Extraction of coherent vortices using wavelets from a rotating shallow water flow in a cylindrical container

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Topic: C and I

We compute by direct numerical simulation (DNS) the time evolution of a shallow water flow in a cylindrical container which is rotating at a constant rate. We integrate the Saint-Venant's equations from a random distribution of the vorticity field, with the surface height computed to be in quasi-geostrophic balance at the initial time. The numerical scheme is based on a Fourier pseudospectral method with a leapfrog time scheme [1], and we impose the no-slip boundary conditions on the wall of the container by using a volume penalisation method [2].

At the final time, we consider the vorticity field and use a wavelet procedure [3] to extract the coherent vortices (Fig. 1). We found that only 2% of the wavelet modes correpond to the coherent vortices and contain 99% of the energy. We check a posteriori that we have extracted all coherent vortices by plotting the coherence function, which is estimated from the scatter plot between the vorticity and the streamfunction (Fig. 2). On this plot each vortex corresponds to an hyperbolic sine function, while the scatter plot between the remaining incoherent vorticity and the streamfunction does not show any functional relation between them, which proves that there are no more coherent vortices there.

Finally, we show that the coherent vortices have the same non-Gaussian probability distribution function (PDF) as the total field, while the remaining incoherent vorticity is Gaussian (Fig. 3). Moreover, we also found that the enstrophy spectrum of the coherent vortices is about the same as for the total flow, while the enstrophy spectrum for the incoherent contribution scales as k^+1 (k being the wavenumber), which corresponds to an enstrophy equipartition (Fig. 3). This proves that we have disentangled the intermittent contribution of the coherent vortices from the non-intermittent background flow.

References

[1] M. Farge and R. Sadourny. *J. Fluid Mech.* **206**, (1989) 433–462.

[2] P. Angot, C.H. Bruneau and P. Fabrie. *Numer. Math.* **81**, (1999) 497–520.

[3] M. Farge, K. Schneider and N. Kevlahan. *Phys. Fluids* **11**, (1999) 2187–2201.

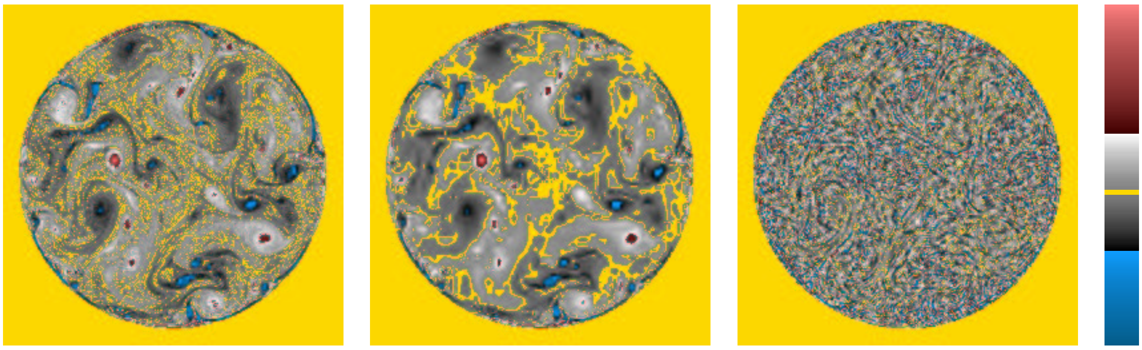

Figure 1: Total vorticity (min=-3.57 / max=4.46) (left), coherent vorticity (min=-3.64 / max=4.37) (middle) and incoherent vorticity (min=-1.25 / max=1.26) (right).

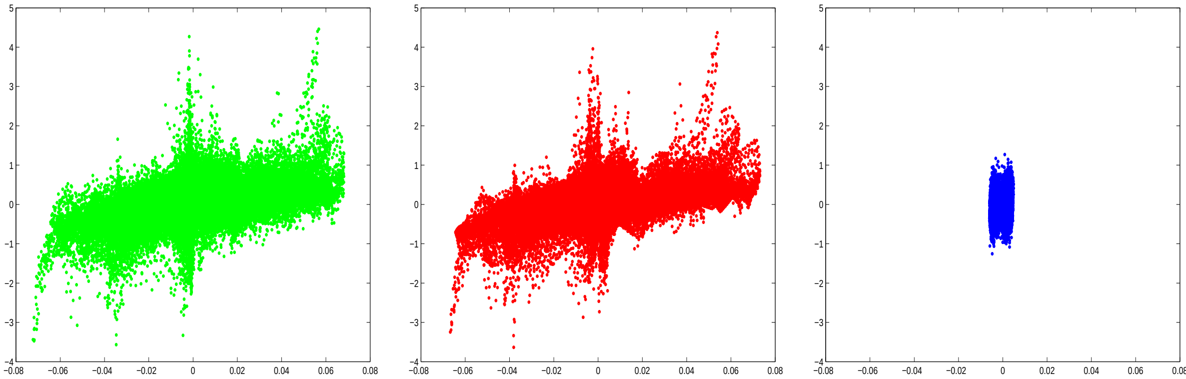

Figure 2: Coherence scatter–plot: vorticity versus stream function for the total (left), the coherent (middle) and the incoherent flows (right).

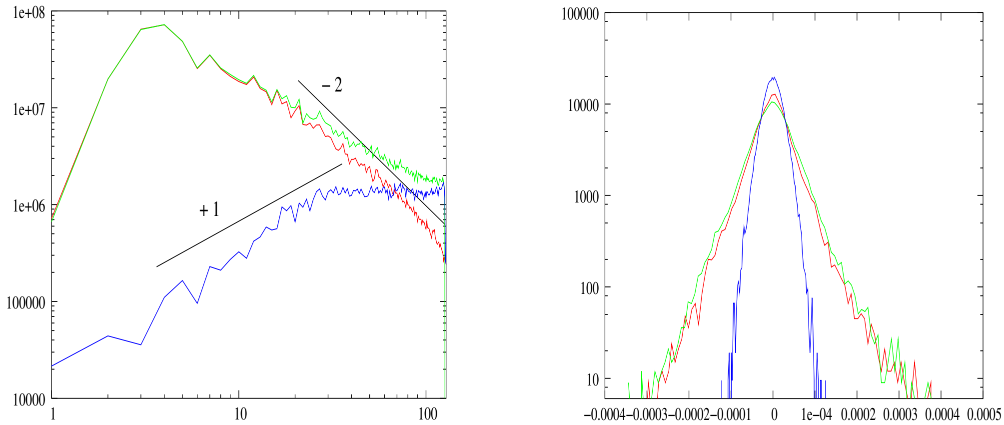

Figure 3: Enstrophy spectra and probability density functions (PDF) of the vorticity ω (total: green, coherent: red, incoherent: blue).