonal spline wavelet basis of Battle-Lemarié type. Figure 1 shows the spatial and spectral support of the symmetric quintic spline wavelets used in the present analysis. In figure 2 we plot the retained enstrophy $Z_>$ as a function of the number of retained wavelet coefficients $N_>$. Note that very few wavelet coefficients retain most of the total enstrophy Z and that after $N_> = 216$ the rate of convergence abruptly changes its behaviour and becomes very slow (figure 2b). The optimal compression rate is obtained for $N_> = 216$, which contain 92.4% of the total enstrophy $Z = 9.2s^{-2}$, although they represent only 1.3% of the total number of coefficients $N = 128^2 = 16384$. The $N_< = 16168$ remaining weaker wavelet coefficients represent 98.7% of the total number of wavelet coefficients but retain only 7.6% of the total enstrophy.

Next we compute the threshold proposed by (Donoho, 1992), (Donoho *et al.* , 1995) for denoising. This criterion depends only on the total number of samples N and on the variance of the vorticity field $< \omega^2 >= 16s^{-2}$. Since we have reached a statistically stationary state $< \omega^2 >$ remains constant, therefore we compute the unique threshold value: $\widetilde{\omega_T} = (2 < \omega^2 > \log_{10} N)^{1/2} = 12s^{-1}$. The coherent structures are then extracted by projecting only the wavelet coefficients having an absolute value larger than $\widetilde{\omega_T}$ back onto grid-points, which gives the coherent vorticity field $\omega_>$. The background flow is found similarly by selecting those wavelet coefficients with absolute value smaller or equal to $\widetilde{\omega_T}$ and then reconstructing the vorticity field $\omega_<$.

In table 1 we compare the first moments up to the 6th order, the skewness and the kurtosis of the uncompressed vorticity ω, the coherent vorticity $\omega_>$ and the background vorticity $\omega_<$. Note that all fields have skewness $S = 0$ and that both the uncompressed vorticity and the coherent vorticity have kurtosis $K = 17$, characteristic of a non-Gaussian probability distribution, while the background vorticity has kurtosis $K = 3$, characteristic of a Gaussian probability distribution. These results are confirmed in figure 6 where we plot the probability distribution functions (PDF) for the three vorticity fields. The uncompressed vorticity and the coherent vorticity have nearly identical PDFs, with heavy tails (extremal values: $-37/+28$) and

TABLE 1. Statistical properties of the vorticity field at $t = 16$s.

quantity	definition	ω total	$\omega_>$ coherent	$\omega_<$ background
# of coefficients	N	16384	216	16128
% of coefficients		100 %	1.3 %	98.7 %
1st moment (mean)	$M_1 = \bar{\omega} = \frac{1}{N}\sum_{i=1}^{N}\omega_i$	0.0	0.0	0.0
2nd moment (variance)	$M_2 =< \omega^2 >= \frac{1}{N}\sum_{i=1}^{N}\omega_i^2$	18.4	17.0	1.4
3rd moment	$M_3 = \frac{1}{N}\sum_{i=1}^{N}\omega_i^3$	-29.6	-31.7	-0.8
4th moment	$M_4 = \frac{1}{N}\sum_{i=1}^{N}\omega_i^4$	5763.7	4966.0	5.8
5th moment	$M_5 = \frac{1}{N}\sum_{i=1}^{N}\omega_i^5$	-55956.0	-50716.3	0.1
6th moment	$M_6 = \frac{1}{N}\sum_{i=1}^{N}\omega_i^6$	3879905.5	3258826.2	39.2
Enstrophy	$Z = \frac{M_2}{2}$	9.2	8.5	0.7
% total enstrophy		100 %	92.4 %	7.6 %
Skewness	$S = \frac{M_3}{M_2^{\frac{3}{2}}}$	-0.4	-0.4	0.0
Kurtosis	$K = \frac{M_4}{M_2^2}$	17.1	17.2	3.0

seem close to a Cauchy distribution. Such a Cauchy distribution has been predicted for the velocity gradients (a quantity similar to the vorticity we consider here) of a system of point-vortices in two and three dimensions (Min *et al.* , 1996). On the contrary, the PDF of the background flow is a parabola (in lin-log coordinates) and does not present heavy tails (extremal values: $-5/+5$), which is characteristic of a Gaussian distribution.

Figures 3 and 4 show that the coherent components retain the precise shape of each coherent structure, and therefore is inhomogeneous, while the incoherent components correspond to the homogeneous background flow.

