Advances in Turbulence IX
Proceedings of the Ninth European Turbulence Conference
I.P. Castro & P.E. Hancock (Eds.)

[id:227,1]

Coherent Vortex Simulation of a three dimensional temporally developing turbulent mixing layer

G. Pellegrino[1,2], K. Schneider[2,3] and M. Farge[4]

[1]ICT, Universität Karlsruhe (TH), Germany
[2]L3M, CNRS & Université d'Aix-Marseille, France.
[3]CMI, Université de Provence, Marseille, France.
[4]LMD-CNRS, Ecole Normale Supérieure, Paris, France.

Contact address: *pellegri@ict.uni-karlsruhe.de*

Introduction

Recently we have introduced a new semi-deterministic method to compute turbulent flows called Coherent Vortex Simulation (CVS) [1], [2]. The main idea is to split the flow into a coherent part and an incoherent background. The first one is mainly responsible for the dynamics and thus its evolution is computed deterministically. The latter is noise like and is characterised by a thermal equilibrium. Hence its influence can be statistically modelled. The separation is based on a non linear filtering of the wavelet representation of vorticity. We find that the coherent part can be described by few wavelets modes. Using an adaptive space discretisation we can reduce the computational cost in terms of memory requirements and CPU time. In this paper we check the potential of this approach by comparing a DNS and a CVS of a temporal developing turbulent mixing layer. This flow can be efficiently computed with a pseudo-spectral code. Furthermore, mixing layers are very interesting test cases because they present nonlinear behaviours that occur in many 3D flows.

Numerical Method and Flow configuration

We integrate the Navier-Stokes equations in terms of the primitive variables, Velocity $\vec{V}$ and pressure P. The vorticity $\vec{\omega}$ is then derived from the velocity to compute the non-linear term and apply the wavelet filter (in the CVS case).

$$\partial_t \vec{V} \;+\; \vec{\omega} \times \vec{V} \;+\; \nabla P \;-\; \frac{1}{Re} \nabla^2 \vec{V} = 0$$
$$\nabla \cdot \vec{V} = 0 \qquad \nabla \times \vec{V} = \vec{\omega}.$$

This set of equations is completed by periodic boundary conditions in the two horizontal directions and a mirror symmetry in the vertical direction.

The pseudo-spectral method employed to solve this system uses Fourier transforms in the horizontal planes and sinus/cosines transforms in the vertical direction. The non-linear term is computed by collocation in physical space. A low storage Runge-Kutta scheme of third order is used for the time integration [3]. The simulations have been done at maximal resolution of $N = 128^3$ and Reynolds number 448 based on the vorticity thickness δ_i, i.e. $Re = \frac{U\delta_i}{\nu}$. The mean stream-wise velocity profile employed as initial condition is $U = \tanh(\frac{2z}{\delta_i})$ where δ_i has been chosen so that the stream-wise length is $Xl = 4X_\lambda\delta_i$ with $X_\lambda = 7$. In this way four Kelvin-Helmholtz instabilities develop. After three dimensional instabilities become more important and stream-wise ribs appear.

Comparison between DNS and CVS

The beginning of the simulation is characterised by the transition to turbulence. In this phase the vortex stretching mechanism is negligible thus there is no production of enstrophy which actually weekly dissipates (see figure 2). The 2D Kelvin-Helmholtz instability starts to develop in the stream-wise direction, creating four rollers which are then deformed by span-wise 3D instabilities ($t = 9\frac{\delta_i}{U}$). In the regions where the rollers get close merging occurs. This corresponds to the mixing transition ($t = 18\frac{\delta_i}{U}$) after which the flow becomes turbulent. Hairpin vortices are generated and stretched by velocity gradients and the production of vorticity becomes important. This first pairing produces two turbulent rollers bigger than the original ones. Once created these two vortices start merging again with a mechanism similar to the previous one ($t = 25\frac{\delta_i}{U}$). Figure 1 shows the iso-surfaces of vorticity modulus of the DNS (left) and CVS (right) at two significative time steps ($t = 18\frac{\delta_i}{U}$ and $t = 25\frac{\delta_i}{U}$) .

Before performing quantitative analysis of the difference between DNS and CVS it is fruitful to look at the vorticity fields in physical space as presented in figure 1. Till the first pairing ($t = 18\frac{\delta_i}{U}$) no differences can be observed between DNS and CVS. The flow is not turbulent thus it is deterministically computed, i.e. the wavelets modes retained describe the flow completely. After this situation changes progressively, the CVS filtered flow appears slightly simpler than the one computed with DNS because the random fluctuations of the incoherent vorticity have been filtered out. Notwithstanding all the main structures of the vorticity are well described by both simulations.

These observations are confirmed by the time evolution of integral quantities like energy and enstrophy. Figure 2 shows that before the mixing transition both energy and enstrophy are the same for the CVS and the DNS. When the turbulent regime is attained ($t = 18\frac{\delta_i}{U}$) incoherent enstrophy, which corresponds to turbulent dissipation, is produced. CVS automatically dissipates this incoherent background thus the filtered flow contains less enstrophy than the DNS one. On the other hand the time evolution of energy of the CVS remains very close to the one of DNS. In figure 2(right) we present the number of wavelet modes retained during the simulation which varies between 8% and 15% N. This graphic shows

