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Etude du transport turbulent dans les plasmas de bord par analyse et filtrage en ondelettes de signaux turbulents mesurés dans la SOL

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Introduction

Nous étudions la contribution des structures cohérentes au transport turbulent radial dans la SOL (scrape off layer) de plusieurs tokamaks, en particulier Tore-Supra à Cadarache et Castor à Prague). Nous avons développé une méthode d'extraction des structures cohérentes dans les écoulements turbulents qui utilise les propriétés de localisation, à la fois en espace ou en temps et en échelles, de la représentation en base d'ondelettes.

La méthode d'extraction que nous avons proposée repose sur deux choix originaux. Le premier consiste à représenter le signal ou le champ en ondelettes, plutôt que dans l'espace physique ou en modes de Fourier, comme on le fait classiquement en turbulence. Le second choix correspond à un changement de point de vue. Comme les chercheurs ne sont pas encore arrivés se mettre d'accord sur une définition précise et opérationnelle des structures cohérentes, nous avons proposé une définition minimale : 'les structures cohérentes ne sont pas du bruit'. Le problème de leur extraction se ramène ainsi à un problème de débruitage. Avec ce point de vue nous n'avons plus besoin des faire des hypothèses sur les structures elles-mêmes, mais seulement sur le bruit que nous cherchons à éliminer pour les extraire. Pour commencer nous avons choisi l'hypothèse la plus simple, à savoir nous supposons que le bruit a une distribution de probabilité Gaussienne et qu'il est décorrélé. Notre travail des années précédentes a consisté à mettre au point cet algorithme et à l'appliquer sur différents signaux turbulents mesurés dans des tokamaks, en particulier sur Tore-Supra.

Cette année fut principalement consacrée à la rédaction d'un article intitulé *'Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets'* dont les auteurs sont : *Marie Farge, Kai Schneider et Pascal Devynck*. Nous l'avons soumis le 8 Mai 2005 au journal *'Physics of Plasmas'*. Le rapport de referee nous a été retourné en Août. Nous avons renvoyé une version révisée le 1er Octobre accompagnée d'un rapport de 8 pages précisant de nombreux points que le referee n'avait pas compris concernant les ondelettes et la notion d'intermittence. L'article a finalement été accepté le 16 Janvier 2006 et a été publié en ligne le 18 Avril 2006. Sa référence est : Physics of Plasmas, 13, 042304 (2006).

Lors de l'année 2005 nous avons participé au programme de l'IPAM (Institute for Pure and Applied Mathematics) à Los Angeles sur *'Multiscale Processes in Fusion Plasmas'*, organisé par Steven Cowley de UCLA du 10 au 14 Janvier. Nous avons fait deux exposés qui sont disponibles sur le site Web http:// www.ipam.ucla.edu/. Nous avons également présenté nos résultats lors de trois conférences internationales:

- Workshop sur *'Multi-scale Interactions in Turbulent Flows'*, organisé par le *'Center for Nonlinear Studies'* du Los Alamos National Laboratories, qui a eu lieu à Santa Fe (USA) du 18 to 21 Juillet 2005.
- 11th European Fusion Theory Conference, Aix-en-Provence, 26-28 Septembre 2005.
- 47th Annual Meeting of the *'Division of Plasma Physics'* of the American Physical Society (APS), qui a eu lieu à Denver (USA) du 24 au 28 Octobre 2005.

Dans les pages suivantes nous présentons les différentes versions de l'article, ainsi que les courriers échangés avec l'éditeur de Physics of Plasmas et le referee.

Correspondance échangée avec l'éditeur de Physics of Plasmas depuis la soumission de l'article le 8 Mai 2005 jusqu'à son acceptation le 2 Janvier 2006

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8 Mai 2005
Soumission de l'article

Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets

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Abstract

A wavelet-based method to extract coherent bursts out of turbulent signals is presented. The signal is projected onto an orthogonal wavelet basis, a threshold is applied to the wavelet coefficients, and the denoised signal is reconstructed in physical space. The threshold value is recursively determined and thus no adjustable parameters are required. The signal is split into two orthogonal components, a coherent and an incoherent one, whose properties can be studied independently. Statistical diagnostics based on the wavelet representation are introduced to compare the scaling behaviour and intermittency of the total signal and its coherent and incoherent components. The extraction method is applied to ion density of edge plasma measured in the scrape-off layer of the tokamak Tore Supra, Cadarache, France. We show that this procedure disentagles the coherent bursts, which contain most of the density variance, are intermittent and long–time correlated with non–Gaussian statistics, from an incoherent background noise, which is much weaker, non-intermittent and almost decorrelated with quasi–Gaussian statistics.

I. INTRODUCTION

A. Coherent bursts

The radial transport at the edge of magnetic fusion devices is known to be dominated by turbulent processes. Understanding them is important, as they will determine the confinement properties of the overall plasma, in the bulk region, and the energy density that must be handled by the limiter or divertor components, in the shadowed region of the plasma where the magnetic field lines are opened. The turbulent transport of ion density has been extensively studied at the edge of plasma by means of Langmuir probes [5, 11, 25], particles beams [21, 22] and more recently 2D visible imaging [28, 29]. All these diagnostics observe a turbulent transport of the ion density in the scrape-off layer (SOL) that can be described as a superposition of convective events, which are responsible for the transport of matter over long radial distances at a fraction (of the order of 10%) of the ion sound speed [1, 6], and of background turbulence.

The convective events are detected as coherent bursts of density, but with a signature different from the one expected for turbulent eddies, since they exhibit a probability distribution function (PDF) which is skewed. Typically, it is found that these convective events account for a small fraction of the time (20%), but transport up to 50 % of the density variance [2], which underlines their importance in the turbulent transport. There is an effort to analyse these bursts independently from the background turbulence. For this purpose different extraction methods have been developed, which are based on signal clipping or conditional averaging. These methods require strong hypotheses on the signal, which has to be statistically steady, and also on the bursts in order to choose the appropriate threshold value. Although they give information about the dynamics [2, 7], other methods to extract the bursts are still needed which require less hypotheses. Moreover, the clipping method does not preserve the smoothness of the signal, since the threshold introduces discontinuities which affect the Fourier spectrum and hence may yield a erroneous scaling.

Since 1988 we have proposed to use the wavelet representation to analyze [12, 13] and extract [16, 17, 20] coherent structures out of turbulent flow fields, as the wavelet representation does not require any hypothesis on the statistical stationarity and homogeneity of the process under study. In this paper we demonstrate the advantages of wavelets to extract coherent bursts in edge plasma. We present the algorithm and apply it to signals of ion density fluctuations measured in the SOL of the tokamak Tore–Supra, Cadarache, France [8].

B. Wavelet representation

The wavelet transform is more appropriate than the Fourier transform to analyze and represent non stationary, non homogeneous and intermittent signals, such as those encountered in turbulence. It uses analyzing functions which are generated by translation and dilation of a so-called 'mother wavelet', which is well localized in both physical and spectral space. In contrast, the Fourier transform uses trigonometric functions, non local in physical space but well localized in spectral space, and the analyzing functions are generated by modulation rather than dilation. The spatial localization of the basis functions and the invariance group of the transform constitute the main differences between wavelet and Fourier representations. For a general presentation of the different types of wavelet transforms and their applications to turbulence, we refer the reader to several review articles [14, 15, 18].

The trigonometric functions used by the Fourier transform oscillate for all time and therefore the information content of the transformed signal is delocalized between all Fourier coefficients. In contrast, the wavelet coefficients preserve the local properties of the signal. Thus, when a wavelet coefficient is filtered out, the effect on the reconstructed signal remains local and therefore does not affect the whole signal, as the Fourier transform does. This property allows to study the behaviour of a limited portion of the signal directly from its wavelet coefficients. It is legitimate to use the Fourier transform when the signal is stationary (in time, or homogeneous in space) and made up of a superposition of waves. Only in this case it is possible to define without ambiguity the associated frequencies. However, if we assume that a turbulent signal is a superposition of elementary structures localized in space and time, and mutually interacting (*e.g.*, vortices), the wavelet representation should be prefered, since it preserves the locality of information in both scale and space. Actually, these two different transforms translate into mathematical language two different interpretations of turbulent signals [14].

In the context of plasma physics the continuous wavelet transform has already been used to analyze signals measured in fusion devices, see *e.g.* [10, 24]. In this paper we propose to use the orthogonal wavelet transform instead, since it is optimal for denoising signals corrupted with additive Gaussian white noise. Donoho and Johnstone [9] have shown that nonlinear thresholding of the wavelet coefficients yields min-max estimators for a large class of functional spaces, in particular for intermittent signals. A generalisation to denoise power spectra where the noise exhibits a non Gaussian behaviour, *i.e.*, χ^2 distribution, can be found in [26]. To improve the choice of the threshold we have proposed a recursive algorithm [3], and we applied it to extract coherent structures out of incompressible turbulent flows [16]. In the present paper we demonstrate its use to study turbulence in the edge plasma of nuclear fusion devices.

The paper is organized as follows. First, we present a recursive wavelet algorithm which allows a separation of a signal into coherent events and a Gaussian background noise. The properties of this algoritm are illustrated by first validating it on an academic signal. Then, we apply it to an ion density signal measured in the SOL of the tokamak Tore Supra. We show that the coherent bursts can thus be efficiently extracted. We then present new statistical diagnostics based on the wavelet representation and use them to compare the scaling behaviour and intermittency of the total signal and its coherent and incoherent components. Finally, some conclusions drawn and perspectives for future work are given.

II. NONLINEAR WAVELET THRESHOLDING

A. Discrete wavelet representation

We use an orthogonal wavelet basis, which constitutes a Multi-Resolution Analysis (MRA) of $L^2(\mathbb{R})$, the space of square integral functions. It represents the time signal $S(t)$, sampled on $N = 2^J$ instants, at different scales $l = 2^{-j}$, for $j = 0$ to $J - 1$ [14, 23]. The signal $S(t)$ is thus developed into a discrete wavelet series,

$$S(t) = \overline{S}_0 \phi_0(t) + \sum_{(j,i)\in\Lambda_J} \widetilde{S}_{ji}\, \psi_{ji}(t) \qquad (1)$$

where ϕ_0 is the scaling function and ψ_{ji} the corresponding wavelets, while j denotes the scale and i the position of each wavelet. The index set Λ_J of the wavelet basis is

$$\Lambda_J = \{(j,i)\,, j = 0, ..., J-1, i = 0, ..., 2^j - 1\}\,. \qquad (2)$$

Due to orthogonality, the scaling coefficients are given by $\overline{S}_0 = \langle S \,,\, \phi_0 \rangle$ and the wavelet coefficients by $\widetilde{S}_{ji} = \langle S \,,\, \psi_{ji} \rangle$, where $\langle \cdot, \cdot \rangle$ denotes the L^2-inner product.

In the following section we use Coifman 12 wavelets which have 4 vanishing moments, *i.e.*, $\int_{I\!R} t^m \, \psi(t) dt = 0$ for $m = 0, ..., 3$. The wavelet $\psi(t)$ and corresponding scaling function $\phi(t)$, together with the modulus of their Fourier transforms $|\widehat{\phi}(\omega)|$ and $|\widehat{\psi}(\omega)|$, are shown in Fig. 1, where the Fourier transform is defined as

$$\widehat{\phi}(\omega) \;=\; \int_{I\!R} \phi(t) \, e^{-i2\pi\omega t} \, dt \qquad (3)$$

B. Selection of the optimal threshold

We consider a signal $S(t)$ sampled on an equidistant grid of size $N = 2^J$. We propose a wavelet based method to separate coherent components $S^C(t)$ from incoherent components $S^I(t)$. This method relies on min–max properties of orthogonal wavelet bases, used to estimate signals from noisy data, which guarantees an optimal estimator of the mean square error for intermittent signals [9]. The value of this threshold, as proposed by Donoho and Johnstone [9], is

$$\epsilon_D = (2 \ln N \sigma^2)^{1/2} \;, \qquad (4)$$

where σ^2 denotes the noise's variance and N the number of samples. This choice is based on theorems [9] proving optimality of wavelet thresholding to denoise signals in presence of Gaussian white noise with variance σ^2, since this wavelet based estimator minimizes the maximal L^2-error for functions with inhomogeneous regularity, *e.g.*, intermittent signals. However, to be able to determine the threshold, the variance of the noise has to be known *a priori*, or to be estimated.

In [3, 16] we have proposed a recursive algorithm to estimate the variance of the noise and extract coherent events. It is based on the conjecture that, given a threshold ϵ_n, the variance of the noise estimated by

$$\sigma_n^2 = \frac{1}{N} \sum_{(j,i) \in \Lambda^J, |\widetilde{S}_{ji}| < \epsilon_n} |\widetilde{S}_j i|^2 \qquad (5)$$

yields a threshold ϵ_{n+1} closer to ϵ_D than ϵ_n. This approach does not require any a priori knowledge of the noise's variance. In [3] we studied the mathematical properties of this algorithm and proved its convergence for signals having sufficiently sparse representation in wavelet space.

The recursive extraction algorithm can be summarized as follows:

Initialization

- given the signal $S(t)$ of duration T, sampled on an equidistant grid $t_i = iT/N$ for $i = 0, N-1$, with $N = 2^J$,
- set $n = 0$ and perform a wavelet decomposition, *i.e.*, apply the Fast Wavelet Transform [23] to S to obtain the wavelet coefficients $\widetilde{S}_{ji}$ for $(j,i) \in \Lambda_J$,
- compute the variance σ_0^2 of S as a rough estimate of the variance of the incoherent signal S^I and compute the corresponding threshold $\epsilon_0 = (2 \ln N\sigma_0^2)^{1/2}$, where $\sigma_0^2 = \frac{1}{N}\sum_{(j,i)\in\Lambda^J} |\widetilde{S}_{ji}|^2$,
- set the number of coefficients considered as noise to $N_I = Card(\Lambda^J) = N$.

Main loop

Repeat

- set $N_I^{old} = N_I$ and count the wavelet coefficients smaller than ϵ_n, which yields N_I
- compute the new variance σ_{n+1}^2 from the wavelet coefficiens smaller than ϵ_n, *i.e.*, $\sigma_{n+1}^2 = \frac{1}{N}\sum_{(j,i)\in\Lambda^J} |\widetilde{S}_{ji}^I|^2$, where

$$\widetilde{S}_{ji}^I = \begin{cases} \widetilde{S}_{ji} & \text{for } |\widetilde{S}_{ji}| \leq \epsilon_n \\ 0 & \text{else,} \end{cases} \qquad (6)$$

  and the new threshold $\epsilon_{n+1} = (2 \ln N\sigma_{n+1}^2)^{1/2}$,

- set n=n+1

until (N_I==N_I^{old}).

Final step

- compute the coherent signal S^C from the coefficients $\widetilde{S}^C_{ji}$ using the inverse Fast Wavelet Transform, where

$$\widetilde{S}^C_{ji} = \begin{cases} \widetilde{S}_{ji} & \text{for } |\widetilde{S}_{ji}| > \epsilon_n \\ 0 & \text{else} \end{cases} \tag{7}$$

- finally, compute pointwise $S^I(t_i) = S(t_i) - S^C(t_i)$ for $i = 0, ..., N-1$.

End

The indices C and I stand for coherent and incoherent, respectively. Note that the decomposition yields $S = S^C + S^I$ and, due to orthogonality, $\langle S^C, S^I \rangle = 0$, which implies that $\sigma^2 = \sigma^2_C + \sigma^2_I$.

The Fast Wavelet Transform (FWT), proposed by Mallat [23], has a complexity of $O(N)$, where N denotes the total number of grid points. Hence, the complexity of the above extraction algorithm is of $O(nN)$, with a number of iterations n typically smaller than $\log_2 N$.

This algorithm defines a sequence of estimated thresholds $(\epsilon_n)_{n \in \mathbb{N}}$ and the corresponding sequence of estimated variances $(\sigma^2_n)_{n \in \mathbb{N}}$. The convergence of these sequences within a finite number of iterations has been demonstrated in [3] applying a fixed point type argument to the iteration function

$$\mathcal{I}_{S,N}(\epsilon_{n+1}) = \left(\frac{2 \ln N}{N} \sum_{(j,i) \in \Lambda^J} |\widetilde{S}^I_{ji}(\epsilon_n)|^2 \right)^{1/2} . \tag{8}$$

Furthermore, we have shown that the convergence rate of the recursive algorithm depends on the signal to noise ratio (SNR), since the smaller the SNR, *i.e.*, the stronger the noise, the faster the convergence. Moreover, if the algorithm is applied to a Gaussian white noise only, it converges in one iteration and removes the noise (in statistical mean). If it is applied to a signal only, the signal is preserved. Finally, we have proven that the algorithm is idempotent, *i.e.*, if it is applied several times, the noise is eliminated the first time, but the coherent signal is not modified anymore in the subsequent applications, as it would have been the case for a Gaussian filter. As a consequence, this algorithm yields a nonlinear projector [3].

C. Application to an academic test signal

To illustrate the properties of the recursive algorithm we apply it to a one-dimensional noisy test signal S (Fig. 2, middle). This signal has been constructed by superposing a Gaussian white noise W, with zero mean and variance $\sigma_W^2 = 1$, to a function F, normalized such that $\left(\frac{1}{N}\sum_i |F_i|^2\right)^{1/2} = 10$, which corresponds to a signal to noise ratio $SNR = 10\log_{10}(\sigma_F^2/\sigma_W^2) = 20\,dB$ (Fig. 2, top). The function F is a piecewise polynomial function which presents several discontinuities, either in the function or in its derivatives. The number of samples is $N = 2^{13} = 8192$.

We apply the recursive extraction algoritm to the test signal S and obtain after $n = 5$ iterations the coherent part S^C and the incoherent noise S^I (cf. Fig. 2, bottom). We observe that S^C yields a denoised version of the test signal S which is very close to F, while the incoherent part S^I is homogeneous and noise like with flatness $\mathcal{F} = 3.03$, which corresponds to quasi–Gaussianity. Note that the flatness $\mathcal{F}$ is defined as the ratio of the centered fourth order moment divided by the square of the variance, and $\mathcal{F} = 3$ for a Gaussian process. Fig. 2 (bottom, left) shows that the coherent signal retains all discontinuities and spikes present in the original function F, without smoothing them as it would have been the case with standard denoising methods, *e.g.*, with low pass Fourier filtering. Nevertheless, we observe slight overshoots in the vicinity of the discontinuities, although they remain much more local than the classical Gibbs phenomena, and could anyway be removed using the translation invariant wavelet transform [23].

III. APPLICATION TO SIGNALS FROM TORE–SUPRA

We study the signal ts28338.dat, that we denote S, of length $N = 2^{13} = 8192$ which corresponds to ion density fluctuations measured in the SOL of tokamak Tore Supra, Cadarache.

The plasma scenario is the following: the shot lasts $18\,s$ and the signal is recorded in the middle of the plasma current plateau. The large radius is $R = 2.33\,m$, the small radius $a = 0.77\,m$, the mean ion density $\overline{n_i} = 1.37 \cdot 10^{19}\,m^{-3}$, the plasma current $I_p = 0.84\,MA$ and the edge safety factor $q = 6.71$. Moreover, $2.1\,MW$ of lower hybrid waves are applied to the plasma. The ion density fluctuations are measured by a fast reciprocating Langmuir probe. The total duration of the probe motion into the plasma is $300\,ms$ and, when the probe reaches $2.8\,cm$ away from the last closed flux surface (LCFS), the signal is recorded at $1\,MHz$ during a $8\,ms$ time window (Fig. 3).

We use the wavelet extraction algorithm to split the signal S (Fig. 4, top) into two orthogonal components, the coherent bursts S^C (Fig. 4, middle) and the incoherent noise S^I (Fig. 4, bottom). The optimal threshold value has been obtained after $n = 12$ iterations of the algorithm (Figure 5). As results, we observe that the coherent signal S^C, made of $5.8\%N$ wavelet coefficients, retains 86.6% of the total variance and the extrema are preserved (Table I). In contrast, the incoherent part, it is made of $94.2\%N$ wavelet coefficients but contributes to only 3.4% of the total variance (Table I), which corresponds to a signal to noise ratio $SNR = 10\log_{10}(\sigma^2/\sigma_I^2) = 8.72\,dB$.

The decomposition shows that the bursty and convective part of the signal dominates over the background turbulence, more strongly than what has been found with previous methods [2]. This can be explained by the position of the probe, which is about $3\,cm$ away from the LCFS. At this radial position, the signal is believed to be composed mostly of fast convective events, that have travelled across the SOL, superposed to a weak turbulent background density.

Fig. 6 shows the PDFs in log–lin coordinates for the total, coherent and incoherent contributions, estimated using histograms with 50 bins and integrals normalized to one. The PDFs of the total signal and the coherent part are skewed and present the same behaviour: positive values have exponential tails with $p(S) \propto \exp(\frac{-5}{2}S)$, while negative values yield a Gaussian behaviour (Fig. 6). In contrast, the PDF of the incoherent component is almost symmetric, with skewness 0.38, instead of 2.56 and 2.84 for the total and coherent part, respectively. It has a quasi–Gaussian shape with flatness 4.03, instead of 12.00 and 14.22 (Fig. 6).

To get more information on the spectral distribution of the density variance for the different components, we consider the Fourier spectrum

$$E(\omega) = \frac{1}{2}\left|\widehat{S}(\omega)\right|^2 , \tag{9}$$

where $\widehat{S}(\omega)$ denotes the Fourier transform defined in equation (3). As estimator for the spectrum we take the periodogram, which is a discrete version of equation (9), although it is known to be a non consistent estimator due to the presence of oscillations. To obtain a consistent estimator we also compute the modified periodogram, by first tapering the data with a raised cosine window (affecting 40 data points at each boundary), and then convolving the periodogram with a Gaussian window (with standard deviation of 40 data points). Figure 7 shows the periodogram and the modified periodogram for S, S^C and S^I, which confirms that the latter yields a stabilized estimator of the spectrum, with no more oscillations. The wavelet decomposition, given in equation (1), yields the distribution of the variance of the signal scale per scale, which is called scalogram [14]. It is defined as

$$\widetilde{E}_j = \frac{1}{2} \sum_{i=0}^{2^j-1} \left(\widetilde{S}_{ji}\right)^2 \ . \tag{10}$$

Parseval's relation implies that $E = \sum_{j \geq 0} \widetilde{E}_j$. Using the relation $\omega_j = \frac{\omega_\psi}{2^j}$ between the scale index j and the frequency ω, where ω_ψ is the centroid frequency of the mother wavelet, the wavelet spectrum can be defined as $\widetilde{E}(\omega_j) = \widetilde{E}_j/\omega_\psi$. It corresponds to a smoothed version of the Fourier spectrum (9), with the modulus square of the Fourier transform of the wavelet as smoothing kernel, since

$$\widetilde{E}(\omega) = \frac{1}{\omega_\psi} \int_0^{+\infty} E(\omega') \left| \widehat{\psi} \left(\frac{\omega_\psi \omega'}{\omega} \right) \right|^2 d\omega' \ . \tag{11}$$

Note that, as the frequency increases, the smoothing interval becomes larger which explains why the wavelet spectrum is a well-conditionned statistical estimator. The advantage of the wavelet spectrum in comparison to the modified periodogram is that the smoothing window is automatically adjusted by the wavelet representation, which corresponds to filters with constant relative bandwidth [14].

