Decaying 2D turbulence in a circular container

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We present high resolution direct numerical simulation of two-dimensional decaying turbulence in a circular geometry with no–slip boundary conditions. We show that starting with random initial conditions the flow rapidly exhibits a self–organization into coherent vortices. We study their formation and the role of the viscous boundary layer of the flow and on the decay of integral quantities.

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Two–dimensional turbulence in a bounded domain plays an important role in oceanographic applications, like in boundary currents and the related formation of vortices.

Many experiments in rotating tanks, e.g. in [1] resulting in quasi-twodimensional geostrophic flows have shown the formation of long–lived coherent vortices. On the other hand only few numerical studies of 2D turbulence in bounded circular domains have been performed so far. Some numerical simulations of decaying 2D turbulence in a circular domain with no–slip boundary conditions have been presented in [5–7]. They used a spectral method with Bessel functions of the first kind, i.e. circular analogues of the Chandrasekhar–Reid functions. Due to the numerical complexity of the full spectral scheme the simulations are limited to low resolution, i.e. $Re < 10^3$, based on the rms initial velocity and the circle radius.

The aim of present paper is to present is to present direct numerical simulation of two-dimensional decaying turbulence in a circular geometry at higher resolution, i.e. 1024^2 corresponding to an initial Re–number of 10^4. The numerical scheme is based on a Fourier pseudospectral method with semi-implicit time discretization and adaptive time-stepping. For details on the numerical scheme we refer to [8]. We solve the Navier-Stokes equations in vorticity–velocity formulation in a square domain and impose the no-slip boundary conditions on the wall of the circular container using a volume penalisation method [2]. The governing equation reads,

$$\partial_t \omega \, + \, \vec{u} \cdot \nabla \omega - \nu \, \nabla^2 \, \omega \, + \, \nabla \times (\frac{1}{\eta} \, \chi_{\Omega_s} \, \vec{u}) \; = \; 0$$

where $\vec{u}$ is the divergence-free velocity field, i.e. $\nabla \cdot \vec{u} = 0$, $\omega = \nabla \times \vec{u}$ the vorticity, ν the kinematic viscosity and $\chi_\Omega(\vec{x})$ the mask function which is 0 inside the cylinder Ω and 1 elsewhere.

Different invariants of the flow can be derived [**?**], i.e. quantities which are time–independent for vanishing viscosity:

- the circulation Γ (total vorticity) is defined as

$$\Gamma = \int_\Omega \omega d\vec{x} = \oint_{\partial\Omega} \vec{u} \cdot ds, \tag{1}$$

- energy E, enstrophy Z and palinstrophy P as

$$E = \frac{1}{2} \int_\Omega |\vec{u}|^2 d\vec{x} \; , \; Z = \frac{1}{2} \int_\Omega |\omega|^2 d\vec{x} \; , \; P = \frac{1}{2} \int_\Omega |\nabla\omega|^2 d\vec{x}, \tag{2}$$

  respectively.

- the energy dissipation is given by $d_t E = -2\nu Z$ and the enstrophy dissipation by

$$d_t Z = -\nu \int_\Omega |\nabla\omega|^2 d\vec{x} + \oint_{\partial\Omega} \omega(\vec{n} \cdot \nabla\omega) ds, \tag{3}$$

  The right hand term reflects the enstrophy production on the boundary.

- the angular mommentum of the flow with respect to the center of the circle is

$$L = 2 \int_\Omega \psi d\vec{x} \tag{4}$$

  where $\psi = \nabla^{-2}\omega$ denotes the stream–function.

As initial condition we choose a correlated Gaussian noise, with zero angular momentum. The corresponding initial Reynolds number based on the kinetic energy E and the cylinder diameter D is $Re = D\sqrt{2E}/\nu = 10^4$. The penalisation parameter η is choosen to be sufficiently small (10^{-3}) and the numerical resolution is 1024^2.

Fig. 1 shows snap shots of the vorticity field at $t = 1$ and 2. We observe the formation of vorticity sheets at

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the cylinder wall and the emergence of coherent vortices. The strong boundary layers persist throughout the simulation. The injection of vorticity and vorticity gradients into the flow leads to a substantial increase of the energy dissipation. We also observe the formation of dipolar vortices which move around and interact with the boundary.

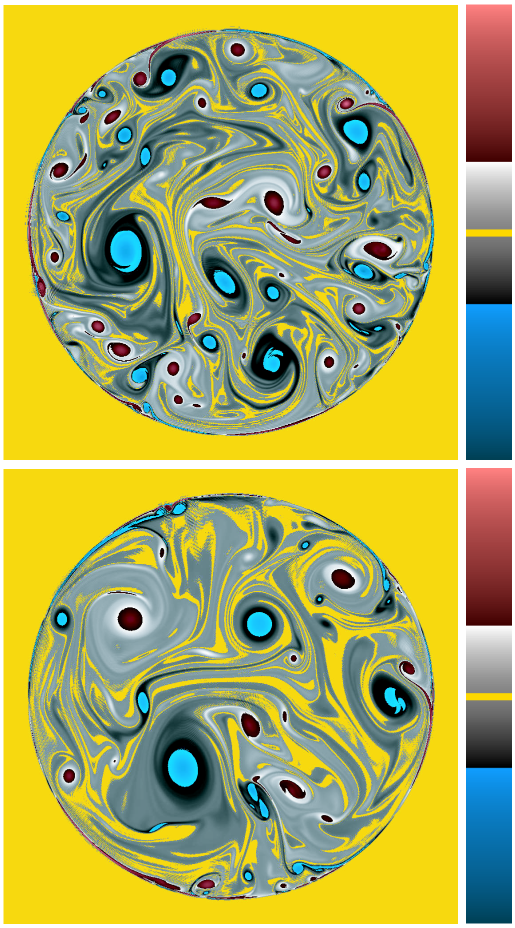

Figure 1: Vorticity fields at $t = 1, 2$.