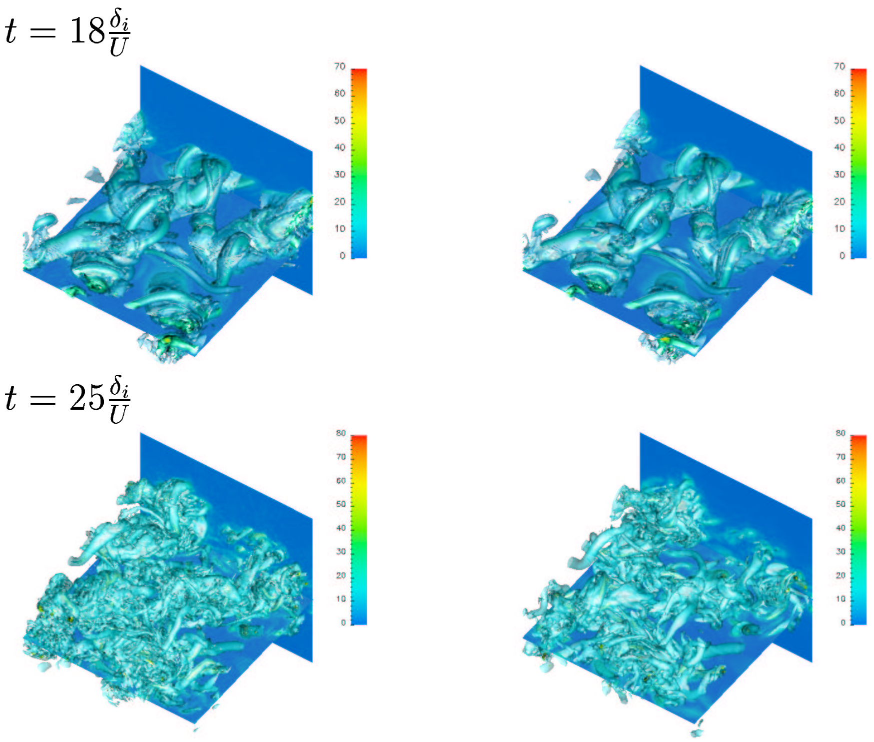

Figure 1: Iso-surfaces of vorticity modulus coloured by span-wise vorticity. DNS (left) and CVS (right).

that the wavelet approximation basis evolves in time, in particular when rolls pairing occurs more wavelet modes are needed, which corresponds to the two

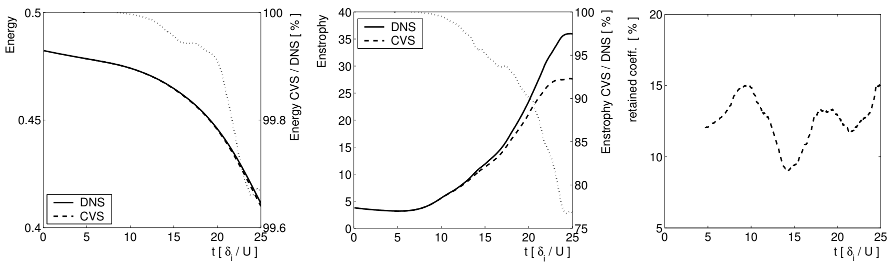

Figure 2: Time evolution of Energy (left) and Enstrophy (middle). Solid line: DNS, dashed line: CVS, dotted line: ratio CVS/DNS.
On the right: number of retained wavelet coefficients during the CVS.

peaks in figure 2(right). In figure 3 we analyse the mean velocity and velocity variance profile. For both we find a very good accordance of the CVS-filtered simulation with the reference DNS.

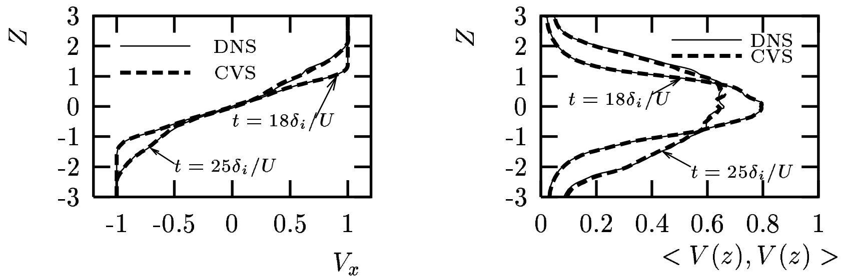

Figure 3: Mean velocity profile (left) and velocity variance profile(right).

Conclusions

- With a Coherent Vortex Simulation it is possible to simulate 3D turbulent flows like a mixing layer by using a reduced number of the wavelet modes.
- The main structures (vortices) developed by the flow are present in the CVS and are characterised by the same dynamical behaviour.
- In a CVS the turbulent dissipation is obtained by the filtering of the incoherent modes.
- CVS also preserves first and second order statistics: mean velocity profile and velocity variance profile are the same for DNS and CVS.

These results give some perspectives to compute turbulent flows at high Reynolds number with reduced computational cost by introducing some adaptive discretisation.

Acknowledgements: we thank Pierre Comte for providing the spectral code and fruitful discussions. We acknowledge financial support from the TMR project 'Wavelets in numerical simulation' contract N^o FMRX-CT98-0184.

References

[1] M. Farge, K. Schneider, N. Kevlahan *Phys. Fluids*, 11(8):2187–2201, 1999.

[2] M. Farge, G. Pellegrino, K. Schneider *Phys. Rev. Lett.*,87, 5, 30 July 2001.

[3] P. Comte, M. Lesieur, E. Lamballais *Phys. Fluids A*,4(12):2761-2778, 1992.

[4] K. Schneider, M. Farge, G. Pellegrino and M. Rogers *Turbulence and Shear Flow Phenomena, ed. E. Lindborg et al., KTH*, Vol. 1, 313-316, 2001.