In Fig. 8 the wavelet spectra together with the modified periodograms are displayed. We observe that the signal and its coherent component present a similar scaling in $\omega^{-5/3}$, which characterizes long–time correlation. As proposed in [2], this may be interpretated as an inverse energy cascade, similar to what is encountered in two-dimensional fluid turbulence. In contrast, the incoherent component has a flat spectrum up to frequency $\omega = 100\,kHz$, which corresponds to decorrelation. For higher frequencies we observe a ω^{-1} scaling, which may be due to experimental noise. We also ascertain that the wavelet spectrum almost coincides with the modified periodogram, and that the higher the frequency the better the stabilization thus obtained. Note that the scalogram and the wavelet spectrum are optimal to characterize scaling laws, as long as the analyzing wavelet is has at least M vanishing moments, with $M > \frac{\beta-1}{2}$, to detect power laws in $\omega^{-\beta}$, see *e.g* [18, 27].

The intermittency of the signal can be quantified using higher order moments of the wavelet coefficients $\widetilde{S}_{ji}$, see *e.g.* [18, 27]. The p-th order moments are obtained by summing up the p-th power of the wavelet coefficients over all positions i

$$\widetilde{\mathcal{M}}_j^p = \frac{1}{2^j} \sum_{i=0}^{2^j-1} \left(\widetilde{S}_{ji}\right)^p . \quad (12)$$

The scale dependent flatness is defined as

$$\widetilde{\mathcal{F}}_j = \frac{\widetilde{\mathcal{M}}_j^4}{\left(\widetilde{\mathcal{M}}_j^2\right)^2} . \quad (13)$$

Similarly to the wavelet spectrum the relation between scale and frequency allows to express the flatness as function of the frequency ω_j. Note that a Gaussian white noise, which is by definition non–intermittent, would yield a flatness equal to three for all frequencies.

To characterize the intermittency of the different contributions we plot in Fig. 9 the flatness $\widetilde{\mathcal{F}}_j$ versus the frequency ω_j. We observe that the flatness of the coherent part increases faster for high frequencies than the one of the total signal, which proves that the coherent contribution is more intermittent than the signal itself. In contrast, the flatness of the incoherent part decreases, up to frequency $\omega = 100\,kHz$, to the value $\mathcal{F} = 3$, which suggests a tendency towards non–intermittent behaviour. For higher frequencies we cannot interpretate the results since the experimental noise seems to dominate, as already observed for the spectrum (Fig. 7, bottom).

IV. CONCLUSION

We presented a wavelet-based recursive method to extract coherent bursts out of turbulent signals. The algorithm decomposes the signal into an orthogonal wavelet basis and reconstructs the incoherent contribution from the wavelet coefficients whose modulus is larger than a given threshold. The threshold value is recursively determined without adjusting any parameter. Moreover, we have shown that this algorithm is fast since it has only linear complexity.

Compared to classical extraction methods, which are based either on thresholding in physical space (*i.e.* in grid point representation), or on conditional averaging, the advantages of working in wavelet space are the following:

- there is no need to suppose the signal be statistically stationary in time (or homogeneous in space),
- the wavelet decomposition preserves the spectral properties of the signal, and thus respects its scaling as long as the analyzing wavelet is smooth enough (which depends on the number of vanishing moments for orthogonal wavelets),
- the wavelet-based extraction method does not require any prior about the shape or the intensity of the bursts to be extracted; the only prior is to assume the noise be Gaussian and white.

We have applied this recursive wavelet algorithm to an ion density signal measured in the SOL of the tokamak Tore Supra. We have thus extracted the coherent bursts from an incoherent background noise. The former contain most of the density variance and are long-time correlated with non-Gaussian statistics, while the latter is almost decorrelated and quasi-Gaussian. We have also observed that the non-Gaussianity of the PDF of the coherent component increases with the frequency, which confirms that the bursts are highly intermittent. In contrast, the incoherent component remains quasi-Gaussian up to high frequencies, which confirms the non intermittency of the background noise. By analogy with previous studies we have made in the context of two-dimensional fluid turbulence [4], we conjecture that the coherent bursts are responsible for the turbulent transport, while the incoherent background flow only contributes to turbulent diffusion.

In [19] we applied this extraction method to both velocity and density signals, measured at different poloidal positions, to study turbulent fluxes and thus characterize the transport properties of the coherent bursts. These results will be subject of a forthcoming paper. We also have already extended this extraction method to treat two and three dimensional, scalar and vector, fields [16, 17, 20], and we plan to apply it to spatio-temporal signals and images of ion density, obtained by fast framing cameras, to improve our characterization of the coherent bursts.

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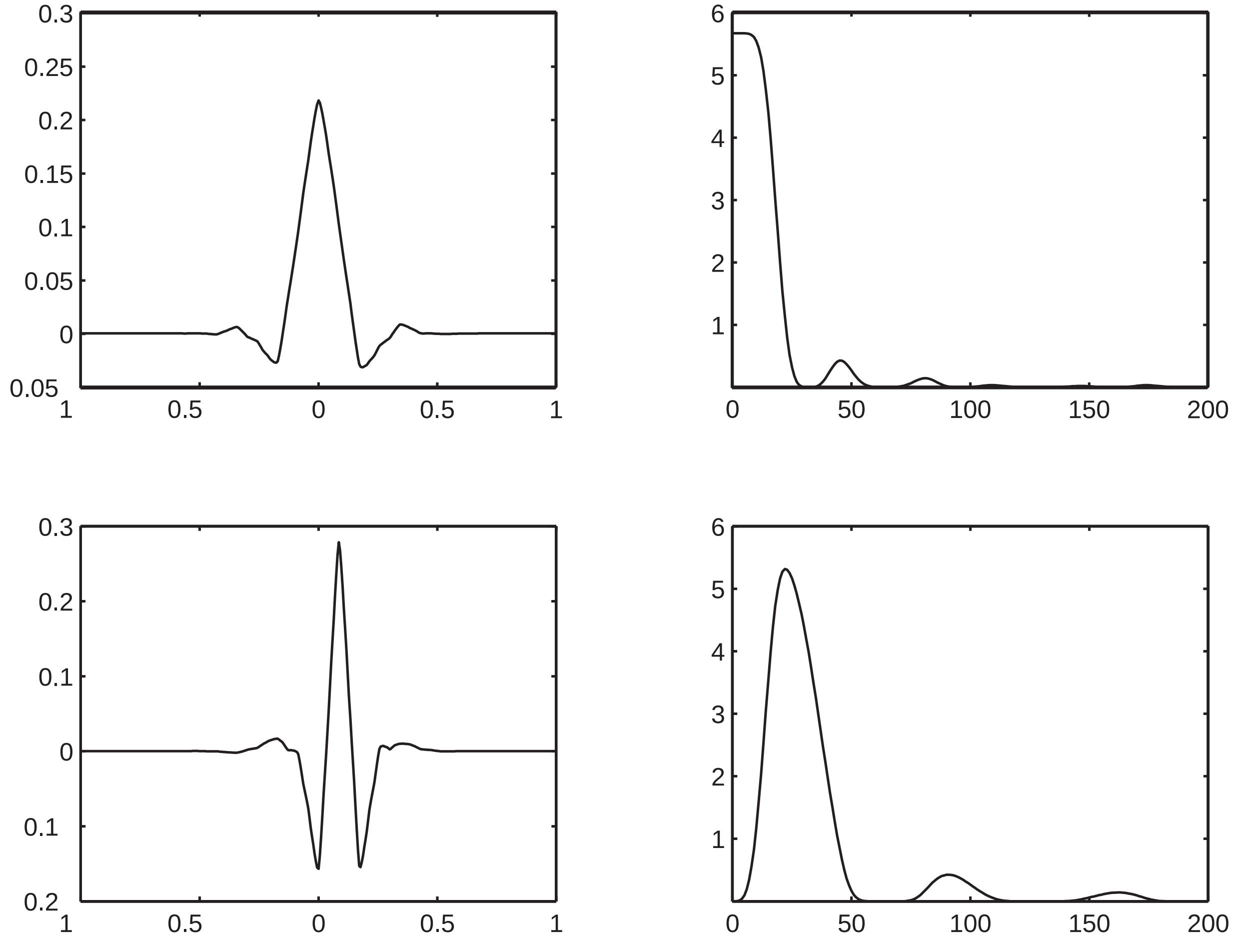

FIG. 1: Coifman 12 wavelet. Top: scaling function $\phi(t)$ and the modulus of its Fourier transform $|\widehat{\phi}(\omega)|$. Bottom: wavelet $\psi(t)$ and the modulus of its Fourier transform $|\widehat{\psi}(\omega)|$.

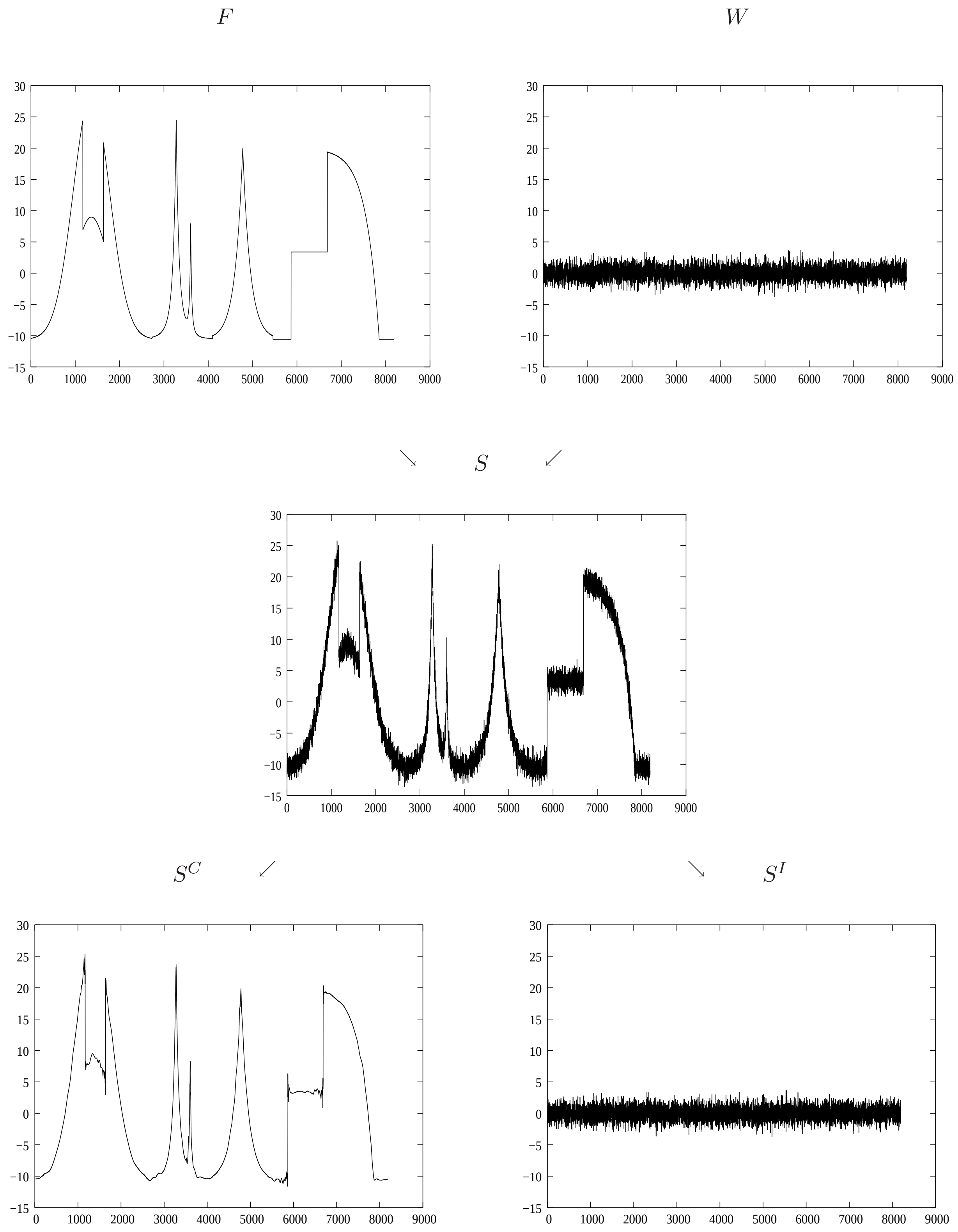

FIG. 2: Construction (top) of a 1D noisy signal $S = F + W$ (middle), and results obtained by the recursive algorithm (bottom), which gives $S = S^C + S^I$.

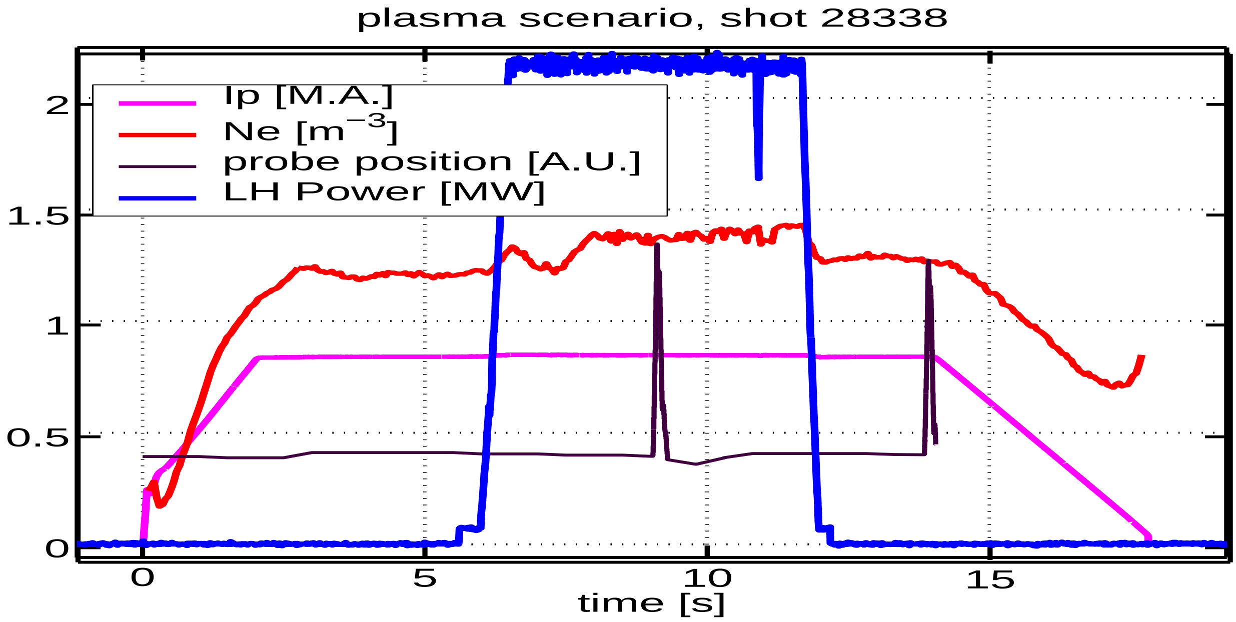

FIG. 3: Plasma scenario of the shot ts28338 from the tokamak Tore Supra, Cadarache. The duration of the shot is $18\,s$. The ion density fluctuations are measured by a fast reciprocating Langmuir probe. When the probe is $2.8\,cm$ away from the LCFS in the SOL, the signal is acquired during time windows of $8\,ms$.

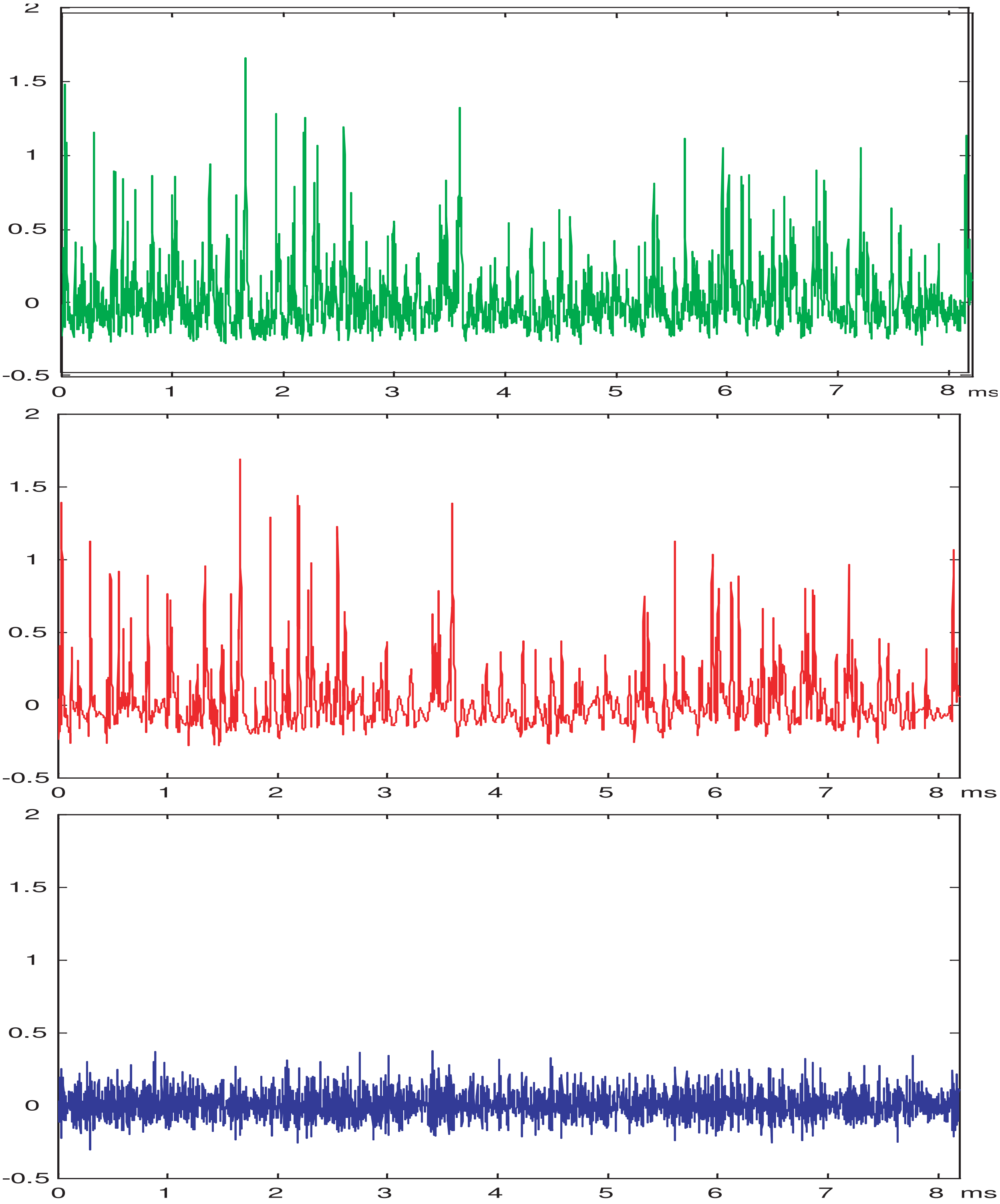

FIG. 4: Signal ts28338 of duration 8.192 ms, corresponding to ion density fluctuations measured at 1 MHz in the SOL of the tokamak Tore Supra. Top: total signal S. Middle: coherent part S^C. Bottom: incoherent part S^I.

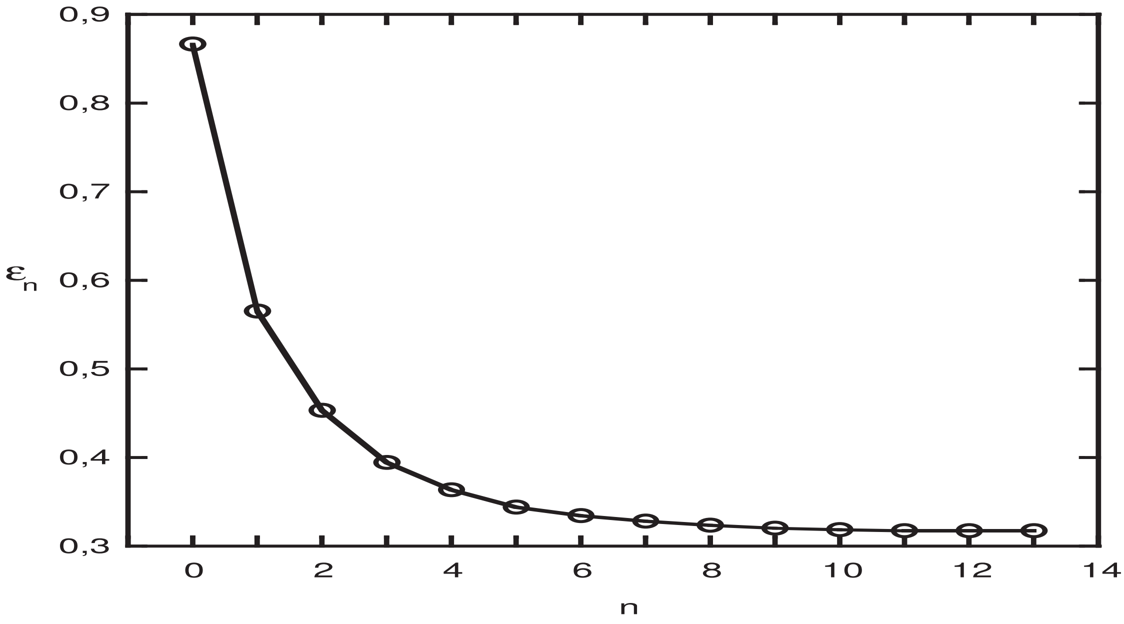

FIG. 5: Threshold value ϵ_n versus iteration number n.

TABLE I: Statistical properties of the signal ts28338 from the tokamak Tore Supra, Cadarache, for the total, coherent and incoherent components using the Coifman 12 orthogonal wavelet.