In Fig. 2 (left) we plot the time evolution of the rms enstrophy ($Z = \frac{1}{2}\int \omega^2 d\vec{x}$) which exhibits a self-similar decay, proportional to $t^{-1/3}$. At later times, we also observe a non monotonic behaviour which is due the formation of vorticity at the no-slip walls. The time evolution of minimum and maximum vorticity is plotted in Fig. 2 (right), which also exhibits a non monotonic behaviour.

The circulation $\Gamma = \frac{1}{2}\int \omega d\vec{x}$ shown in Fig. 3 (left) oscillates randomly around zero. In Fig. 3 (right) we plot the time evolution of the angular momentum, $L = 2\int \Psi d\vec{x}$ where Ψ is the stream function, which confirms previous studies of decaying turbulence with no slip boundary conditions in square and circular domains [3, 6].

Fig 4 shows the pdfs of vorticity at different time instants. Starting with a Gaussian shape at $t = 0$ the

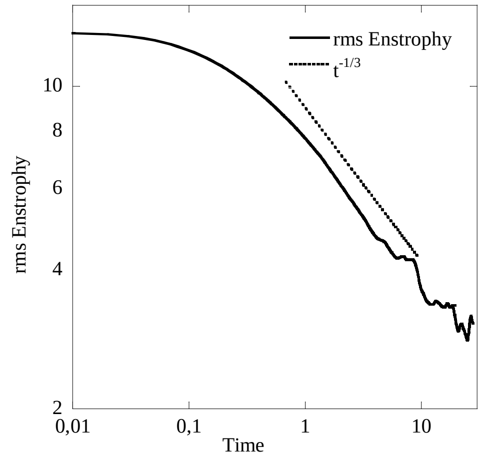

Figure 2: Time evolution of rms enstrophy Z.

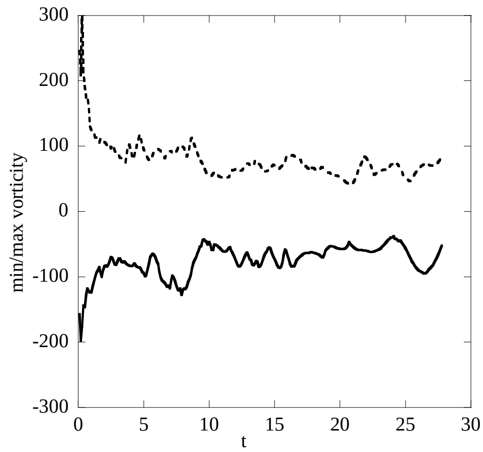

Figure 3: Time evolution min/max vorticity ω.

vorticity pdf becomes more and more non-Gaussian and exhibits heavy tails, which confirms the formation of vortex sheets and the emergence of coherent vorticies.

Discuss: Energy spectra? Decay of Energy and palinstrophy.

In conclusion, we have shown that

We have introduced a new numerical method to compute 2D Navier–Stokes turbulence within bounded domains. Discuss decay properties.

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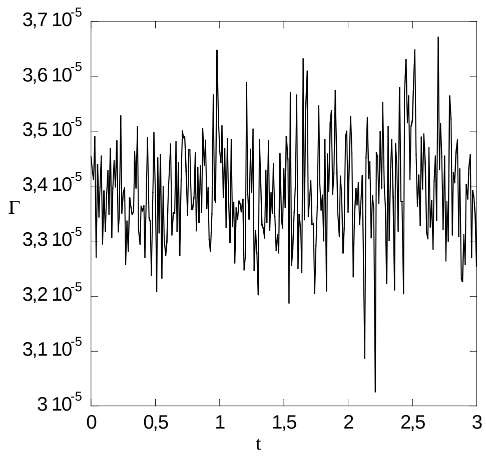

Figure 4: Time evolution of circulation Γ.

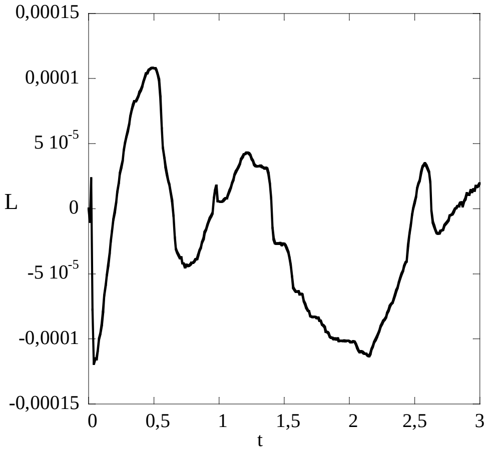

Figure 5: Time evolution of angular momentum L.

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