In figure 5 we show that both components have a broad band energy spectrum, although the coherent components dominate in the low wavenumbers while the incoherent components dominate in the high wavenumbers. This is due to the fact that the spatial support of the coherent structures decreases with scale and therefore their weight in the high wavenumbers of the energy spectrum becomes negligible. However, since the background flow is homogeneous, the spatial support of the incoherent components is dense in space and therefore the background flow conditions the high wavenumber range of the energy spectrum, where we observe a k^{-4} scaling for the energy and therefore a k^{-2} (pink noise) scaling for the enstrophy. The fact that there is a break in the power-law spectra of both the coherent and incoherent components at $k = 16$ is due to the wavelet-forcing which has been limited to scales smaller than $L = 32 = 2^5$ which corresponds to $k = 16$.

In figure 7 we show that the coherence function of the coherent components $\omega_>$ is similar to that of the uncompressed vorticity field ω. The coherence function is a superposition of functions $\omega_> = F(\Psi_>)$ (corresponding to the lines drawn on figure 7), each one corresponding to a coherent structure. In contrast, the background flow does not exhibit such a correlation between $\omega_<$ and $\Psi_<$ and is therefore incoherent. This is further proof that the nonlinear wavelet thresholding technique is appropriate for coherent structures eduction in turbulent flows.

5. Conclusion

The goal for modelling or computing the evolution of turbulent flows is to take a coarse-graining point of view, namely to keep the essential information and discard the details then considered as noise. For two-dimensional turbulent flows we propose dividing the relevant dynamical field, i.e. the vorticity field ω, into its inhomogeneous, intermittent and organized components $\omega_>$, which correspond to the coherent vortices and are characterized by non-Gaussian statistics, and its homogeneous, non-intermittent and random components $\omega_<$, which correspond to the well-mixed background flow and are characterized by Gaussian statistics. Both components are multiscale and this is why the Fourier transform, whose modulus loses the spatial information, is not the optimal functional basis to study turbulence. The grid-point representation is not suitable either, because we want to be able to detect the characteristic scaling of the two regions and therefore must avoid introducing spurious discontinuities.

By applying such an eduction technique to wavelet-forced two-dimensional turbulent flow, we have shown that the coherent vorticity is highly non-Gaussian while the background vorticity is Gaussian. This result has strong implications for two-dimensional turbulence modelling. The coherent vorticity corresponds to the dynamical components which are out of statistical equilibrium and have a very large variance characteristic of non-Gaussian behaviour. Therefore we are unable to model the coherent vorticity by a simple stochastic process. The number of degrees of freedom in the coherent vorticity is less than 2% of the total number of degrees of freedom required for a full direct numerical simulation (DNS). Therefore we suggest using a pseudo-wavelet scheme (Schneider *et al.* , 1997) to compute only these few coherent modes. The remaining degrees of freedom represent more than 98% of the total and correspond to the incoherent components having Gaussian statistics. The incoherent part may therefore be modelled by any standard statistical turbulence model, such as Reynolds averaged or $k - \epsilon$ models (Mohammadi and Pironneau, 1993).

In this paper we have proposed using orthogonal wavelets, which play the role of phase-space atoms independently defined in space and scale, to extract coherent structures from the vorticity field. This eduction method uses an objectively defined, universal threshold. The Navier–Stokes dynamics combines these "phase-space atoms" into "phase-space molecules" which correspond to the coherent structures, whose formation is observed during the flow evolution. Thus each vortex can be computed as the superposition of well-localized functions, namely wavelets, which describe its internal degrees of freedom. This method may allow us to drastically reduce the number of degrees of freedom necessary to compute the turbulent flow evolution (Fröhlich and Schneider, 1996), (Schneider *et al.* , 1997). We have already shown that this approach (Farge *et al.* , 1990) may be extended to the case of three-dimensional turbulent flows where vorticity tubes, often called vorticity filaments, play the role of coherent structures moving in an homogeneous random background flow. We believe that turbulence, both two-dimensional and three-dimensional, is a random superposition of a set of meta-stable vortices whose interactions give rise to its characteristic unpredictable behaviour, therefore the statistical tools we use in turbulence should be based on the recognition of vortices as the basis elements of turbulent flows.

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Figure 1. Quintic spline wavelet $psi_{7,0}$ in physical and in Fourier space.

Figure 2. Compression rate: enstrophy versus the number of wavelet coefficients.

Figure 3. Surface plots of the vorticity fields ω, $\omega_>$ and $\omega_<$.

Figure 4. Cuts of the vorticity fields ω, $\omega_>$ and $\omega_<$ at $y = 2\pi 77/128$.

Figure 5. Energy spectra of the vorticity fields ω, $\omega_>$ and $\omega_<$.

Figure 6. Probability density functions (histograms using 100 bins) of the vorticity fields ω, $\omega_>$ and $\omega_<$.

Figure 7. Scatter plots of the vorticity fields ω, $\omega_>$ and $\omega_<$ versus the corresponding stream functions Ψ, $\Psi_>$ and $\Psi_<$.