Signal	total S	coherent S^C	incoherent S^I
# of coefficients	8192	479	7713
% of coefficients	100 %	5.8 %	94.2 %
min value	-0.284	-0.282	-0.307
max value	1.689	1.686	0.374
mean value	0.019	0.019	$< 10^{-11}$
Variance σ^2	0.0417	0.0361	0.0056
% of variance	100 %	86.6 %	3.4 %
Skewness	2.564	2.842	0.383
Flatness	12.001	14.224	4.026

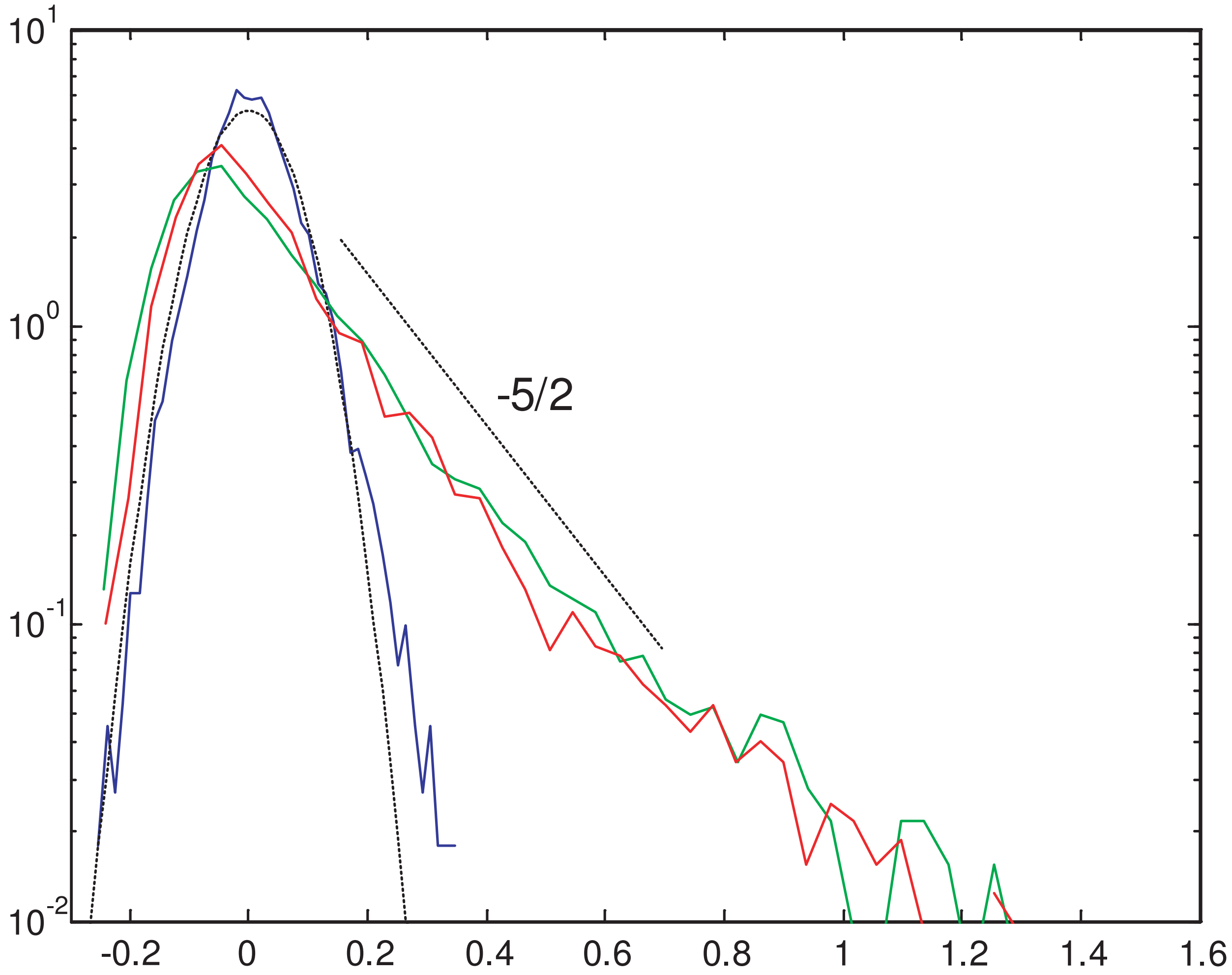

FIG. 6: Probability density function $p(S)$, estimated using histograms with 50 bins. PDF of the total signal S (green), of the coherent component S^C (red) and of the incoherent component S^I (blue), together with a Gaussian fit with variance σ_I^2 (black dotted line).

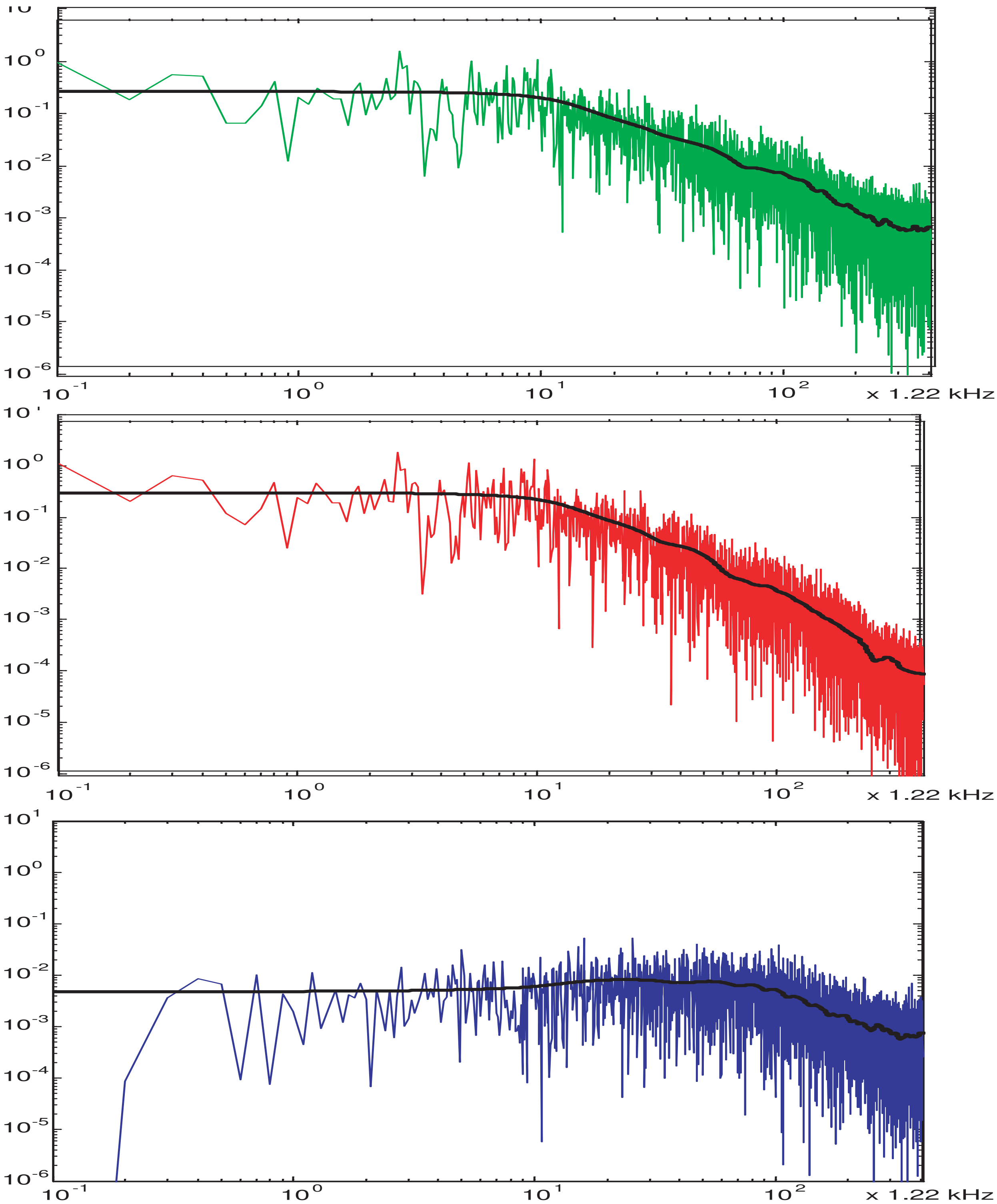

FIG. 7: Fourier spectrum $E(\omega)$ of the signal Tore Supra, ts28338. Top: spectrum of the total signal S. Middle: spectrum of the coherent signal S^C. Bottom: spectrum of the incoherent signal S^I. Note that the periodogram is plotted in green, red and blue for the total, coherent and incoherent signal, respectively. Superimposed are the modified periodograms (black thick line).

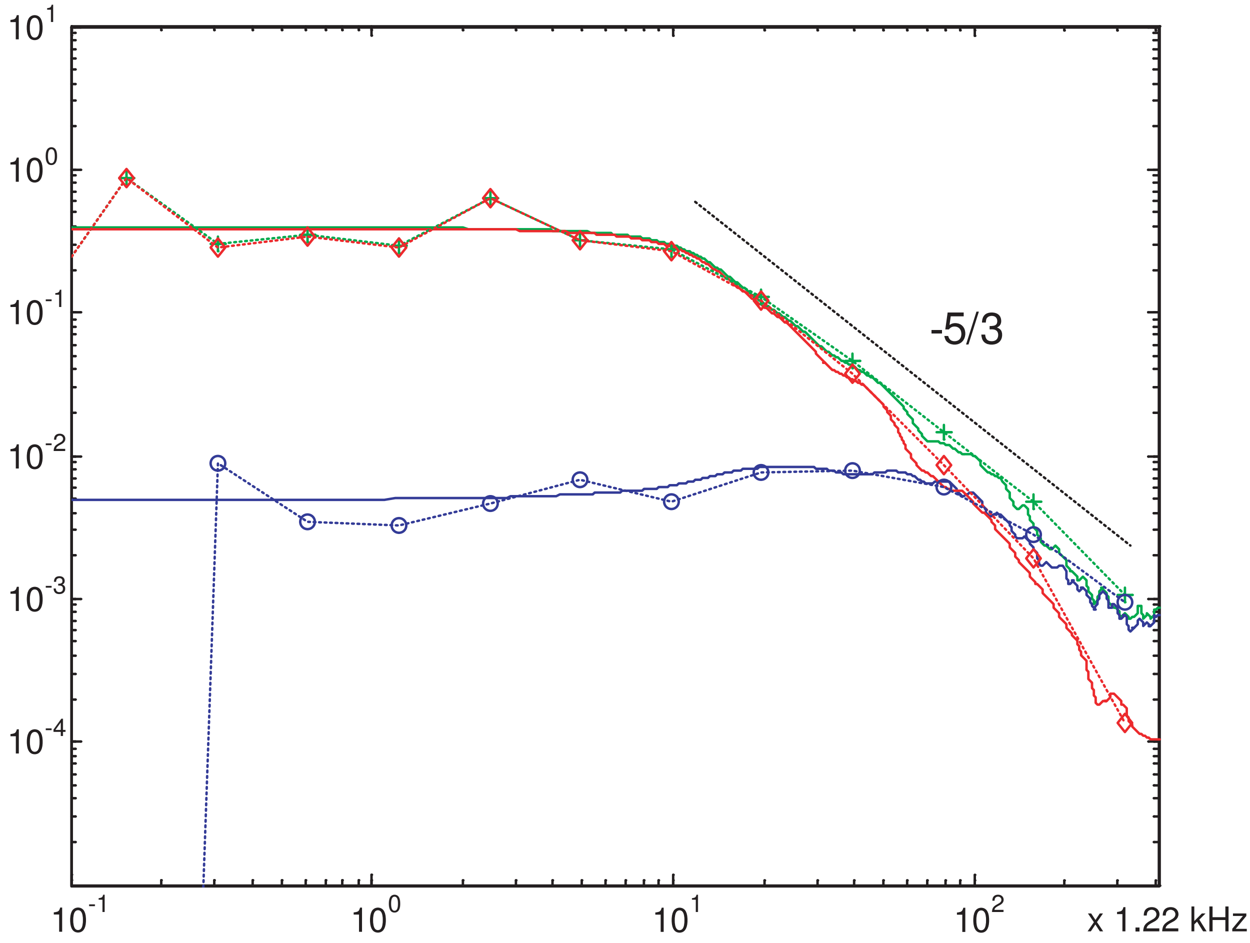

FIG. 8: Wavelet spectra $\widetilde{E}(\omega_j)$ (lines with symbols) and modified periodograms $E(\omega)$ (lines) of the total signal S (green and $+$), of the coherent signal S^C (red and $\diamond$) and of the incoherent signal S^I (blue and $\circ$).

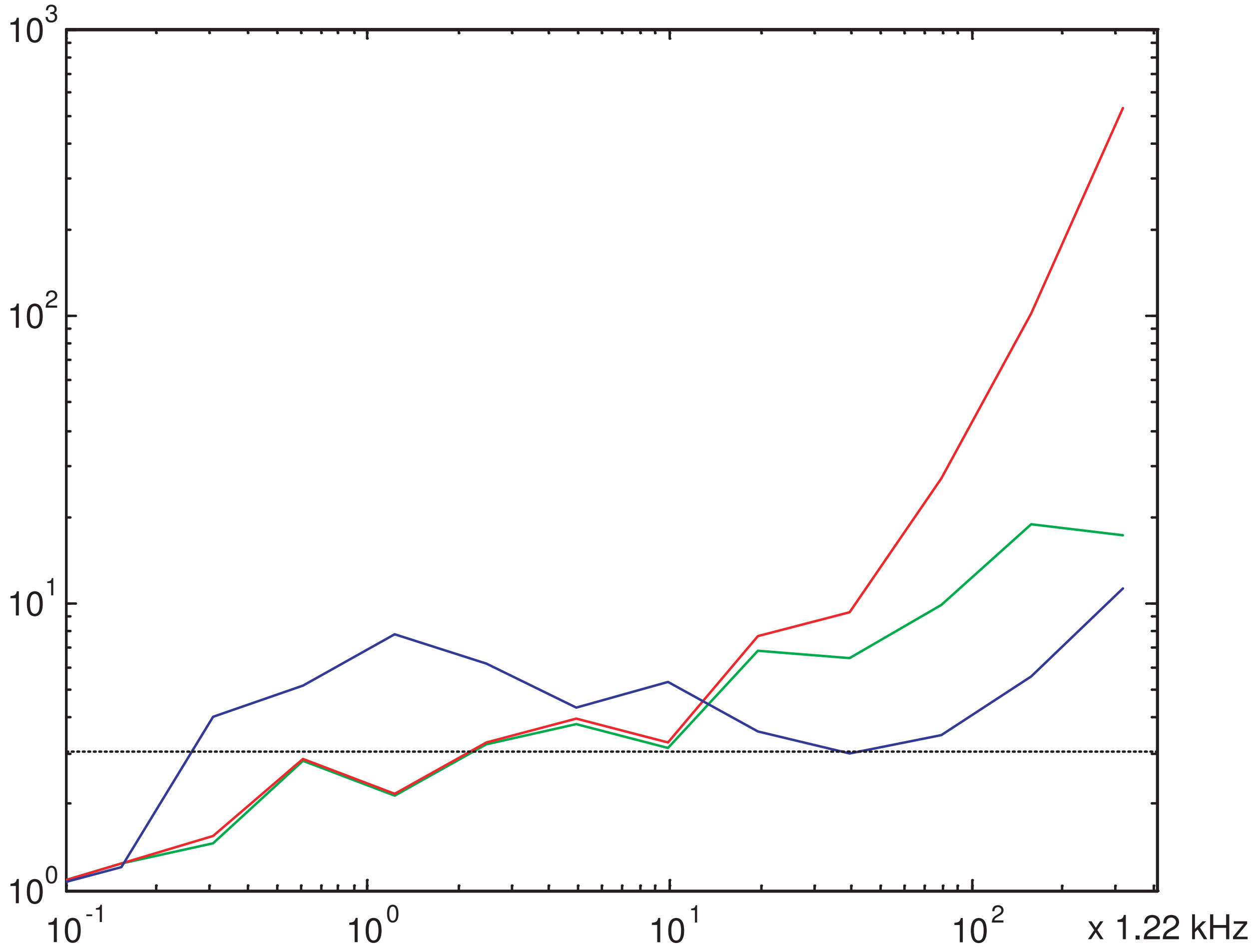

FIG. 9: Flatness $\widetilde{\mathcal{F}}$ versus frequency ω_j for the total signal S (green), of the coherent signal S^C (red) and of the incoherent signal S^I (blue). The horizontal dotted line $\mathcal{F}(\omega_j) = 3$ corresponds to the flatness of a Gaussian process.

7 Juin 2005
Premier rapport de referee

Date: Tue, 7 Jun 2005 13:47:24 UT
From: physplas@pppl.gov
To: farge@lmd.ens.fr
Subject: PoP: MS #POP28974 Editor

Dear Prof. Farge,

We have received the referee's comments on your paper "Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets", which indicate that there are questions regarding its appropriateness for publication in Physics of Plasmas. Please revise your manuscript and submit a detailed response to the referee. The revised manuscript is due by August 6, 2005. The revised manuscript will then be sent back to the referee for further review.

Please feel free to contact the Editorial Office if you have any questions or concerns.

Sincerely,

Sandra Schmidt

Assistant Editor
Physics of Plasmas Office
Princeton Plasma Physics MS 20
James Forrestal Campus
Sayre Drive at Route 1
Princeton, NJ 08543
fax:609-243-2427
sschmidt@pppl.gov

Manuscript #POP28974:

Editor's Comments:

Please make the following editorial corrections in the revised paper:

1. Add the general reference for Tore Supra to the abstract. This should be the actual citation, enclosed in brackets after the machine name.

2. Remove personal pronouns (we, our) from the abstract.

3. When referring to a reference as part of a sentence, do not use just the reference number, but a phrase such as:

. . . as given in Ref. 2, or (see, for example, Ref. 23)

4. Please use the following book citation format:

Authors, Title (Publisher, City, Year) page.

5. Create a separate listing of the captions at the end of the text file.
It is preferred that you do not call the figures into the text, but if it is necessary to do this for the file to typeset properly, then it is acceptable to do so. Please note that the figures are to be online only color in the captions, as, e.g.,:

Figure 1 (Color online)

6. If you have used your text software to size or place your figures, or to arrange the parts of figures, please note that the figures will be processed for publication separately from the text. Please size and place the figures in the figures files themselves. All parts of a multiple part figure should also be contained in a single electronic file for that figure.

7. The table appears in the middle of the figures. It should appear after the references.

Premier rapport de referee en fichier attaché

The authors of the paper entitled "Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets" (PoP 28974) present interesting data processing using wavelet transform to decompose the signals taken in the SOL of the Tore Supra tokamak into its coherent and incoherent components. The method is interesting with convincing results. Having said that, I do have some critiques that are listed below that need to be addressed before this article is suitable for publication.

General comments

- I invite the authors to explain more their method. The reason is that while in fluid turbulence these methods are more known and more frequently used, in plasma physics however, this is not the case. Accordingly, in order for this article to have the desired impact on the plasma community, the method (I will be more specific below) should be explained more thoroughly.
- The authors use the word *noise* to describe the incoherent part of the signal. My suggestion is to either prove this fact or replace this word with, say for example, turbulent fluctuations.
- The authors use color lines, unless they are willing to pay the color figures, I invite them to modify that into different types of lines.
- Some of the figures do not have labels, please provide them.

The following comments are listed in order of their appearance in the text.

- A distinction should made between fluid and SOL plasma. In fluid turbulence intermittent structures are at small scales close to that of Kolmogorov and their contribution is dominated by the incoherent part of the signal. In SOL plasma, it is actually the opposite, intermittent structures occur at large scales (close to the macro-scale of turbulence see Ref. 2) and they dominate the signal. I believe that the authors at different locations in the article do not make this distinction quite clearly and I invite them to.
- In the introduction section, I believe that there is a specific definition for the "smoothness of the signal". Please either clarify by explaining more or remove it from the text.
- Page 4, Please explain what is meant by "min-max estimator".
- Page 4, second paragraph, I guess what is meant is a separation between coherent and incoherent, where incoherent may not be necessarily noise, it could be turbulent fluctuations.
- Page 5, Please take the time and space to explain what is the Coifman "12"(?) wavelets, give the analytical expression if there is one, what are its properties, what makes it different from say a Morlet wavelet, why the

fourth vanishing order, etc. I am convinced that few plasma turbulence researchers are familiar with this type of wavelet transform. Moreover, this method being to my knowledge the first time presented to the plasma community, it is desirable to have more explanation and clarification even though references exist and are cited.

- Page 5, please explain what do you mean by “min-max properties”.
- Page 5, The threshold expression (ε_D) as the authors explain is based on assuming noisy data. However, and as Fig. 4 clearly shows, this is not the case for SOL turbulence. The signal being well above the “noise” level, please explain what does this mean for the threshold choice.
- Please rectify equations 3 and 4 for minor mistakes.
- In the algorithm paragraph, please explain more step 4, in the initialization, and clarify by what do you mean by “count the wavelet coefficients” in step 1 main loop, and the final line in the main loop (until $N_I == N_I^{old}$). Also is there a reason, why $S^I(t)$ is obtained by subtraction rather then by inverse FWT? Also, are the signals components orthogonal ($\langle S^C S^I \rangle$) or their wavelet coefficients?
- Page 7, define complexity and please explain its practical consequences and what it means to have an algorithm with $O(nN)$ order.
- Please explain more what is meant by a “fixed point type argument”.
- Remove ts28338.dat just mention the shot number.
- Page 9, Even though the signal has been successfully divided into a coherent and incoherent part, this does not mean, and it is not shown here, that the coherent part is caused by convective event only. I guess this would be a future investigation. Please rectify.
- Page 9, the authors calculate the Fourier spectrum using the periodogram. As I found out, the definition of a periodogram is not simply a discrete version of equation (9). Later on, they describe a complicated way to get to the average power spectrum plotted in black lines in Fig. 7. This seems to me very complicated to just get the power spectrum. So, my suggestion is why not just using an FFT to calculate Eq. (9)? Could the authors explain what is gained in this process. Also please supply a reference for the periodogram as it is done for the scalogram.
- Page 10, the scaling exponent of the power spectrum characterizes all the frequencies where self-similarity is achieved. This mainly happen at high-frequencies, *i.e.* short time-scales. I do not understand how the authors come to the conclusion that it characterizes long-time correlation! Please rectify.
- Page 10, the authors explain the incoherent power spectrum by noise (maybe).

I suggest that this is very easy to check by plotting the power spectrum of noise taken without plasma. I think it is important to know if the incoherent power spectrum reflects fluctuations that are of turbulent origin.

- The authors found that the flatness factor increases with frequency (Fig. 9). This is surprising since it is rather admitted and shown on several occasions that intermittency in the SOL of tokamaks is caused by large scale events. Hence, one would expect that the flatness to have a maximum at lower frequencies. Could the authors provide an explanation of this apparent inconsistency?
- In the conclusion, page 12, as I already mentioned, the incoherent part of the signal may and may not be caused only by noise, so please rectify or/and clarify. Also, I do not believe that the non-Gaussianity dependence on the frequency is indicative of how *high* is intermittency, please rectify. Also, it is rather widely admitted that both the turbulent diffusion by eddies as well as the coherent bursts cause radial transport, so the authors conjecture does not make sense to me at least in the way it is written.

As the authors have remarked most of the above comments reflect more explanations that are needed in order to make this article reader friendly for the plasma community. I have no doubt that the authors will be able to address all of the above issues, and looking forward at reading the new version.

12 Oct 2005
Envoi de la première version révisée
et de notre réponse au referee

ANSWER TO REFEREE

Extraction of coherent bursts
from turbulent edge plasma in magnetic fusion devices
using orthogonal wavelets

Marie Farge, Kai Schneider and Pascal Devynck

General comments

Comment 1
'…the method should be explained more thoroughly.'

We thank the referee for his advice. Following his remark we have explained in more details the orthogonal wavelet transform and the algorithm for coherent structure extraction. In particular we have stated in more details the principle on which our method is based.

We summarize the main points concerning our approach:

- We want to study coherent structures encountered in fluid turbulence, *e.g.* vortices, or plasma turbulence, *e.g.* bursts. Since turbulent signals are highly fluctuating, one studies them statistically using classical diagnostics such as correlation functions, spectra or structure functions. Unfortunately those diagnostics loose the temporal structure of the signal, since they are computed with time integrals and the Fourier modes used as basis functions used are not localized in time.

- The first improvement we propose to study coherent structures in turbulent flows is to replace the Fourier representation by the wavelet representation, which keeps track of both time and scale (related to the inverse frequency), instead of frequency only.

- The second improvement is to change our viewpoint concerning the extraction of coherent structures out of turbulent flows. Since there is not yet an universal model to characterize them, *e.g.* Gaussian bumps, chirps or spikes, we prefer to start from a more consensual statement: coherent structures are different from noise. Consequently, we define them as what remains after denoising.

- We thus split a turbulent signal into a coherent component (made of organized structures produced by nonlinear instabilities) and an incoherent noise-like component (made of turbulent fluctuations whose variance actually gives a good estimation of the turbulence level). Since we use the orthogonal wavelet representation both components are orthogonal and therefore the L^2-norm, *e.g.* energy or enstrophy, is thus split into coherent and incoherent contributions.

- Assuming that coherent structures are what remains after denoising, we need a model, not for the structures, but for the noise. As a first guess we choose the simplest model and suppose the noise to be additive, Gaussian and white, *i.e.* uncorrelated. Having this model in mind, we then rely on Donoho and Johnstone's theorem to compute the value used to threshold the wavelet coefficients.

- Since the threshold value depends on the variance of the noise, which in the case of turbulence is not *a priori* known, we propose a recursive method to estimate it from the variance of the weakest wavelet coefficients, *i.e.* those whose modulus is below the threshold value.

- We finally apply our method to a turbulent signal measured in the SOL of the tokamak Tore Supra in Cadarache. We then checked *a posteriori* that the incoherent turbulent fluctuations are indeed noise-like, quasi-Gaussian and quasi-uncorrelated, which thus confirms the hypotheses we have chosen for the noise.

Comment 2
'... the word noise ...'

By definition a noise cannot be contracted in any basis.
Concerning the incoherent contribution that we consider to be noise-like, we have checked that it is homogeneous in physical space (fig. 4 bottom) and that energy is spread over most of the spectral range, since we observe energy equipartition up to the cutoff frequency. The fact that the incoherent signal does not contract, neither in physical space nor in Fourier space, gives evidence that it actually corresponds to a noise.
Nevertheless, we appreciate the suggestion of the referee and we replace the word 'noise' by 'noise-like turbulent fluctuations'.

Comment 3
'... color lines ...'

We are ready to pay for color figures.
For clarity we have modified the line styles on the figures using dotted, dashed-dotted and solid lines, to be able to distinguish them in black and white.

Comment 4
'Some of the figures do not have labels ...'

We added labels to all figures, besides Figure 2, which is a purely academic signal whose units do not matter.

Specific comments

Comment 1
'A distinction should be made between fluid and SOL plasma…'

We think that the definitions of intermittency in plasma and fluid turbulence are similar, since both fields are using random functions to study turbulence.

Let us recall that the first definition of intermittency was introduced by Townsend (Austr. J. Sci. Res., 1, 161, 1948), who pictured turbulence as a sequence of active bursts separated by quiescent regions. To quantify it, he introduced an 'intermittency factor' which is the ratio of the time supports of active regions and quiescent regions.

Uriel Frisch gave a very similar definition in his book (Turbulence, CUP, page 122, 1995): «The function is intermittent if it displays activity during only a fraction of the time which decreases with the scale under consideration». He then explained that the intermittency of a function can be quantified by computing the variation of the flatness when the scale decreases. If it remains constant, the signal is non-intermittent, if it increases when scale decreases, it is intermittent. We use the same diagnostics of intermittency when we compute the scale dependent flatness.

Dieter Biskamp (Nonlinear Magnetohydrodynamics, CUP, page 217, 1993) stated that the «spottiness of the dissipative eddies is a special feature of what is now believed to be a general property of fully developed turbulence that with decreasing scale turbulent fluctuations become less and less space-filling, *i.e.* are concentrated in regions of smaller and smaller volume but increasingly complicated shape. This phenomenon is called intermittency, which is a central topic in actual turbulence research».

Assuming the above definitions of intermittency, we do not understand the point made by the referee in his remark: «In fluid turbulence intermittent structures are at small scales close to that of Kolmogorov and their contribution is dominated by the incoherent part of the signal. In SOL plasma, it is actually the opposite, intermittent structures occur at large scales (close to the macro-scale of turbulence see Ref. 2) and they dominate the signal». In agreement with the previously mentioned authors, we think that the time support of an intermittent structure decreases with the scale and is therefore necessarily multiscale. This is why we do not understand the notion of 'large scale intermittency', which sounds for us as an oxymoron, although we know that several authors (cf. Tabeling, Paret, Jullien) have used it. We think that this confusion is due to the clipping method they use. The advantage of using wavelets instead is that we now have a precise definition of scale. For more details about wavelets and intermittency, you can download our paper: 'Spatial intermittency in two-dimensional turbulence: a wavelet approach', Perspectives in Mathematics and Physics, 34, World scientific, from our web site: //wavelets.ens.fr.

This discrepancy between intermittency in fluids and in plasmas may come from the use of 'clipping' to measure intermittency, since it introduces a strong bias. Arkady Tsinober mentioned in his book (An informal Introduction to Turbulence, Kluwer, page 154, 2001): «Loosely, an intermittency factor is defined as a fraction of volume (time) where the variable is 'active'. The main deficiency is that intermittency factors depend on the choice of the threshold below which the variable is considered 'inactive' (Kuo and Corrsin JFM, 50, 285, 1971)». As we mentioned in our paper our motivation to use wavelets is to avoid such a clipping, since the retained bursts and the corresponding intermittency factor depend for the latter on the choice of the threshold.

Comment 2
'In the introduction …smoothness of the signal'

Smoothness characterizes the regularity of a function which states that the function and/or its high order derivatives are continuous, *i.e.* do not present discontinuities.
We replaced 'smoothness' by 'regularity', which may be better known, and gave its definition as reference in the text. We also explained that the clipping method introduces discontinuities in the signal, which thus looses its regularity.

Comment 3
'Page 4 … min-max estimator'

We have discarded this reference to min-max estimator which may be too technical.
We have just stated the following sentence: «…wavelet threshold is optimal to denoise signals in presence of additive Gaussian white noise, because it minimizes the maximal L^2-error (between the denoised signal and the noise-free signal) for functions with inhomogeneous regularity, such as intermittent signals». This phrase actually defines what is a min-max estimator.

Comment 4
'Page 4… incoherent may not be necessarily noise…'

The definition of 'noise' has already been given in Comment 2. Here we should keep the term 'Gaussian background noise' since it is the hypothesis made on the noise as required by our approach. Indeed, the originality of our method is to make hypotheses on what is discarded, rather than what is kept. Since there is not yet a consensus to define what a coherent structure is, we could at least agree on what a coherent structure is not. Our algorithm is based on the assumption that a coherent structure is not a noise and therefore our methodology is based on denoising. As the simplest model on what we will discard, we suppose it to be an additive, Gaussian and white, *i.e.,* uncorrelated, noise. We then perform the extraction of the coherent contribution and check *a posteriori* if the discarded component, *i.e.* the incoherent contribution, verifies those hypotheses.

Comment 5
'Page 5 … Coifman 12 wavelet…'

We have added some precisions about Coifman 12 wavelets , but discarded the reference to the high-order moment cancellation, since it may be too technical here. Incidently, most orthogonal wavelets do not have analytical expression and they are insstead defined by their associated quadrature mirror filters (QMFs). The Fourier transform of the wavelet is thus defined as the limit of an infinite product of the corresponding QMFs . Actually, we even did not mention this and prefer to refer the reader to Mallat's book
(A wavelet tour of signal processing, Academic Press, 1998) .

Comment 6
'Page 5 … min-max properties'

See Comment 3.

Comment 7
'Page 5 … the signal being well above the noise level'

Noisy data do not necessary imply that noise dominates the signal.
On figure 4 we agree that the signal is well above the noise level. As we demonstrate in the results that our algorithm succeeds to eliminate this noisy part and to retain the coherent part. We can thus compute the signal to noise ratio and obtain SNR=8.72 dB in this case. In Donoho's approach one needs to know *a prioi* the noise's variance. This is not the case with our algorithm since we use an iterative procedure to find the noise's variance, which thus supplement Donoho's approach and makes it useful in practice.
Let us also notice that the experimental noise remains weaker than the incoherent background noise-like turbulence.

Comment 8
'Please rectify equations 3 and 4 for minor mistakes'

We do not find any errors in both equations. We actually found a typo in equation (5) concerning the index ij, which has been corrected.

Comment 9
'… clarify what do you mean…explain step 4… are the signals components orthogonal'

We replaced 'count the wavelet coefficients' by 'count the number of wavelet coefficients'. This is thus clearlier.
The final line in the main loop (until $N = = N_I^{old}$) means that we stop looping when N is equal to N_I^{old}, which is when the number of coefficients considering to be noise does not change anymore.
'Also there is no reason why $S^I(t)$ is obtained by substraction…'
We agree with the referee that S^I can also be obtained by inverse wavelet transform, however it is computationally simpler to perform a substraction which yields the same result. Note that due the fact that the wavelet transform is an isometry, coherent and incoherent components are orthogonal both in physical space and in wavelet space.

Comment 10
'Page 7… algorithm with O(nN) order'

This characterizes the number of multiplications needed to compute the wavelet direct and inverse transforms. To avoid this notation which is used by applied mathematicians, we have written instead the sentence: 'The operation count of the algorithm is (2 n m N) multiplications'.

Comment 11
'…fixed point type argument'

We have added an explanation stating that the optimal threshold is a fixed point of the iteration function defined in eq. (8) leading to $I(\varepsilon) = \varepsilon$.

Comment 12
'Remove ts28338'

This has been done in most occurrences. We kept it only once since it is the number of the shot of Tore Supra we are studying in this paper.

Comment 13
'Page 9…this does not mean…that the coherent part is caused by convective events only'

We replaced 'convective' by 'coherent' in the sentence on page 9 which now reads : 'The decomposition shows that the bursty and coherent part of the signal dominates…'

Comment 14
'Page 9 ... periodogram'

The standard textbook explaining periodograms is Priestley's textbook (Spectral analysis and time series, 2 volumes, Academic Press, 1988). The method we use to compute a consistent estimator of the spectrum is the modified periodogram which is explained in this text book. We have thus tapered the signal with a cosine window and smoothed its periodogram using a Gaussian window, as we have already explained in our paper. In the revised version we added the reference to Priestley's textbook.

Comment 15
'Page 10 ... scaling exponent… characterizes long-time correlation'

A time series with a spectrum having a negative scaling exponent corresponds to correlation and a flat spectrum to decorrelation (cf. Priestley's textbook quoted above).
If the maximum of the energy is at a low wavenumber it corresponds to long-time correlation. If it is at a high wavenumber it corresponds to short-time correlation. We discarded this comment in the text, since it may be confusing to a reader not accustomed to signal processing of stochastic processes.

Comment 16
'Page 10 ... plotting the power spectrum of noise taken without plasma'

We thank the referee for this useful comment. Following his advice, we have measured the experimental noise level without the plasma and found that it is in the range 10^{-4}-10^{-3}, while the signal, even at high frequency, remains above (see Figure 1 below). Therefore the incoherent fluctuations correspond to turbulence and not to experimental noise, at least up to 120 kHz. We observe that at higher frequencies the spectrum of the incoherent component is no more flat and present ω^{-1} power-law behaviour, similar to the experimental noise in the same frequency range. Therefore, we add the following precision in our text: «For higher frequencies we observe a ω^{-1} scaling, which may be due to experimental noise, since it presents the same scaling at high frequencies, although its amplitude remains smaller than the incoherent fluctuations».

Comment 17

'…Fig. 9… intermittency … caused by large scale events … flatness'

See Comment 1 of the specific comments section.

Comment 18
'…page 12… non-Gaussianity dependence on the frequency … intermittency'

See Comment 1 of the specific comments section.

'…turbulent diffusion by eddies as well as coherent bursts cause radial transport…''

Our text states: 'By analogy with previous studies we have made in the context of two-dimensional turbulence (Comm. Nonlinear Sci. Num. Simulation, 8 (3-4), 2003) we conjecture that the coherent bursts are responsible for the turbulent transport, while the incoherent background flow contributes to turbulent diffusion' . In the paper we refer to we have shown that for 2D Navier-Stokes equations all the convective transport is made by the coherent vortices, while the incoherent background is only responsible for turbulent dissipation characterized by classical diffusion. You can download this paper from our website: //wavelets.ens.fr (publication, n° 178).

We thank the referee for all his remarks and for the improvements he suggested us.

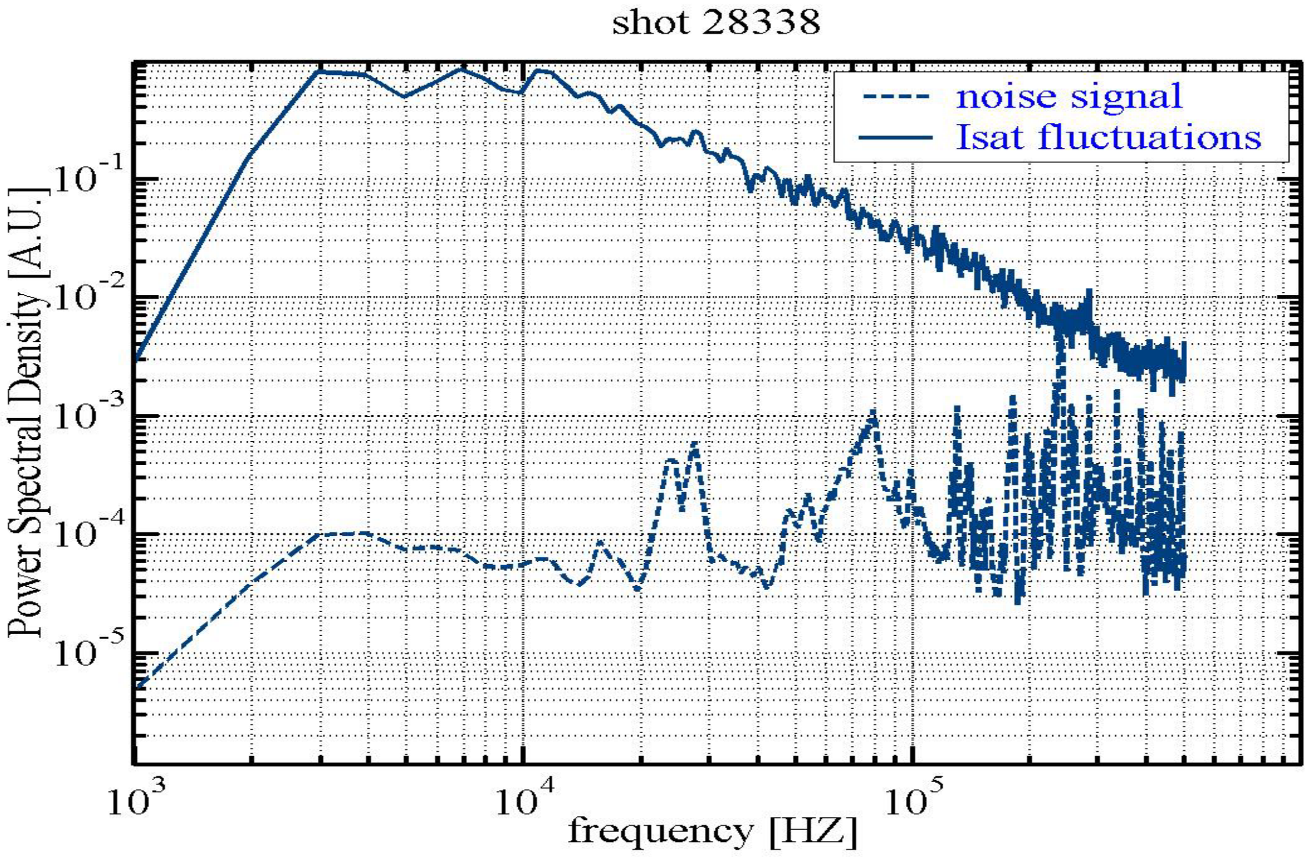

Figure 1

Energy spectrum of the signal (solid line)
and the experimental noise (dashed line)

20 Oct 2005
Second rapport de referee

From physplas@pppl.gov Thu Oct 20 22:44:49 2005
Date: Thu, 20 Oct 2005 20:37:03 UT
From: physplas@pppl.gov
To: farge@lmd.ens.fr
Subject: PoP: MS #POP28974A Editor

Dear Prof. Farge,

We have received the referee's comments on your revised paper "Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets". Though largely positive, there are recommendations for revision. Please revise your manuscript and submit a detailed response to the referee. The revised manuscript and response are due by December 19, 2005. The Editors will then make the decision as to the next step in the review process.

Please feel free to contact the Editorial Office
if you have any questions or concerns.

Sincerely,

Sandra Schmidt

Assistant Editor
Physics of Plasmas Office
Princeton Plasma Physics MS 20
James Forrestal Campus
Sayre Drive at Route 1
Princeton, NJ 08543
fax:609-243-2427
sschmidt@pppl.gov

Manuscript #POP28974A:

Editor's Comments:
It would most helpful if you could create a list of the figure captions after the references in the latex file.

Reviewer Comments:
Please see the attached file.

Second rapport de referee en fichier attaché

The authors of the paper entitled "Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets" (PoP 28974) responded to most of the comments and suggestions successfully. Just a couple of points that I would like to stress before this paper is accepted for publication.

- In Eq. 3, it is the frequency f and not ω that should be in the exponential.
- In Eq. 4, it is hard to tell what is under the logarithm, is it N or $N\sigma^2$.
- I beleive that the authors did not respond to the only physics comments of my first report and that is about Fig. 9. Therefore I rephrase my comment: The authors found that the flatness factor increases with frequency (Fig. 9). This is surprising since it is rather admitted and shown on several occasions that intermittency in the SOL of tokamaks is caused by large scale events. Hence, one would expect that the flatness to have a maximum at lower frequencies. Could the author provide an explanation of this apparent inconsistency?

21 Décembre 2005
Notre réponse au second rapport de referee
et seconde version révisée

From farge@berlioz.ens.fr Wed Dec 21 08:59:54 2005
Date: Wed, 21 Dec 2005 09:59:51 +0100 (CET)
From: FARGE Marie <farge@berlioz.ens.fr>
To: Dianne E.Nunes <physplas@pppl.gov>
Cc: FARGE Marie <farge@lmd.ens.fr>, Schneider Kai <kschneid@cmi.univ-mrs.fr>, Devynck Pascal <devynck@pegase.cad.cea.fr>
Subject: PoP: MS #POP28974A Editor

Dear Dianne Nunes,

I thank you very much for your response to my previous mail.

I am sending you as attached files:

1. the tex file with the new version of our paper,
2. our answer to the editor,
3. our answer to the referee,
4. the eps file of the new figure we have added to our paper.

I have accessed to our paper from the POP website and tried to modify it to replace the present version by the revised one. Unfortunately I have not been able to do that since, after clicking on 'Revise Manuscript', the system was asking me to reload all the files. I am presently in Japan and I have just been able to borrow someone else's computer for few minutes, without being able to resend all the files since there are 11 figures plus the text. Could you please kindly substitute this revised version and the new figure (Figure 10), together with our answer to referee on your website?

I thank you very much for your kind help,
Yours Sincerely,

Marie Farge

Réponse au second rapport de referee en fichier attaché

Manuscript #POP2897A (revised version)

Reply to the referee :

We thank the referee for his useful comments. We modified the paper accordingly and hope that it is now suitable for publication.

We now answer the three points addressed by the referee.

1. We checked eq. (3) and explained in the text that ω actually denotes a frequency.

2. We modified eq. (4) to avoid any confusion which quantity is under the logarithm.

3. In addition to the scale dependent flatness shown in Fig.9, we also studied the flatness of the low-pass filtered signal, namely we have partially reconstructed the signal up to a cut-off scale that we then vary. The result is presented in the new Fig.10 which shows that the large scale signal is already intermittent. This is in agreement with the result shown in the paper by Antar et al. (Physics of Plasmas 8, 2001) observing intermittency in the large scales, however we also found that intermittency is still increasing up to the frequency 20 kHz (see figure 9 and 10). We have also added two sentences to clarify our definition of intermittency. We hope that these additions resolve the 'apparent inconsistency' mentioned in the referee report.

2 Janvier 2006
Acceptation de l'article
et demande de renvoi de la Figure 10
que nous avions déjà envoyée le 21 Décembre

From sschmidt@pppl.gov Mon Jan 2 09:16:18 2006
Date: Fri, 30 Dec 2005 19:10:26 UT
From: sschmidt@pppl.gov
To: farge@lmd.ens.fr
Subject: PoP: POP28974B (Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devi...)

Dear Dr. Farge,

I am happy to inform you that your paper has been accepted for publication. I just need to clear up one thing before we can proceed with publication. In your paper, there are ten figure captions, but there appear to be files for only 9 figures in the PXP system. If you need to send another figure file to us, you can do so via email to plasmas@pppl.gov and I will upload it.

Thank you and Happy New Year!

Sandy Schmidt

Assistant Editor
Physics of Plasmas
Princeton Plasma Physics Laboratory
Princeton, NJ 08540
fax: 609-243-2427

16 Janvier 2006
Accord du referee
pour publication

From physplas@pppl.gov Mon Jan 16 17:50:52 2006
Date: Mon, 16 Jan 2006 17:43:09 UT
From: physplas@pppl.gov
To: farge@lmd.ens.fr
Subject: PoP: MS #POP28974B Acceptance Notice

Dear Prof. Farge,

We have successfully uploaded the final figure for your paper. So, I am pleased to confirm that your revised manuscript, referenced below, has been accepted for publication in Physics of Plasmas and is tentatively scheduled for publication in the February 2006 issue:

"Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets"

Here are the final comments of the referee:

The authors successfully answered all of my comments. I thus consider this article suitable for publication in Physics of Plasmas.

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Thank you for your contribution to the Journal. If you have any questions, feel free to contact us at physplas@pppl.gov.

Sincerely,

Dr. Ronald C. Davidson

Editor
Physics of Plasmas Office
Princeton Plasma Physics MS 20
James Forrestal Campus
Sayre Drive at Route 1
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609-243-2424

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8 Février 2006
Envoi des épreuves
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Date: Wed, 8 Feb 2006 11:34:18 -0500 (EST)
From: php@aip.org
To: farge@lmd.ens.fr
Subject: AUTHOR PROOF for PHP

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Article Title: EXTRACTION OF COHERENT BURSTS FROM TURBULENT EDGE PLASMA IN MAGNETIC FUSION DEVI
Section: ARTICLES
Sub-section: Nonlinear Phenomena, Turbulence, Transport

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PHYSICS OF PLASMAS **13**, 1 (2006)

Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets

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(Received 9 May 2005; accepted 16 January 2006)

A new method to extract coherent bursts from turbulent signals is presented. It uses the wavelet representation which keeps track of both time and scale and thus preserves the temporal structure of the analyzed signal, in contrast to the Fourier representation which scrambles it among the phases of all Fourier coefficients. Using orthogonal wavelets, turbulent signals can be decomposed into coherent and incoherent components, which are orthogonal and whose properties can thus be studied independently. Diagnostics based on the wavelet representation are also introduced to compare the statistical properties of the original signals with their coherent and incoherent components. The wavelet-based extraction method is applied to the saturation current fluctuations measuring the plasma density fluctuations at the edge of the tokamak Tore Supra, Cadarache, France. This procedure disentangles the coherent bursts, which contain most of the density variance, are intermittent and correlated with non-Gaussian statistics, from the incoherent background fluctuations, which are much weaker, nonintermittent, noise-like and almost decorrelated with quasi-Gaussian statistics. We conjecture that the coherent bursts are responsible for the turbulent transport, whereas the remaining incoherent fluctuations only contribute to the turbulent diffusion. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2172350]

I. INTRODUCTION

A. Coherent bursts

The radial transport at the edge of tokamaks is known to be dominated by turbulent processes. Understanding them is important, as they will determine the confinement properties of the overall plasma in the bulk region and the energy density that must be handled by the limiter or divertor components in the shadowed region of the plasma where the magnetic field lines are opened. The turbulent transport of plasma density has been extensively studied at the edge of plasma by means of Langmuir probes,[1–3] particles beams,[4,5] and more recently two-dimensional (2D) visible imaging.[6,7] All these diagnostics observe a turbulent transport of the plasma density in the scrape-off layer (SOL) that can be described as a superposition of convective events, which are responsible for the transport of matter over long radial distances at a fraction (of the order of 10%) of the ion sound speed,[8,9] and of background turbulence.

The convective events are detected as coherent bursts of plasma density, but with a signature different from the one expected for turbulent eddies, since they exhibit a probability distribution function (PDF) which is skewed. Typically, it is found that these convective events account for a small fraction of the time and substantial proportion of the turbulence intensity,[10] which underlines their importance in the turbulent transport. There are many efforts to analyze these bursts independently from the background turbulence. For this purpose different extraction methods have been developed, which are based on signal clipping (see, e.g., Ref. 10), correlation with given templates or conditional averaging. These methods require strong hypotheses on the signal, which has to be statistically steady, and also on the bursts in order to choose the appropriate threshold value. Actually the clipping method presents two drawbacks. First, the duration of the bursts and their turbulent intensity strongly varies depending on the threshold value (e.g., from 4% to 20% of the total time and between 20% and 50% of the total turbulent intensity[10]), which unfortunately cannot be estimated *a priori*. Second, the clipping method does not preserve the regularity[11] of the signal, since the threshold introduces discontinuities which affect the Fourier spectrum and hence yields an erroneous scaling. Although these methods give some information about the dynamics,[10,12] other methods requiring less hypotheses to extract the bursts are needed.

Since 1988 we have proposed to use the wavelet representation to analyze[13,14] and extract[15–17] coherent structures out of turbulent flow fields, as the wavelet representation does not require any hypothesis on the statistical stationarity and homogeneity of the process under study. In this article we demonstrate the advantages of wavelets to separate coherent bursts from turbulent fluctuations in edge plasma. We present a wavelet-based extraction algorithm, which does not even require any parameter, such as threshold value, to be adjusted. We then apply it to study the plasma density fluctuations measured in the SOL of the tokamak Tore-Supra, Cadarache, France.[18]

B. Wavelet representation

Since turbulent signals are highly fluctuating, one studies them statistically, using classical diagnostics such as correlation functions, spectra or structure functions. Unfortunately those diagnostics lose the temporal structure of the signal, since they are computed with time integrals and the Fourier modes used as basis functions are not localized in time.

The wavelet transform is more appropriate than the Fourier transform to analyze and represent nonstationary, nonhomogeneous, and intermittent signals, such as those encountered in turbulence. It uses analyzing functions which are generated by translation and dilation of a so-called "mother wavelet," well localized (i.e., have a finite support) in both physical and spectral space. In contrast, the Fourier transform uses trigonometric functions, which are nonlocal (having an infinite support) in physical space but well localized in spectral space, and the analyzing functions are generated by modulation rather than dilation. The localization of the basis functions and the invariance group of the transform constitute the main differences between wavelet and Fourier representations. For a general presentation of the different types of wavelet transforms and their applications to turbulence, we refer the reader to several review articles.[19–21]

Trigonometric functions used by the Fourier transform oscillate for all times and the temporal information of the transformed signal is scrambled among the phases of all Fourier coefficients. In contrast, the wavelet coefficients preserve the temporal properties of the signal. Thus, when a wavelet coefficient is filtered out, the effect on the reconstructed signal remains local in time and does not affect the overall signal, as the Fourier transform does. This property allows one to study the behavior of a limited portion of the signal directly from its wavelet coefficients.

If a turbulent signal is stationary, nonintermittent and supposed to be made up of a superposition of waves, not having any nonlinear behavior such as chirps, solitons, or shocks, only in this case one can define without ambiguity the associated frequencies. However, if a turbulent signal is supposed to be a superposition of elementary structures localized in space and time, and nonlinearly interacting (e.g., vortices, shocklets), the wavelet representation should be preferred, because it preserves the locality of information in both space and scale. Actually, these two different transforms translate into mathematical language two different interpretations of turbulent signals.[19]

In the context of plasma physics the continuous wavelet transform has already been used to analyze signals measured in magnetic fusion devices, see e.g., Refs. 22 and 23. In this article we propose to use the orthogonal wavelet transform instead, since it has been proved to be optimal for de-noising signals corrupted with additive Gaussian white noise.[24] A generalization to correlated noise is straightforward, and a similar method has been developed[25] to treat non-Gaussian noises, i.e., χ^2 distribution. To improve the choice of the threshold we have proposed a recursive algorithm,[26] that we have applied to extract coherent structures out of incompressible turbulent flows.[15] In the present article we demonstrate its use to study turbulence in edge plasmas of magnetic fusion devices, such as tokamaks or stellarators.

C. Content

This article is organized as follows. First, we present the wavelet-based extraction method. We then explain the recursive algorithm and validate it on an academic signal. We finally apply it to a saturation current signal measured in the SOL of the tokamak Tore Supra, Cadarache, France. We thus show that the coherent bursts can be efficiently extracted. We also present new statistical diagnostics based on the wavelet representation that we use to compare the original signal with its coherent and incoherent components. Finally, some conclusions are drawn and perspectives for future work are given.

II. EXTRACTION OF COHERENT BURSTS

A. Principle

We propose a new method to extract coherent structures from turbulent flows, as encountered in fluids (e.g., vortices, shocklets) or plasmas (e.g., bursts), in order to study their role in transport and mixing.

As already mentioned, we first replace the Fourier representation by the wavelet representation, which keeps track of both time and scale, instead of frequency only. The second improvement consists in changing our viewpoint about coherent structures. Since there is not yet an universal definition of coherent structures in turbulent flows, we prefer starting from a minimal but more consensual statement about them, that everyone hopefully could agree with: *coherent structures are not noise*. Using this apophatic method we propose the following definition: *coherent structures correspond to what remains after de-noising*. APC: #1

For the noise we use the mathematical definition stating that a noise cannot be compressed in any functional basis. Another way to say this is to observe that the shortest description of a noise is the noise itself. Notice that plasma physicists typically call "noise" what is actually "experimental noise," measured when there is no plasma. Their definition includes what we define as noise, plus possibly some organized features (e.g., parasite waves) that we do not consider as noise according to the above-mentioned mathematical definition.

This new way of thinking about coherent structures presents the advantage of being able to process "incomplete fields." What does it mean? A typical example of incompleteness is encountered in the experimental setting, where typically one measures the time evolution of a three-dimensional (3D) field using a probe located in one point, thus obtaining a one-dimensional (1D) cut of a four-dimensional space-time field. Notice that incompleteness is different from discretization, i.e., sampling, that one should consider in addition. If the algorithm used to extract coherent structures requires templates of typical structures, it becomes intractable when the measured field is incomplete, because, in order to define the template, one should then consider how the probe sees all possible motions and distortion of the coherent structures passing by in order to define the templates. Since our algorithm requires a model of the noise, but not of the coherent structures themselves (no templates are needed), it treats any field, complete or incomplete, the same way.

Considering our definition of coherent structures, turbulent signals are split into two contributions: coherent bursts, corresponding to that part of the signal which can be compressed in a wavelet basis, plus incoherent noise, corresponding to that part of the signal which cannot be compressed, neither in wavelets nor in any other basis. We will then check *a posteriori* that the incoherent contribution is spread, and therefore does not compress, in both Fourier and grid point basis. Since we use the orthogonal wavelet representation, both coherent and incoherent components are orthogonal and therefore the L^2 norm, e.g., energy, is the sum of coherent and incoherent contributions.

Assuming that coherent structures are what remains after de-noising, we need a model, not for the structures, but for the noise. As a first guess, we choose the simplest model and suppose the noise to be additive, Gaussian and white, i.e., uncorrelated. Having this model in mind, we then rely on the theorem of Donoho and Johnstone[24] to compute the value used to threshold the wavelet coefficients. Since the threshold value depends on the variance of the noise, which in the case of turbulence is not *a priori* known, we propose a recursive method to estimate it from the variance of the weakest wavelet coefficients, i.e., those whose modulus is below the threshold value.

After applying our algorithm to a turbulent signal, we then check *a posteriori* that the incoherent component is indeed noise-like, spread in physical space, quasi-Gaussian and quasi-uncorrelated (i.e., spread in Fourier space, which thus confirms the hypotheses we have chosen for the noise.

B. Orthogonal wavelet representation

The construction of orthogonal wavelet bases and the associated fast numerical algorithm are based on the mathematical concept of multiresolution analysis, which considers approximations at different scales. A function or a signal (sampled function) can thus be decomposed into a set of embedded coarser and coarser approximations. The originality of the wavelet representation is to encode the differences between successive finer approximations, instead of the approximations themselves. The amount of information needed to go from a coarse approximation to a finer approximation is then described using orthogonal wavelets. A function or a signal is thus represented by its coarsest approximation, encoded by the scaling coefficients, plus the differences between the successive finer approximations, encoded by the wavelet coefficients.

We consider a signal $S(t)$ of duration T sampled on $N = 2^J$ equidistant instants $t_i = iT/N$, with $i=0,\dots,N-1$. We project it onto an orthogonal wavelet basis[19,27] to represent it at different instants t_i and different time scales $\tau = 2^{-j}$, with $j=0,\dots,J-1$.

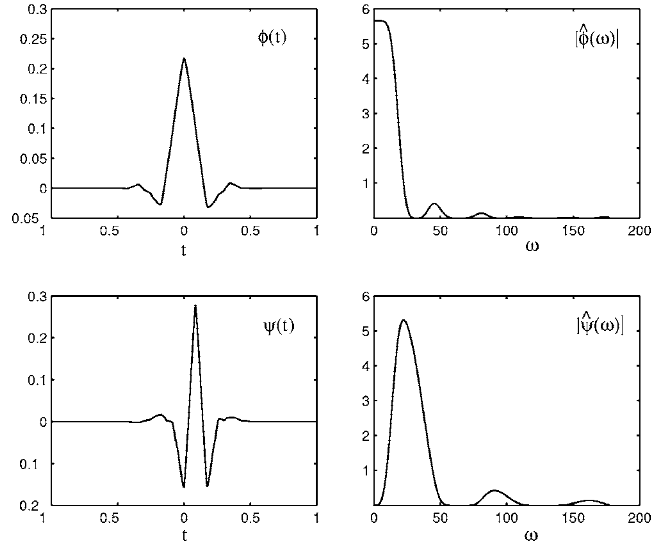

FIG. 1. Coifman 12 wavelet. (Top) Scaling function $\phi(t)$ and the modulus of its Fourier transform $|\hat{\phi}(\omega)|$. (Bottom) Wavelet $\psi(t)$ and the modulus of its Fourier transform $|\hat{\psi}(\omega)|$.

The signal is thus developed into an orthogonal wavelet series,

$$S(t) = \bar{S}_{00}\phi_{00}(t) + \sum_{(j,i)\in\Lambda_J} \tilde{S}_{ji}\psi_{ji}(t), \quad (1)$$

where ϕ_{00} is the scaling function and ψ_{ji} is the corresponding wavelets, i is the index for the instant t and j is the index for the time scale τ. To simplify the notation, we introduce Λ_J, which indexes all wavelets constituting the basis, defined as

$$\Lambda_J = \{(j,i), \quad j=0,\dots,J-1, \quad i=0,\dots,2^j-1\}. \quad (2)$$

Due to orthogonality of the basis functions, the coefficients are computed using the L^2 inner product, denoted by $\langle f,\psi\rangle = \int_{-\infty}^{\infty} f(t)\psi(t)dt$. The scaling coefficients are $\bar{S}_{00}=\langle S,\phi_{00}\rangle$ and the wavelet coefficients are $\tilde{S}_{ji}=\langle S,\psi_{ji}\rangle$. The scaling coefficients encode the approximation of the function S at the largest scale $\tau_0 = 2^0 = 1$, which corresponds to the mean value, whereas the wavelet coefficients encode the differences between approximations at two different time scales which correspond to the details added to get a finer time resolution. In this article we use the Coifman 12 wavelet, which generates all functions of the wavelet basis from a set of two discrete filters, a low-pass and a band-pass filter, each of length 12.[27] The scaling function $\phi(t)$, defined by the low-pass filter, and the corresponding wavelet $\psi(t)$, defined by the band-pass filter, together with the modulus of their Fourier transforms $|\hat{\phi}(\omega)|$ and $|\hat{\psi}(\omega)|$, are shown in Fig. 1. The Fourier transform we use is defined by

$$\hat{\phi}(\omega) = \int_{-\infty}^{\infty} \phi(t) e^{-\iota 2\pi\omega t} dt, \quad (3)$$

with $\iota = \sqrt{-1}$.

C. Wavelet de-noising

As explained previously, we define the coherent bursts to be what remain after de-noising the turbulent signal $S(t)$. We then propose a wavelet-based method to split the signal $S(t)$ into two orthogonal components: the coherent signal $S^C(t)$, which retains the coherent bursts, and the incoherent signal $S^I(t)$, which corresponds to the turbulent fluctuations assumed to be noise-like. For this we first project $S(t)$ onto an orthogonal wavelet basis and we compute a threshold value ϵ. We then separate the wavelet coefficients $\widetilde{S}_{ij}$ into two classes: those whose modulus is larger than the threshold value ϵ correspond to the coherent coefficients $\widetilde{S}^C_{ij}$, whereas the remaining coefficients correspond to the incoherent coefficients $\widetilde{S}^I_{ij}$. Finally, the coherent component is reconstructed in physical space using the inverse wavelet transform to get $S^C(t)$, whereas the incoherent components is easily obtained as $S^I(t)=S(t)-S^C(t)$. It could also be obtained by applying the inverse wavelet transform to $\widetilde{S}^I_{ij}$.

We choose the simplest model for the noise to be eliminated, we suppose it to be additive, Gaussian and white. If we know *a priori* the noise's variance σ^2, the optimal threshold value is given by

$$\epsilon_D = (2 \ln N\sigma^2)^{1/2}. \tag{4}$$

Indeed, Donoho and Johnstone[24] have proven that such a wavelet thresholding is optimal to de-noise signals in the presence of additive Gaussian white noise, because it minimizes the maximal L^2 error (between the de-noised signal and the noise-free signal) for functions with inhomogeneous regularity, such as intermittent signals. However, to compute the threshold ϵ_D the variance of the noise has to be known.

In Refs. 26 and 15 we have proposed a recursive algorithm to estimate the variance of the noise when it is not known *a priori*, which is the case for most practical applications, in particular for coherent burst extraction. The recursive algorithm is based on the observation that, given a threshold ϵ_n, the variance of the noise estimated using Parseval's theorem

$$\sigma_n^2 = \frac{1}{N} \sum_{(ji)\in\Lambda^J, |\widetilde{S}_{ji}|<\epsilon_n} |\widetilde{S}_{ji}|^2 \tag{5}$$

yields a new variance σ^2_{n+1} and hence a threshold ϵ_{n+1} closer to the optimal threshold ϵ_D than ϵ_n. In Ref. 26 we studied the mathematical properties of this algorithm and proved its convergence for signals having sufficiently sparse representation in wavelet space, such as intermittent signals.

D. Algorithm

The recursive extraction algorithm can be summarized as follows.

(1) Initialization:

- Given the signal $S(t)$ of duration T, sampled on an equidistant grid $t_i=iT/N$ for $i=0, N-1$, with $N=2^J$;
- set $n=0$ and perform a wavelet decomposition, i.e., apply the fast wavelet transform (FWT)[27] to S to obtain the wavelet coefficients $\widetilde{S}_{ji}$ for $(j,i)\in\Lambda_J$;
- compute the variance σ_0^2 of S as a rough estimate of the variance of the incoherent signal S^I and compute the corresponding threshold $\epsilon_0=(2\ln N\sigma_0^2)^{1/2}$, where $\sigma_0^2 = 1/N\Sigma_{(j,i)\in\Lambda^J}|\widetilde{S}_{ji}|^2$; and
- set the number of coefficients considered as noise to $N_I = N$, i.e., to the total number of wavelet coefficients.

(2) Main loop:
Repeat the following until $(N_I = = N_I^{\text{old}})$

- set $N_I^{\text{old}}=N_I$ and count the number of wavelet coefficients smaller than ϵ_n, which yields a new value for N_I;
- compute the new variance σ^2_{n+1} from the wavelet coefficients smaller than ϵ_n, i.e., $\sigma^2_{n+1}=1/N\Sigma_{(j,i)\in\Lambda^J}|\widetilde{S}^I_{ji}|^2$, where

$$\widetilde{S}^I_{ji} = \begin{cases} \widetilde{S}_{ji} & \text{for } |\widetilde{S}_{ji}| \leq \epsilon_n \\ 0 & \text{else,} \end{cases} \tag{6}$$

and the new threshold $\epsilon_{n+1}=(2\ln N\sigma^2_{n+1})^{1/2}$; and
- set $n=n+1$.

(3) Final step:

- Reconstruct the coherent signal S^C from the coefficients $\widetilde{S}^C_{ji}$ using the inverse FWT, where

$$\widetilde{S}^C_{ji} = \begin{cases} \widetilde{S}_{ji} & \text{for } |\widetilde{S}_{ji}| > \epsilon_n \\ 0 & \text{else} \end{cases} \tag{7}$$

and
- finally, compute pointwise the incoherent signal $S^I(t_i) = S(t_i) - S^C(t_i)$ for $i=0,\ldots,N-1$.

(4) End:
Note that the decomposition yields $S(t)=S^C(t)+S^I(t)$ and orthogonality implies that energy is split into $\sigma^2=\sigma_C^2+\sigma_I^2$, since $\langle S^C, S^I\rangle=0$.

The FWT, proposed by Mallat,[27] requires $(2mN)$ multiplications for its computation, where m is the length of the discrete filter defining the orthogonal wavelet used. Hence, the extraction algorithm we propose is computed in $(2nmN)$ operations, with a number of iterations n very small, typically less than $\log_2 N$. Recall that the operation count for the fast Fourier transform is proportional to $N\log_2 N$ operations.

This algorithm defines a sequence of estimated thresholds $(\epsilon_n)_{n\in\mathbb{N}}$ and the corresponding sequence of estimated variances $(\sigma_n^2)_{n\in\mathbb{N}}$. The convergence of these sequences within a finite number of iterations has been demonstrated in Ref. 26 applying a fixed point type argument to the iteration function

$$\mathcal{I}_{S,N}(\epsilon_{n+1}) = \left(\frac{2\ln N}{N} \sum_{(j,i)\in\Lambda^J} |\widetilde{S}^I_{ji}(\epsilon_n)|^2 \right)^{1/2}. \tag{8}$$

The algorithm thus stops after n iterations when $\mathcal{I}_{S,N}(\epsilon_n) = \epsilon_{n+1}$.

Further, we have shown that the convergence rate of the recursive algorithm depends on the signal to noise ratio $(\text{SNR}=10\log_{10}(\sigma^2/\sigma_I^2))$, since the smaller the SNR, i.e., the stronger the noise, the faster the convergence. Moreover, if the algorithm is applied to a Gaussian white noise only, it converges in one iteration and removes the noise (in statistical mean). If it is applied to a signal without noise, the signal

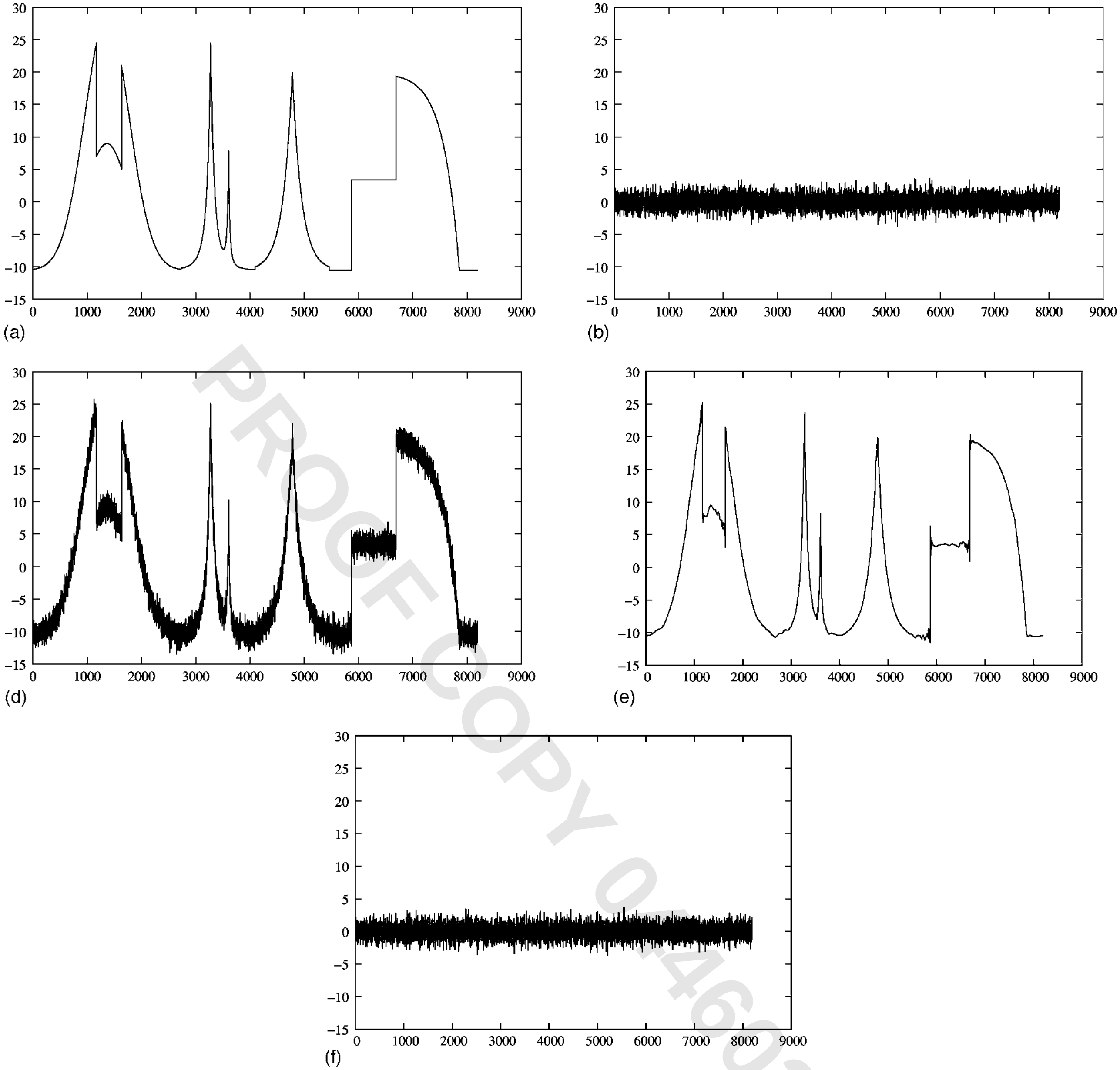

FIG. 2. (Top) Construction of a (middle) 1D noisy signal $S=F+W$, and (bottom) results obtained by the recursive algorithm, which gives $S=S^C+S^I$.

is fully preserved. Finally, we have proven that the algorithm is idempotent, i.e., if we apply it several times, the noise is eliminated the first time, and the coherent signal is no more modified in the subsequent applications, as it would have been the case for a Gaussian filter. As a consequence, this algorithm yields a nonlinear projector.[26]

E. Application to an academic test signal

To illustrate the properties of the recursive algorithm we apply it to a 1D noisy test signal S (Fig. 2, middle). This signal has been constructed by superposing a Gaussian white noise W, with zero mean and variance $\sigma_W^2=1$, to a function F, normalized such that its variance yields 10, which corresponds to a signal to noise ratio $\mathrm{SNR}=10\log_{10}(\sigma_F^2/\sigma_W^2)=20$ dB (Fig. 2, top). The function F is a piecewise polynomial function which presents several discontinuities, either in the function or in its derivatives. The number of samples is $N=2^{13}=8192$.

We apply the recursive extraction algorithm to the test signal $S(t)$ and obtain after $n=5$ iterations the coherent part $S^C(t)$ and the incoherent noise $S^I(t)$ (cf. Fig. 2, bottom). We observe that $S^C(t)$ yields a de-noised version of the test signal $S(t)$ which is very close to $F(t)$, whereas the incoherent part $S^I(t)$ is homogeneous and noise like with flatness $\mathcal{F}=3.03$, which corresponds to quasi-Gaussianity. Note that the flatness $\mathcal{F}$ is defined as the ratio of the centered fourth order moment divided by the square of the variance, and $\mathcal{F}=3$ for a Gaussian process. Fig. 2 (bottom, left) shows that the coherent signal retains all discontinuities and spikes present in the original function $F(t)$, without smoothing them as it would have been the case with standard de-noising methods,

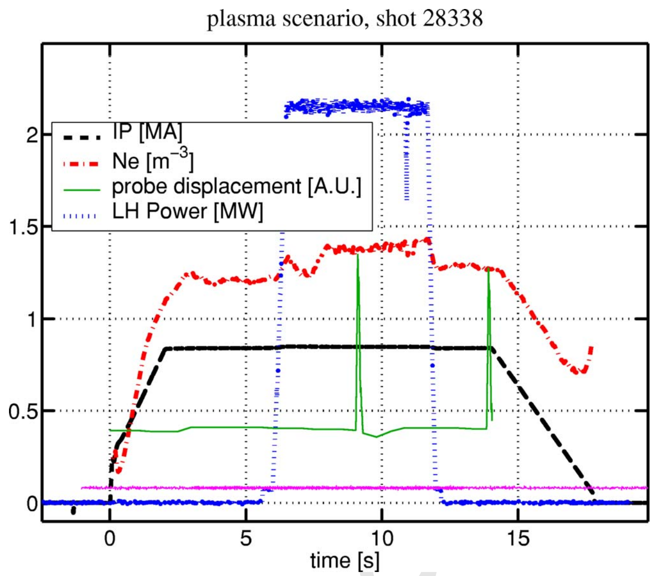

FIG. 3. Plasma scenario of the shot 28338 from the tokamak Tore Supra, Cadarache. The duration of the shot is 18 s. The plasma density fluctuations are measured by a fast reciprocating Langmuir probe. When the probe is 2.8 cm away from the LCFS in the SOL, the signal is acquired during time windows of 8 ms.

e.g., with low pass Fourier filtering. Nevertheless, we observe slight overshoots in the vicinity of the discontinuities, although they remain much more local than the classical Gibbs phenomena, and could easily be removed using the translation invariant wavelet transform.[27]

III. APPLICATION TO TURBULENT EDGE PLASMA

A. Density fluctuations

We have measured the time evolution of the ion saturation current during 8 ms in the SOL of the tokamak Tore Supra in Cadarache (France). This signal, denoted $S(t)$, gives an approximation of the density fluctuations.

The measure was taken according to the following plasma scenario: the shot 28338 lasted 18 s and the signal has been recorded in the middle of the plasma current plateau. The large radius was $R=2.33$ m, the small radius $a=0.77$ m, the mean plasma density $\bar{n}_i=1.37\times10^{19}$ m^{-3}, the plasma current $I_p=0.84$ MA and the edge safety factor $q=6.71$. Moreover, 2.1 MW of lower hybrid waves were applied to the plasma.

The ion saturation current fluctuations were measured by a fast reciprocating Langmuir probe. The total duration of the probe motion into the plasma was 300 ms and, when the probe reached 2.8 cm away from the last closed flux surface (LCFS). The signal has been recorded at 1 MHz during 8 ms (Fig. 3), which gives $N=2^{13}=8192$ samples. A high-pass filter at frequency 0.1 kHz and a low-pass filter at frequency 500 kHz have been applied to eliminate both low frequencies and aliasing.

B. Extraction of coherent bursts

We use the wavelet extraction algorithm to split the signal $S(t)$ (Fig. 4, top) into two orthogonal components, the

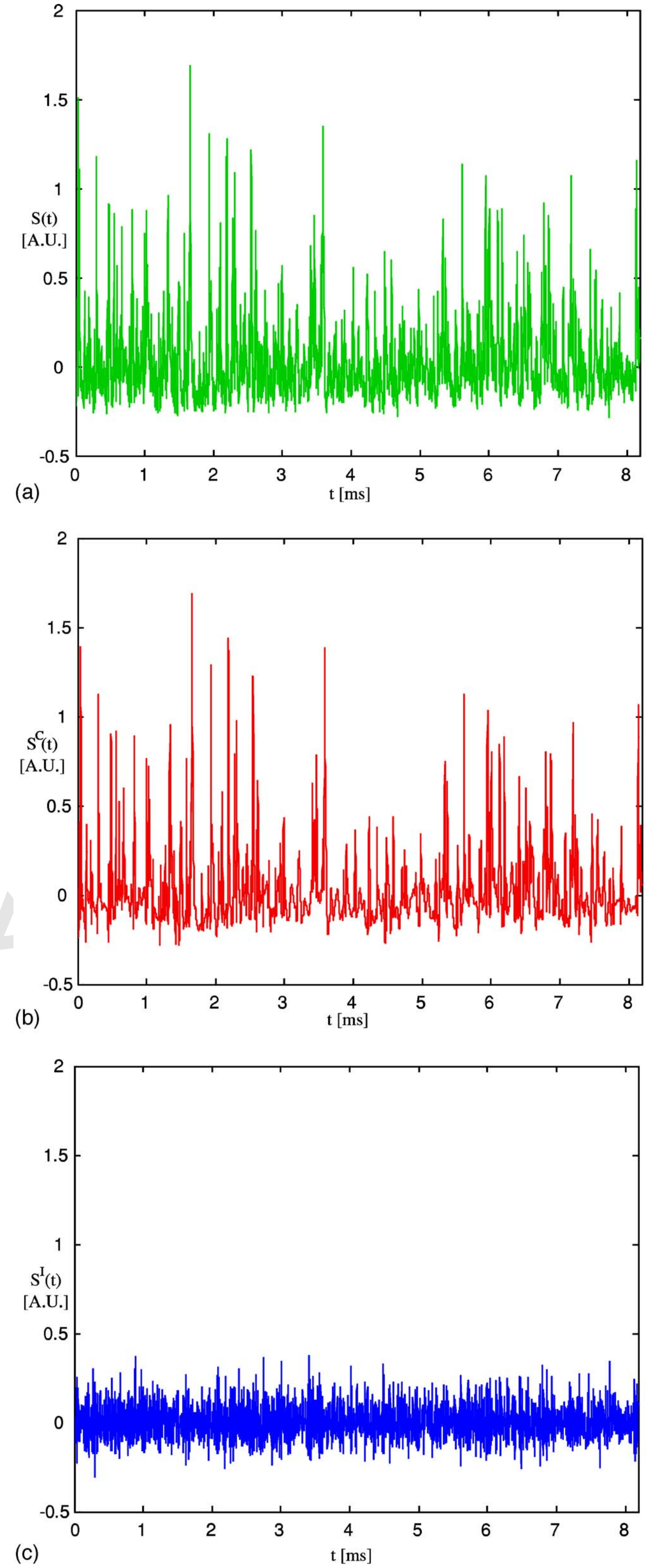

FIG. 4. Signal $S(t)$ of duration 8.192 ms, corresponding to saturation current fluctuations measured at 1 MHz in the SOL of the tokamak Tore Supra, Cadarache. (Top) Total signal S, (middle) coherent part S^C, and (bottom) incoherent part S^I.

coherent bursts $S^C(t)$ (Fig. 4, middle) and the incoherent turbulent fluctuations $S^I(t)$ (Fig. 4, bottom). The optimal threshold value has been obtained after $n=12$ iterations of the algorithm (Fig. 5).

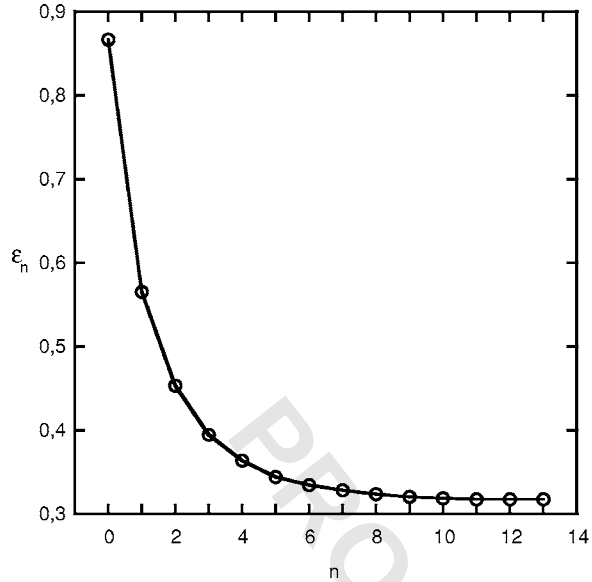

FIG. 5. Threshold value ϵ_n vs iteration number n.

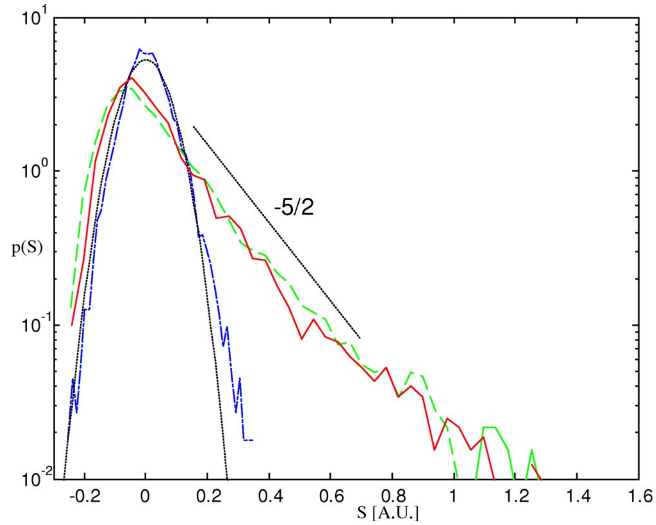

FIG. 6. Probability density function $p(S)$, estimated using histograms with 50 bins. PDF of the total signal S (green dashed line), of the coherent component S^C (red solid line), and of the incoherent component S^I (blue dotted-dashed line), together with a Gaussian fit with variance σ_I^2 (black dotted line).

As results, we observe that the coherent signal $S^C(t)$, made of 5.8%N wavelet coefficients, retains 86.6% of the total variance and the extrema are preserved (Table I). In contrast, the incoherent contribution $S^I(t)$, is made of 94.2%N wavelet coefficients but contributes to only 3.4% of the total variance (Table I), which corresponds to a signal to noise ratio $\mathrm{SNR}=10\log_{10}(\sigma^2/\sigma_I^2)=8.72$ dB.

The decomposition shows that the bursty and coherent part of the signal dominates over the turbulent fluctuations of the background, this more strongly than what has been found with previous methods based on clipping.[10]

APC: #2 Figure 6 shows the PDFs in log-lin coordinates for the total, coherent and incoherent contributions, estimated using histograms with 50 bins with integrals normalized to one. The PDFs of the total signal and the coherent part are skewed and present the same behavior: positive values have exponential tails with $p(S)\propto\exp(-5/2S)$, whereas negative values yield a Gaussian behavior (Fig. 6). In contrast, the PDF of the incoherent component is almost symmetric, with skewness 0.38, instead of 2.56 and 2.84 for the total and coherent part, respectively. It has a quasi-Gaussian shape with flatness 4.03, instead of 12.00 and 14.22, respectively (Fig. 6).

TABLE I. Statistical properties of the signal $S(t)$ from the tokamak Tore Supra, Cadarache, for the signal and its coherent and incoherent components using the Coifman 12 orthogonal wavelet.

Properties	Total S	Coherent S^C	Incoherent S^I
Number of coefficients	8192	479	7713
Percent of coefficients	100	5.8	94.2
Minimum value	−0.284	−0.282	−0.307
Maximum value	1.689	1.686	0.374
Mean value	0.019	0.019	$<10^{-11}$
Variance σ^2	0.0417	0.0361	0.0056
Percent of variance	100	86.6	3.4
Skewness	2.564	2.842	0.383
Flatness	12.001	14.224	4.026

C. Fourier spectrum and modified periodogram

To get more information on the spectral distribution of the density variance for the different components, we consider the Fourier spectrum

$$E(\omega)=\frac{1}{2}|\hat{S}(\omega)|^2, \tag{9}$$

where $\hat{S}(\omega)$ denotes the Fourier transform as defined in Eq. (3). As estimator for the spectrum we take the periodogram, which is a discrete version of Eq. (9), although it is known to be a inconsistent estimator due to the presence of oscillations.[28] To obtain a consistent estimator we also compute the modified periodogram, by first tapering the data with a raised cosine window (affecting 40 data points at each boundary), and then convolving the periodogram with a Gaussian window (with standard deviation of 40 data points). Figure 7 shows the periodogram and the modified periodogram for S, S^C, and S^I, which confirms that the latter yields a stabilized estimator of the spectrum, with no more oscillations.

D. Wavelet spectrum

The wavelet decomposition, given in Eq. (1), yields the distribution of the variance of the signal scale per scale, which is called scalogram.[19] It is defined as

$$\widetilde{E}_j=\frac{1}{2}\sum_{i=0}^{2^j-1}(\widetilde{S}_{ji})^2. \tag{10}$$

Parseval's theorem implies that $E=\Sigma_{j\geqslant 0}\widetilde{E}_j$. Using the relation $\omega_j=\omega_\psi/2^j$ between the scale index j and the frequency

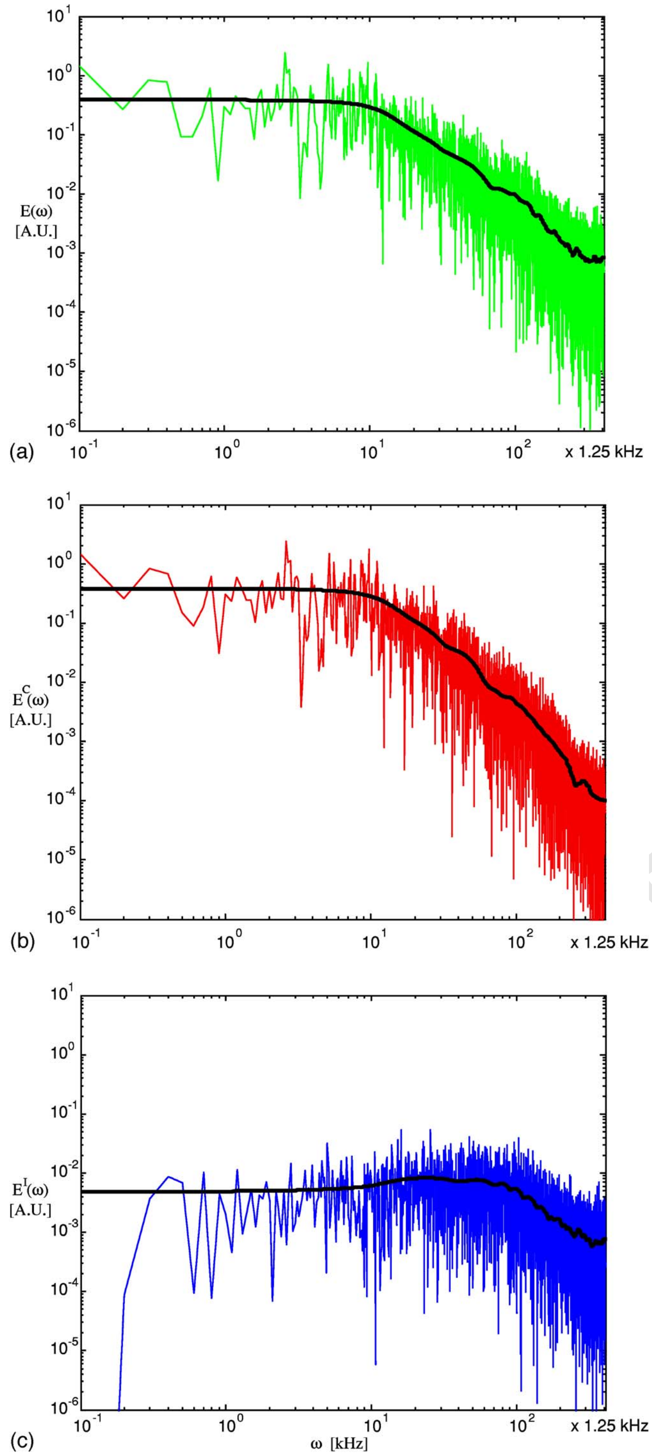

FIG. 7. Fourier spectrum $E(\omega)$. (Top) Spectrum of the total signal $S(t)$, (middle) spectrum of the coherent component $S^C(t)$, and (bottom) spectrum of the incoherent component $S^I(t)$. Note that the periodogram is plotted in green, red, and blue for the total, coherent, and incoherent signals, respectively. Superimposed are the modified periodograms (black thick line).

ω, the wavelet spectrum can be defined as $\widetilde{E}(\omega_j)=\widetilde{E}_j/\omega_\psi$, with ω_ψ being the centroid frequency of the mother wavelet whose value is $w_\psi=1.3$ for the Coifman 12 wavelet used here. It corresponds to a smoothed version of the Fourier spectrum (9), the smoothing kernel being the square of the Fourier transform of the wavelet, since

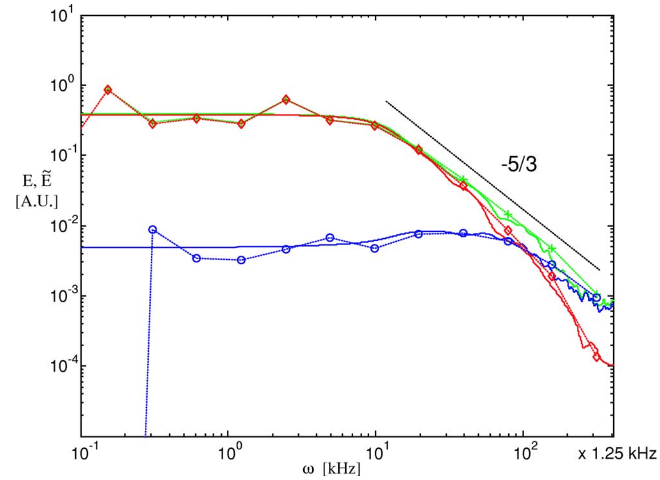

FIG. 8. Wavelet spectra $\widetilde{E}(\omega_j)$ (lines with symbols) and modified periodograms $E(\omega)$ (lines) of the total signal S (green and +), of the coherent signal S^C (red and ◇) and of the incoherent signal S^I (blue and ○).

$$\widetilde{E}(\omega)=\frac{1}{\omega_\psi}\int_0^{+\infty} E(\omega')\left|\hat{\psi}\left(\frac{\omega_\psi\omega'}{\omega}\right)\right|^2 d\omega'. \qquad (11)$$

Note that, as the frequency increases, i.e., when one goes to small scale, the smoothing interval becomes larger which explains why the wavelet spectrum is a well-conditioned statistical estimator. The advantage of the wavelet spectrum in comparison to the modified periodogram is that the smoothing window is automatically adjusted by the wavelet representation, since wavelets correspond to filters with constant relative bandwidth $\Delta\omega/\omega$.[19]

In Fig. 8 the wavelet spectra, together with the modified periodograms, are displayed.

We observe that the signal and its coherent component present a similar scaling in $\omega^{-5/3}$, which characterizes correlation since the spectral slope is negative. As proposed in Ref. 10 this may be interpreted as an inverse energy cascade, similar to what is encountered in 2D fluid turbulence. In contrast, the incoherent component has a different scaling with a flat spectrum up to frequency $\omega=120$ kHz, corresponding to decorrelation. For higher frequencies we observe a ω^{-1} scaling, which may be due to experimental noise, which presents the same scaling at high frequencies, although its amplitude remains smaller than the incoherent fluctuations. Figure 8 also shows that the wavelet spectrum almost coincides with the modified periodogram, and that the higher the frequency the better the stabilization thus obtained.

Note that the scalogram and the wavelet spectrum are optimal to characterize scaling laws, as long as the analyzing wavelet has at least M vanishing moments, with $M>\beta-1/2$, to detect power laws in $\omega^{-\beta}$, see, e.g., Refs. 21 and 29.

E. Intermittency

Intermittency characterizes the fact that the time support of the fluctuations decreases with scale.[30,31] It therefore quantifies how bursty a signal is. Townsend[32] has proposed the "intermittency factor" as the ratio between the time supports of active and quiescent regions. But the main deficiency is that intermittency factors depend on the choice of the threshold below which the variation is considered to be inactive.[33] As we have already mentioned previously, one of the drawbacks of such a clipping method is that the active bursts, and the corresponding intermittency factor, depend on the choice of the threshold, which can be avoided by using the wavelet representation.

Biskamp stated in[30] that "the spottiness of the dissipative eddies is a special feature of what is now believed to be a general property of fully developed turbulence that with decreasing scale turbulent fluctuations become less and less space-filling, i.e., are concentrated in regions of smaller and smaller volume but increasingly complicated shape. This phenomenon is called intermittency, which is a central topic in actual turbulence research." Frisch explained in Ref. 31 that intermittency can be quantified by computing the variation of the flatness when scale decrease: if flatness remains constant the signal is nonintermittent, if it increase when scale decreases it is intermittent. We use the same definition of intermittency and compute the scale dependent flatness from the higher order moments of the wavelet coefficients $\widetilde{S}_{ji}$, as introduced in Refs. 21 and 29. By summing up the pth power of the wavelet coefficients over all positions i, one obtains the pth order moments

$$\widetilde{\mathcal{M}}_j^p = \frac{1}{2^j}\sum_{i=0}^{2^j-1} (\widetilde{S}_{ji})^p . \qquad (12)$$

The scale dependent flatness is then defined as

$$\widetilde{\mathcal{F}}_j = \frac{\widetilde{\mathcal{M}}_j^4}{(\widetilde{\mathcal{M}}_j^2)^2} . \qquad (13)$$

The relation between scale and frequency allows one to express the flatness as a function of the frequency ω_j, similarly to the wavelet spectrum. Note that Gaussian white noise, which is by definition nonintermittent, would yield a flatness equal to three for all frequencies.

To characterize the intermittency of the signal and its different contributions we plot in Fig. 9 the flatness $\widetilde{\mathcal{F}}_j$ versus the frequency ω_j. We observe that the flatness of the coherent contribution increases faster for high frequencies than that of the total signal. This proves that the coherent contribution is more intermittent than the signal itself, which is obvious since it only retains the bursts. In contrast, the flatness of the incoherent contribution decreases to the value $\widetilde{\mathcal{F}}_j=3$, up to frequency $\omega=120$ kHz, which gives evidence for its nonintermittent behavior. The wavelet based flatness corresponds to the flatness of the band-pass filtered signal, as typically used in the fluid turbulence community.[31] Note that the signal reconstructed from its wavelet coefficients at given scale j corresponds to the band-pass filtered signal around the frequency $\omega_j=\omega_\psi/2^j$.

For comparison we also show in Fig. 10 the flatness of the low-pass filtered signal, for dyadically increasing cutoff frequencies $\omega_C=\omega_\psi/2^{J_C}$. Therefore, we reconstruct the signal in physical space on N grid points using only wavelet coefficients up to a given scale J_C, corresponding to the filter cutoff. The wavelet coefficients for scales $j \geqslant J_C$ are set to zero and the low-pass filtered signal is computed by the inverse wavelet transform using Eq. (1).

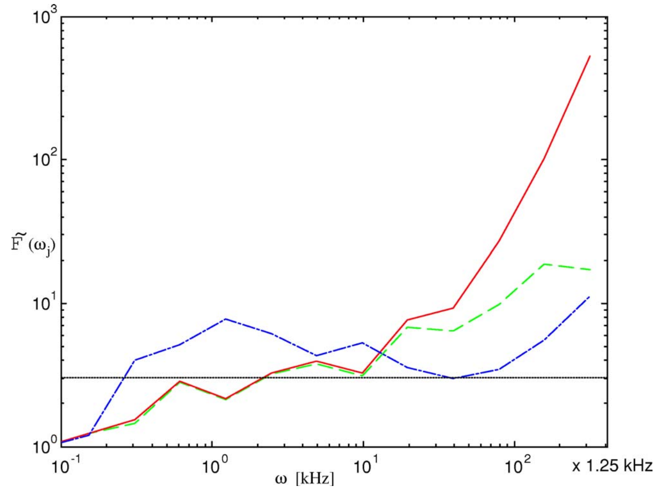

FIG. 9. Flatness of the band-pass filtered signal $\widetilde{\mathcal{F}}$ vs frequency ω_j for the total signal S (green dashed line), of the coherent signal S^C (red solid line), and of the incoherent signal S^I (blue dotted-dashed line). The horizontal dotted line $\mathcal{F}(\omega_j)=3$ corresponds to the flatness of a Gaussian process.

Similarly to Fig. 9, we observe in Fig. 10 that the flatness of the total and coherent signal increases with frequency for $\omega > 3$ kHz. Considering the signal filtered at 20 kHz we observe that its flatness is just above 7, however the signal contains only large bursts, since all smaller details have been filtered out. This shows that the signal is already intermittent at large scales. For the small scales, i.e., for $\omega \geqslant 20$ kHz, the flatness of the total and the coherent signal is above 10. This shows that adding small details to the large scale bursts increases the flatness, and hence the signal's intermittency as quantified by its flatness.

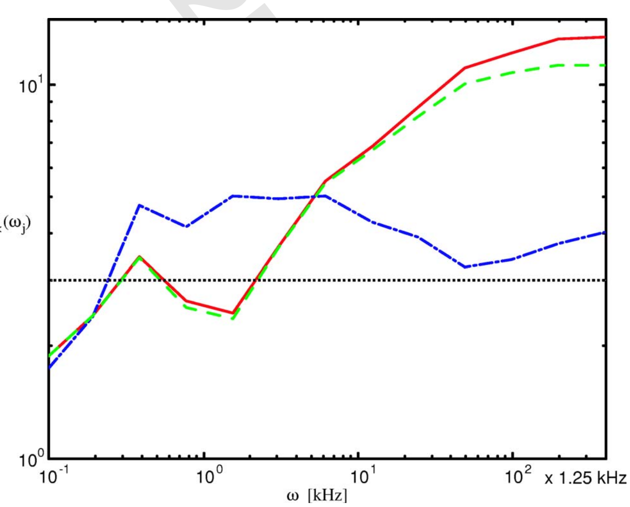

FIG. 10. Flatness of the low-pass filtered signal $\mathcal{F}_<$ vs frequency ω_j for the total signal S (green dashed line), of the coherent signal S_C (red solid line), and of the incoherent signal S^I (blue dotted-dashed line). The horizontal dotted line $\mathcal{F}_>(\omega_j)=3$ corresponds to the flatness of a Gaussian process.

The flatness $\mathcal{F}_<$ of the low-pass filtered signal, considered for increasing cutoff frequencies, quantifies the intermittency of the signal reconstructed up to the corresponding cutoff scales, whereas the flatness $\widetilde{\mathcal{F}}$ of the band-pass filtered signal, considered for bands of increasing wave number, yields incremental information on the flatness of the signal scale by scale. The latter quantity can be compared with the energy spectrum which gives the energy distribution scale by scale, whereas the former gives some cumulative information, i.e., information on the flatness of the lower frequency contributions of the signal is included in the flatness of the higher frequency contributions. Hence, both quantities do not yield the same values if the PDF of the signal varies with scale.

IV. CONCLUSION

We presented a wavelet-based recursive method to extract coherent bursts out of turbulent signals. The algorithm decomposes the signal into an orthogonal wavelet basis and reconstructs the coherent contribution from the wavelet coefficients whose modulus is larger than a given threshold. The threshold value is recursively determined without any adjustable parameter. Moreover, we have shown that this algorithm is fast since it has only linear complexity.

Compared to classical extraction methods, which are based, either on thresholding in physical space ("clipping"), or on conditional averaging, working in wavelet space presents the following advantages:

(i) there is no need to suppose the signal to be statistically stationary in time,
(ii) the wavelet decomposition preserves the spectral properties of the signal, and thus respects its scaling as long as the analyzing wavelet is smooth enough (which depends on the number of vanishing moments for orthogonal wavelets),
(iii) the wavelet-based extraction method does not require any prior about the shape or the intensity of the bursts to be extracted; the only prior is to assume the noise to be Gaussian and white.

We have applied this recursive wavelet algorithm to ion saturation current measured in the SOL of the tokamak Tore Supra. We have thus extracted the coherent bursts from an incoherent background noise. The former contains most of the density variance and are correlated with non-Gaussian statistics, whereas the latter is almost decorrelated and quasi-Gaussian. We have also observed that the non-Gaussianity of the PDF of the coherent component increases with the frequency, which confirms that the bursts are highly intermittent. In contrast, the incoherent component remains quasi-Gaussian up to high frequencies, which confirms the nonintermittency of the background noise. By analogy with previous studies we have made in the context of 2D fluid turbulence,[34] we conjecture that the coherent bursts are due to organized structures produced by nonlinear interactions and responsible for turbulent transport. On the other hand, the incoherent background corresponds to the turbulent fluctuations which only contribute to turbulent diffusion. Moreover the variance of the incoherent fluctuations yields a good estimation of the turbulence level.

In Ref. 35 we applied this extraction method to both plasma velocity and density signals, measured at different poloidal positions, to study turbulent fluxes and thus characterize the transport properties of the coherent bursts. These results will be subject of a forthcoming article. We also have already extended this extraction method to treat 2D and 3D, scalar and vector, fields,[15–17] and we plan to apply it to spatiotemporal signals and images of plasma density fluctuations, obtained by fast framing cameras, to improve the characterization of coherent bursts.

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AUTHOR QUERIES — 044602PHP

#1 Author: Can you please double check the word apophatic as it cannot be found in the dictionary.
#2 Author: Please clarify what "lin" stands for in the phrase "log-lin."
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#6 Author: For Ref. 23, please double check the accuracy of the volume, page, and year.

3 Mars 2006
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6 Mars 2006
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Joanne Hensel nous renvoie la liste des règles à suivre pour les auteurs

Date: Mon, 06 Mar 2006 14:54:37 -0500
From: Joanne Hensel <jhensel@aip.org>
To: farge@lmd.ens.fr
Subject: Re: Physics of Plasmas paper 044602PHP: Figure 2

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8 Mars 2006
Notre réponse au mail de Joanne Hensel
suite à notre remarque sur le manque de soutien technique

From farge@lmd.ens.fr Wed Mar 8 20:34:54 2006
Date: Wed, 8 Mar 2006 20:34:54 +0100 (CET)
From: farge <farge@lmd.ens.fr>
To: jhensel@aip.org

Cc: dbrzozow@aip.org, mcarrich@aip.org, mlynch@aip.org, Schneider Kai <kschneid@cmi.univ-mrs.fr>, Devynck Pascal <devynck@pegase.cad.cea.fr>
Subject: Re: Physics of Plasmas paper 044602PHP

Dear Joanne Hensel,

> Date: Mon, 06 Mar 2006 14:54:37 -0500
> From: Joanne Hensel <jhensel@aip.org>
> To: farge@lmd.ens.fr
> Subject: Re: Physics of Plasmas paper 044602PHP: Figure 2
>
> Dear Dr. Farge,
>
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It may not be your intention to cause difficulty, but you must realise that you do by your unreasonable requirements cause great difficulty, and intense irritation, for your authors, without whom your journals would cease to exist. I appreciate of course that you "are attempting to provide authors and their reading audience with a quality product"; please be aware that this is what your authors are doing all the time in carrying out their research work and writing their papers for submission to your journals. For this, of course, they receive no payment; **if any profit is made, it is the journal that benefits. It is not unreasonable therefore for authors to expect that your editorial staff should be competent to carry out whatever editing of submitted figures may be required to fit page layout etc. I repeat: we are authors, not printers. We expect you, the Publishers, to have the specialised knowledge to ensure good page layout of the final product.**

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Marie Farge

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11 Mars 2006
Envoi des corrections
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From farge@lmd.ens.fr Sat Mar 11 19:41:47 2006
Date: Sat, 11 Mar 2006 19:41:44 +0100 (CET)
From: farge <farge@lmd.ens.fr>
To: Joanne Hensel <jhensel@aip.org>
Cc: Schneider Kai <kschneid@cmi.univ-mrs.fr>, Devynck Pascal <devynck@pegase.cad.cea.fr>
Subject: Proofs : Physics of Plasmas paper 044602PHP

Dear Joanne Hensel,

> Attached please find the revised version of your article to be published in
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Thank you very much for sending me a new version of the proofs. Unfortunately there are stil quite many problems, since my handwriting may be difficult to read. Here is the list of errors to be corrected.

Thank you very much for your help,
Yours Sincerely,

Marie Farge

- Page 3, paragraph B

... between approximations at two succession scales,...
should be replaced by:

successive scales

- Page 4, paragrath D

the headers (1) Initialization
(2) Main loop
(3) Final step
(4) End
should be enhanced, either by being in italic or in bold characters.

In my first corrections I have asked you to add some blank lines to let them appear as separated from the text. Please, try to find a better solution, since the present one is not good.

- Page 5, paragraph E

... its variance yields 60, which...
should be replaced by

yields 100

- Page 6, paragraph A

... gives an extimation of the density...
should be replaced by:

estimation

- Page 6, caption of Figure 4

... corresponding to saturation current fluctuations...
should be replaced by:

to the saturation current fluctuations

- Page 7, paragraph B

... dominates over the turbulent fluctuations background...
should be replaced by:

turbulent background fluctuations

- Page 7, paragraph B

... in semi logarithmatic corrdinates...
should be replaced by:

semi-logarithmic

- Page 7, caption of Figure 6

...estimated using histograms with...
should be replaced by:

histogram ------ no s

- Page 8, paragraph D

... smoothing interval becomes larger which...
shoould be replaced by:

larger, which ------ add a coma

- Page 8, paragraph D

...the higher the frequency the better the stabilization...
should be replaced by:

frequency, the better ----- add a coma

- Page 9, paragraph E

In the formula for the scale dependent flatness \tilde{F_j} the position of the full stop is weird. You should move it up on the right. I have already asked you that in the first check of the proofs, do you have a technical reason for not being able to do this?

- Page 10, Conclusion
... spatio-temporal signals and its images...
should be replaced by:

signals and to images

- Page 10, Acknowledgments
... is grateful to Trinity College, Cambridge (U.K.), for its hospitality.
Two of the authors...

should be replaced by:

grateful to the fellows of Trinity College, Cambridge (U.K.), for their hospitality. Two of the authors

--------- please discard the carriage return after 'hospitality', as I have already asked in my check.

- Page 10, Reference 3
... Plasma Physics 38, Special Issue, 74-79 (1998).
should be replaced by:

38, 74-79 -------discard 'Special Issue' as previously asked.

- Page 10, Reference 13
... Paris, 30, II, 1479 (1988).
should be replaced by:

Paris, II, 30 ______invert '30' and 'II', as prveviously asked

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29 Mars 2006
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des corrections du second jeu d'épreuves

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Version publiée de l'article

Extraction of coherent bursts from turbulent edge plasma in magnetic fusion devices using orthogonal wavelets

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LMD-IPSL-CNRS, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05, France

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LMSNM-CNRS & CMI, Université de Provence, 39 rue F. Joliot-Curie, 13453 Marseille Cedex 13, France

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(Received 9 May 2005; accepted 16 January 2006; published online 18 April 2006)

A new method to extract coherent bursts from turbulent signals is presented. It uses the wavelet representation which keeps track of both time and scale and thus preserves the temporal structure of the analyzed signal, in contrast to the Fourier representation which scrambles it among the phases of all Fourier coefficients. Using orthogonal wavelets, turbulent signals can be decomposed into coherent and incoherent components, which are orthogonal and whose properties can thus be studied independently. Diagnostics based on the wavelet representation are also introduced to compare the statistical properties of the original signals with their coherent and incoherent components. The wavelet-based extraction method is applied to the saturation current fluctuations measuring the plasma density fluctuations at the edge of the tokamak Tore Supra, Cadarache, France. This procedure disentangles the coherent bursts, which contain most of the density variance, are intermittent and correlated with non-Gaussian statistics, from the incoherent background fluctuations, which are much weaker, non-intermittent, noise-like and almost decorrelated with quasi-Gaussian statistics. We conjecture that the coherent bursts are responsible for turbulent transport, whereas the remaining incoherent fluctuations only contribute to turbulent diffusion.
 [DOI: 10.1063/1.2172350]

I. INTRODUCTION

A. Coherent bursts

The radial transport at the edge of tokamaks is known to be dominated by turbulent processes. Understanding them is important, as they determine the confinement properties of the overall plasma in the bulk region and the energy density to be handled by the limiter or divertor components in the shadowed region of the plasma, where the magnetic field lines are opened. The turbulent transport of plasma density has been extensively studied at the edge of plasma by means of Langmuir probes,[1–3] particles beams,[4,5] and more recently two-dimensional (2D) visible imaging.[6,7] All these diagnostics observe a turbulent transport of the plasma density in the scrape-off layer (SOL) that can be described as a superposition of convective events, which are responsible for the transport of matter over long radial distances at a fraction of the ion sound speed,[8,9] and of background turbulence.

The convective events are detected as coherent bursts of plasma density, but with a signature different from the one expected for turbulent eddies, since they exhibit a probability distribution function (PDF) which is skewed. Typically, it is found that these convective events account for a small fraction of the time and substantial proportion of the turbulence intensity,[10] which underlines their importance in the turbulent transport. There are many efforts to analyze these bursts independently from the background turbulence. For this purpose different extraction methods have been developed, which are based on, either signal clipping (see, e.g., Ref. 10), correlation with given templates, or conditional averaging. These methods require strong hypotheses on the signal, which has to be statistically steady, and also on the bursts, in order to choose the appropriate threshold value. Actually, the clipping method presents two drawbacks. First, the duration of the bursts and their turbulent intensity strongly varies depending on the threshold value (e.g., from 4% to 20% of the total time and between 20% and 50% of the total turbulent intensity[10]), which unfortunately cannot be estimated *a priori*. Second, the clipping method does not preserve the regularity[11] of the signal, since the threshold introduces discontinuities which affect the Fourier spectrum and hence yields an erroneous scaling. Although these methods give some information about the dynamics,[10,12] other methods requiring less hypotheses to extract the bursts are needed.

Since 1988 we have proposed to use the wavelet representation to analyze[13,14] and extract[15–17] coherent structures out of turbulent flow fields, as the wavelet representation does not require any hypothesis on the statistical stationarity and homogeneity of the process under study. In this article we demonstrate the advantages of wavelets to separate coherent bursts from turbulent fluctuations in edge plasma. We present a wavelet-based extraction algorithm, which does not even require any parameter, such as threshold value, to be adjusted. We then apply it to study the plasma density fluctuations measured in the SOL of the tokamak Tore-Supra, Cadarache, France.[18]

B. Wavelet representation

Since turbulent signals are highly fluctuating, one studies them statistically, using classical diagnostics such as correlation functions, spectra or structure functions. Unfortunately those diagnostics lose the temporal structure of the signal, since they are computed with time integrals and the Fourier modes used as basis functions are not localized in time.

The wavelet transform is more appropriate than the Fourier transform to analyze and represent non-stationary, nonhomogeneous, and intermittent signals, such as those encountered in turbulence. It uses analyzing functions which are generated by translation and dilation of a so-called "mother wavelet," which is well localized (i.e., having a finite support) in both physical and spectral space. In contrast, the Fourier transform uses trigonometric functions, which are nonlocal (having an infinite support) in physical space but well localized in spectral space, and the analyzing functions are generated by modulation rather than dilation. The localization of the basis functions and the invariance group of the transform constitute the main differences between wavelet and Fourier representations. For a general presentation of the different types of wavelet transforms and their applications to turbulence, we refer the reader to several review articles.[19–21]

Trigonometric functions used by the Fourier transform oscillate for all times, and the temporal information of the transformed signal is scrambled among the phases of all Fourier coefficients. In contrast, the wavelet coefficients preserve the temporal properties of the signal. Thus, when a wavelet coefficient is filtered out, the effect on the reconstructed signal remains local in time and does not affect the overall signal, as the Fourier transform does. This property allows one to study the behavior of a limited portion of the signal directly from its wavelet coefficients.

If a turbulent signal is stationary, non-intermittent and supposed to be made up of a superposition of waves, not having any nonlinear behavior such as chirps, solitons, or shocks, only in this case one can define without ambiguity the associated frequencies. However, if a turbulent signal is supposed to be a superposition of elementary structures localized in space and time, and nonlinearly interacting (e.g., vortices, shocklets), the wavelet representation should be preferred, because it preserves the locality of information in both space and scale. Actually, these two different transforms translate into mathematical language two different interpretations of turbulent signals.[19]

In the context of plasma physics the continuous wavelet transform has already been used to analyze signals measured in magnetic fusion devices, see e.g., Refs. 22 and 23. In this article we propose to use the orthogonal wavelet transform instead, since it has been proved to be optimal for de-noising signals corrupted with additive Gaussian white noise.[24] A generalization to correlated noise is straightforward, and a similar method has been developed[25] to treat non-Gaussian noises, i.e., χ^2 distribution. To improve the choice of the threshold we have proposed a recursive algorithm,[26] that we have applied to extract coherent structures out of incompressible turbulent flows.[15] In the present article we demonstrate its use to study turbulence in edge plasmas of magnetic fusion devices, such as tokamaks or stellarators.

C. Content

This article is organized as follows. First, we present the wavelet-based extraction method. We then explain the recursive algorithm and validate it on an academic signal. We finally apply it to a saturation current signal measured in the SOL of the tokamak Tore Supra, Cadarache, France. We thus show that the coherent bursts can be efficiently extracted. We also present new statistical diagnostics based on the wavelet representation that we use to compare the original signal with its coherent and incoherent components. Finally, some conclusions are drawn and perspectives for future work are given.

II. EXTRACTION OF COHERENT BURSTS

A. Principle

We propose a new method to extract coherent structures from turbulent flows, as encountered in fluids (e.g., vortices, shocklets) or plasmas (e.g., bursts), in order to study their role in transport and mixing.

As already mentioned, we first replace the Fourier representation by the wavelet representation, which keeps track of both time and scale, instead of frequency only. The second improvement consists in changing our viewpoint about coherent structures. Since there is not yet an universal definition of coherent structures in turbulent flows, we prefer starting from a minimal but more consensual statement about them, that everyone hopefully could agree with: *coherent structures are not noise*. Using this apophatic method we propose the following definition: *coherent structures correspond to what remains after denoising*.

For the noise we use the mathematical definition stating that a noise cannot be compressed in any functional basis. Another way to say this, is to observe that the shortest description of a noise is the noise itself. Notice that plasma physicists typically call "noise" what is actually "experimental noise", measured when there is no plasma. Their definition includes what we define as noise, plus possibly some organized features (e.g., parasite waves) that we do not consider as noise according to the above-mentioned mathematical definition.

This new way of thinking about coherent structures presents the advantage of being able to process "incomplete fields". What does it mean? A typical example of incompleteness is encountered in the experimental setting, where typically one measures the time evolution of a three-dimensional (3D) field using a probe located in one point, thus obtaining a one-dimensional (1D) cut of a four-dimensional space-time field. Notice that incompleteness is different from discretization, i.e., sampling, that one should consider in addition. If the algorithm used to extract coherent structures requires templates of typical structures, it becomes intractable when the measured field is incomplete, because, in order to define the template, one should then consider how the probe sees all possible motions and distortions of the coherent structures passing by, in order to define the templates. Since our algorithm requires a model of the noise, but not of the coherent structures themselves (no templates are needed), it treats any field, complete or incomplete, the same way.

Considering our definition of coherent structures, turbulent signals are split into two contributions: coherent bursts, corresponding to that part of the signal which can be compressed in a wavelet basis, plus incoherent noise, corresponding to that part of the signal which cannot be compressed, neither in wavelets nor in any other basis. We will then check *a posteriori* that the incoherent contribution is spread, and therefore does not compress, in both Fourier and grid point bases. Since we use the orthogonal wavelet representation, both coherent and incoherent components are orthogonal and therefore the total energy is the sum of coherent and incoherent energies.

Assuming that coherent structures are what remains after denoising, we need a model, not for the structures, but for the noise. As first guess, we choose the simplest model and suppose the noise to be additive, Gaussian and white, i.e., uncorrelated. Having this model in mind, we then rely on the theorem of Donoho and Johnstone[24] to compute the value used to threshold the wavelet coefficients. Since the threshold value depends on the variance of the noise, which in the case of turbulence is not *a priori* known, we propose a recursive method to estimate it from the variance of the weakest wavelet coefficients, i.e., those whose modulus is below the threshold value.

After applying our algorithm to a turbulent signal, we then check *a posteriori* that the incoherent component is indeed noise-like, spread in physical space, quasi-Gaussian and quasi-uncorrelated (i.e., spread in Fourier space), which thus confirms the hypotheses we have chosen for the noise.

B. Orthogonal wavelet representation

The construction of orthogonal wavelet bases and the associated fast numerical algorithm are based on the mathematical concept of multiresolution analysis, which considers approximations at different scales. A function or a signal (sampled function) can thus be decomposed into a set of embedded coarser and coarser approximations. The originality of the wavelet representation is to encode the differences between successive finer approximations, instead of the approximations themselves. The amount of information needed to go from a coarse approximation to a finer approximation is then described using orthogonal wavelets. A function or a signal is thus represented by its coarsest approximation, encoded by the scaling coefficients, plus the differences between the successive finer approximations, encoded by the wavelet coefficients.

We consider a signal $S(t)$ of duration T sampled on $N=2^J$ equidistant instants $t_i=iT/N$, with $i=0,\ldots,N-1$. We project it onto an orthogonal wavelet basis[19,27] to represent it at different instants t_i and different time scales $\tau=2^{-j}$, with $j=0,\ldots,J-1$.

The signal is thus developed into an orthogonal wavelet series,

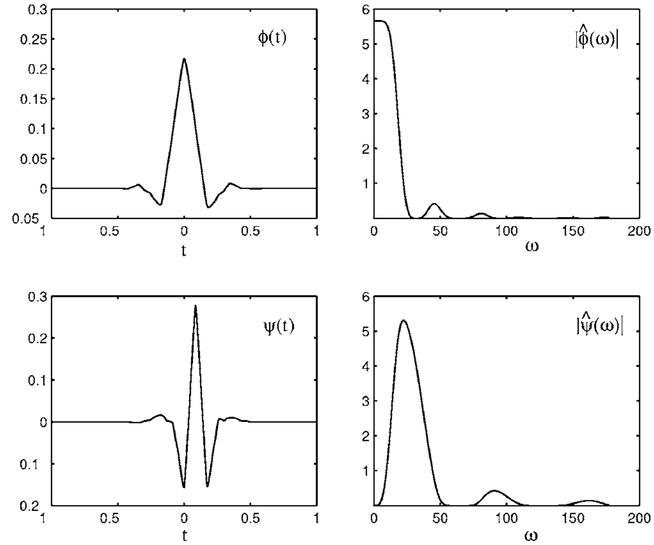

FIG. 1. Coifman 12 wavelet. (Top) Scaling function $\phi(t)$ and the modulus of its Fourier transform $|\hat{\phi}(\omega)|$. (Bottom) Wavelet $\psi(t)$ and the modulus of its Fourier transform $|\hat{\psi}(\omega)|$.

$$S(t) = \bar{S}_{00}\phi_{00}(t) + \sum_{(j,i)\in\Lambda_J} \tilde{S}_{ji}\psi_{ji}(t), \tag{1}$$

where ϕ_{00} is the scaling function and ψ_{ji} the corresponding wavelets, i the index for the instant t and j the index for the time scale τ. To simplify notation, we introduce Λ_J, which indexes all wavelets constituting the basis, defined as

$$\Lambda_J = \{(j,i), \quad j=0,\ldots,J-1, \quad i=0,\ldots,2^j-1\}. \tag{2}$$

Due to orthogonality of the basis functions, the coefficients are computed using the L^2 inner product, denoted by $\langle f,\psi\rangle = \int_{-\infty}^{\infty} f(t)\psi(t)dt$. The scaling coefficients are $\bar{S}_{00}=\langle S,\phi_{00}\rangle$ and the wavelet coefficients are $\tilde{S}_{ji}=\langle S,\psi_{ji}\rangle$. The scaling coefficients encode the approximation of the function S at the largest scale $\tau_0=2^0=1$, which corresponds to the mean value, whereas the wavelet coefficients encode the differences between approximations at two successive scales, which correspond to the details added to get a finer time resolution. In this article we use the Coifman 12 wavelet, which generates all functions of the wavelet basis from a set of two discrete filters, a low-pass and a band-pass filter, each of length 12.[27] The scaling function $\phi(t)$, defined by the low-pass filter, and the corresponding wavelet $\psi(t)$, defined by the band-pass filter, together with the modulus of their Fourier transforms $|\hat{\phi}(\omega)|$ and $|\hat{\psi}(\omega)|$, are shown in Fig. 1. The Fourier transform we use is defined by

$$\hat{\phi}(\omega) = \int_{-\infty}^{\infty} \phi(t)e^{-\iota 2\pi\omega t}dt, \tag{3}$$

with $\iota=\sqrt{-1}$.

C. Wavelet denoising

As explained previously, we define the coherent bursts as what remains after denoising the turbulent signal $S(t)$. We then propose a wavelet-based method to split the signal $S(t)$

into two orthogonal components: the coherent signal $S^C(t)$, which retains the coherent bursts, and the incoherent signal $S^I(t)$, which corresponds to the turbulent fluctuations assumed to be noise-like. For this, we first project $S(t)$ onto an orthogonal wavelet basis and we compute a threshold value ϵ. We then separate the wavelet coefficients $\widetilde{S}_{ij}$ into two classes: those whose modulus is larger than the threshold value ϵ correspond to the coherent coefficients $\widetilde{S}^C_{ij}$, whereas the remaining coefficients correspond to the incoherent coefficients $\widetilde{S}^I_{ij}$. Finally, the coherent component is reconstructed in physical space using the inverse wavelet transform to get $S^C(t)$, whereas the incoherent component is easily obtained as $S^I(t)=S(t)-S^C(t)$. It could also be computed by applying the inverse wavelet transform to $\widetilde{S}^I_{ij}$.

We choose the simplest model for the noise to be eliminated, therfore we suppose it to be additive, Gaussian and white. If we know *a priori* the noise's variance σ^2, the optimal threshold value is given by

$$\epsilon=(2\ln N\sigma^2)^{1/2}. \tag{4}$$

Indeed, Donoho and Johnstone[24] have proven that such a wavelet thresholding is optimal to denoise signals in the presence of additive Gaussian white noise, because it minimizes the maximal L^2 error (between the denoised signal and the noise-free signal) for functions with inhomogeneous regularity, such as intermittent signals. However, to compute the threshold ϵ the variance of the noise has to be known.

In Refs. 26 and 15 we have proposed a recursive algorithm to estimate the variance of the noise when it is not known *a priori*, as it is the case for most practical applications, in particular for coherent bursts extraction. The recursive algorithm is based on the observation that, given a threshold ϵ_n at interation n, the variance of the noise estimated using Parseval's theorem

$$\sigma_n^2=\frac{1}{N}\sum_{(j,i)\in\Lambda_J,|\widetilde{S}_{ji}|<\epsilon_n}|\widetilde{S}_{ji}|^2 \tag{5}$$

yields a new variance σ^2_{n+1}, and hence a threshold ϵ_{n+1} closer to the optimal threshold ϵ than ϵ_n. In Ref. 26 we studied the mathematical properties of this algorithm and proved its convergence for signals having sufficiently sparse representation in wavelet space, such as intermittent signals.

D. Algorithm

The recursive extraction algorithm can be summarized as follows.

(1) Initialization

- Given the signal $S(t)$ of duration T, sampled on an equidistant grid $t_i=iT/N$ for $i=0, N-1$, with $N=2^J$;
- set $n=0$ and perform a wavelet decomposition, i.e., apply the fast wavelet transform (FWT)[27] to S to obtain the wavelet coefficients $\widetilde{S}_{ji}$ for $(j,i)\in\Lambda_J$;
- compute the variance σ_0^2 of S as a rough estimate of the variance of the incoherent signal S^I and compute the corresponding threshold $\epsilon_0=(2\ln N\sigma_0^2)^{1/2}$, where $\sigma_0^2=1/N\Sigma_{(j,i)\in\Lambda_J}|\widetilde{S}_{ji}|^2$;
- set the number of coefficients considered as noise to $N_I=N$, i.e., to the total number of wavelet coefficients.

(2) Main loop

Repeat the following until $(N_I==N_I^{\text{old}})$:

- set $N_I^{\text{old}}=N_I$ and count the number of wavelet coefficients smaller than ϵ_n, which yields a new value for N_I;
- compute the new variance σ^2_{n+1} from the wavelet coefficients smaller than ϵ_n, i.e., $\sigma^2_{n+1}=\frac{1}{N}\Sigma_{(j,i)\in\Lambda_J}|\widetilde{S}^I_{ji}|^2$, where

$$\widetilde{S}^I_{ji}=\begin{cases}\widetilde{S}_{ji} & \text{for } |\widetilde{S}_{ji}|\leq\epsilon_n\\ 0 & \text{else,}\end{cases} \tag{6}$$

and the new threshold $\epsilon_{n+1}=(2\ln N\sigma^2_{n+1})^{1/2}$;
- set $n=n+1$.

(3) Final step

- Reconstruct the coherent signal S^C from the coefficients $\widetilde{S}^C_{ji}$ using the inverse FWT, where

$$\widetilde{S}^C_{ji}=\begin{cases}\widetilde{S}_{ji} & \text{for } |\widetilde{S}_{ji}|>\epsilon_n\\ 0 & \text{else}\end{cases} \tag{7}$$

- finally, compute pointwise the incoherent signal $S^I(t_i)=S(t_i)-S^C(t_i)$ for $i=0,\dots,N-1$.

(4) End.

Note that the decomposition yields $S(t)=S^C(t)+S^I(t)$ and orthogonality implies that energy is split into $\sigma^2=\sigma_C^2+\sigma_I^2$, since $\langle S^C,S^I\rangle=0$.

The FWT, proposed by Mallat,[27] requires $(2mN)$ multiplications for its computation, where m is the length of the discrete filter defining the orthogonal wavelet used. Hence, the extraction algorithm we propose is computed in $(2nmN)$ operations, with a number of iterations n very small, typically less than $\log_2 N$. Recall that the operation count for the fast Fourier transform is proportional to $N\log_2 N$ operations.

This algorithm defines a sequence of estimated thresholds $(\epsilon_n)_{n\in\mathbb{N}}$ and the corresponding sequence of estimated variances $(\sigma_n^2)_{n\in\mathbb{N}}$. The convergence of these sequences within a finite number of iterations has been demonstrated in Ref. 26 applying a fixed point type argument to the iteration function

$$\mathcal{I}_{S,N}(\epsilon_{n+1})=\left(\frac{2\ln N}{N}\sum_{(j,i)\in\Lambda_J}|\widetilde{S}^I_{ji}(\epsilon_n)|^2\right)^{1/2}. \tag{8}$$

The algorithm thus stops after n iterations when $\mathcal{I}_{S,N}(\epsilon_n)=\epsilon_{n+1}$.

Furthermore, we have shown that the convergence rate of the recursive algorithm depends on the signal to noise ratio ($\text{SNR}=10\log_{10}(\sigma^2/\sigma_I^2)$), and the smaller the SNR, i.e., the stronger the noise, the faster the convergence. Moreover, if the algorithm is applied to a Gaussian white noise only, it converges in one iteration and removes the noise (in statistical mean). If it is applied to a signal without noise, the signal is fully preserved. Finally, we have proven that the algorithm is idempotent, i.e., if we apply it several times, the noise is eliminated the first time, and the coherent signal is no more modified in the subsequent applications, as it would have been the case for a Gaussian filter. As a consequence, this algorithm yields a nonlinear projector.[26]

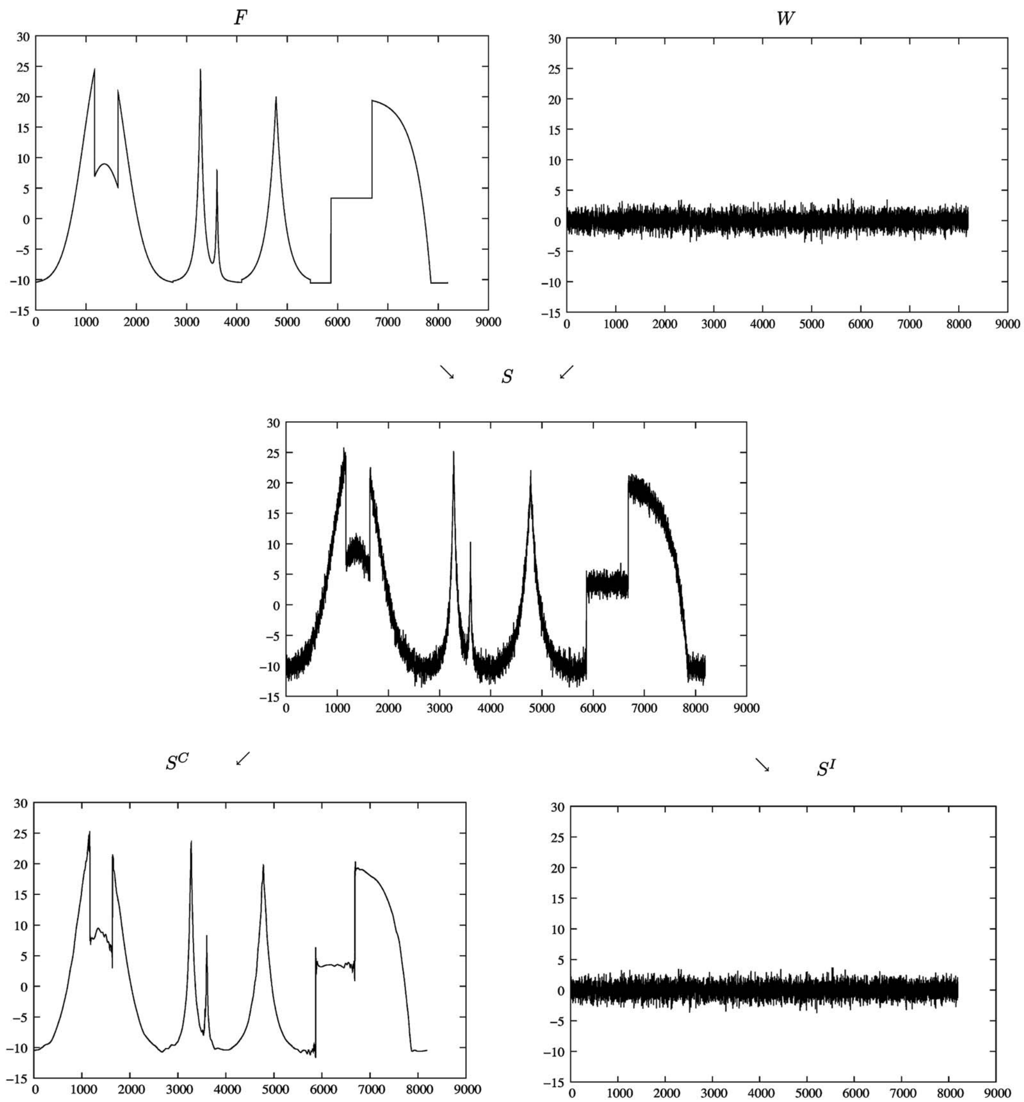

FIG. 2. (Top) Construction of a (middle) 1D noisy signal $S=F+W$, and (bottom) results obtained by the recursive algorithm, which gives $S=S^C+S^I$.

E. Application to an academic test signal

To illustrate the properties of the recursive algorithm we apply it to a 1D noisy test signal S (Fig. 2, middle). This signal has been constructed by superposing a Gaussian white noise W, with zero mean and variance $\sigma_W^2=1$, to a function F, normalized such that its variance yields 100, which corresponds to a signal to noise ratio $\mathrm{SNR}=10\log_{10}(\sigma_F^2/\sigma_W^2)=20$ dB (Fig. 2, top). The function F is a piecewise polynomial function which presents several discontinuities, either in the function or in its derivatives. The number of samples is $N=2^{13}=8192$.

We apply the recursive extraction algorithm to the test signal $S(t)$ and it converges after $n=5$ iterations, giving the coherent compnent $S^C(t)$ and the incoherent noise $S^I(t)$ (cf. Fig. 2, bottom). We observe that $S^C(t)$ yields a denoised version of the test signal $S(t)$ which is very close to $F(t)$, whereas the incoherent part $S^I(t)$ is homogeneous and noise-like with flatness $\mathcal{F}=3.03$, which corresponds to quasi-Gaussianity. Note that the flatness $\mathcal{F}$ is defined as the ratio of the centered fourth order moment divided by the square of the variance, and $\mathcal{F}=3$ for a Gaussian process. Fig. 2 (bottom, left) shows that the coherent signal retains all discontinuities and spikes present in the original function $F(t)$, without smoothing them as it would have been the case with standard denoising methods, e.g., with low-pass Fourier filtering. Nevertheless, we observe slight overshoots in the vicinity of the discontinuities, although they remain much more local than the classical Gibbs phenomena, and could easily be removed using the translation invariant wavelet transform.[27]

III. APPLICATION TO TURBULENT EDGE PLASMA

A. Density fluctuations

We have measured the time evolution of the ion saturation current during 8 ms in the SOL of the tokamak Tore Supra in Cadarache (France). This signal, denoted $S(t)$, gives an estimation of the density fluctuations.

The measure was taken according to the following plasma scenario: the shot 28338 lasted 18 s and the signal

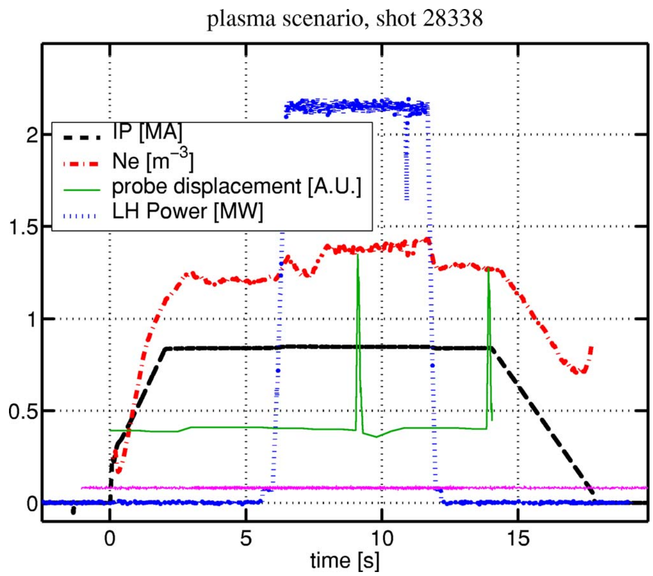

FIG. 3. Plasma scenario of the shot 28338 from the tokamak Tore Supra, Cadarache. The duration of the shot is 18 s. The plasma density fluctuations are measured by a fast reciprocating Langmuir probe. When the probe is 2.8 cm away from the LCFS in the SOL, the signal is acquired during time windows of 8 ms.

has been recorded in the middle of the plasma current plateau. The large radius was $R=2.33$ m, the small radius $a=0.77$ m, the mean plasma density $\bar{n}_i=1.37\times 10^{19}$ m^{-3}, the plasma current $I_p=0.84$ MA and the edge safety factor $q=6.71$. Moreover, 2.1 MW of lower hybrid waves were applied to the plasma.

The ion saturation current fluctuations were measured by a fast reciprocating Langmuir probe. The total duration of the probe motion into the plasma was 300 ms. When the probe reached 2.8 cm away from the last closed flux surface (LCFS), the signal was recorded at 1 MHz during 8 ms (Fig. 3), which gave $N=2^{13}=8192$ samples. A high-pass filter at frequency 0.1 kHz and a low-pass filter at frequency 500 kHz have been applied to eliminate both low frequencies and aliasing.

B. Extraction of coherent bursts

We use the wavelet extraction algorithm to split the signal $S(t)$ (Fig. 4, top) into two orthogonal components, the coherent bursts, $S^C(t)$ (Fig. 4, middle), and the incoherent turbulent fluctuations, $S^I(t)$ (Fig. 4, bottom). The optimal threshold value has been obtained after $n=12$ iterations of the algorithm (Fig. 5).

As results, we observe that the coherent signal $S^C(t)$, made of $5.8\%N$ wavelet coefficients, retains 86.6% of the total variance and the extrema are preserved (Table I). In contrast, the incoherent contribution $S^I(t)$, is made of $94.2\%N$ wavelet coefficients but contributes to only 13.4% of the total variance (Table I), which corresponds to a signal to noise ratio $\mathrm{SNR}=10\log_{10}(\sigma^2/\sigma_I^2)=8.72$ dB.

The decomposition shows that the bursty and coherent contribution to the signal dominates over the turbulent background fluctuation, and this more strongly than what has been found with previous methods based on clipping.[10]

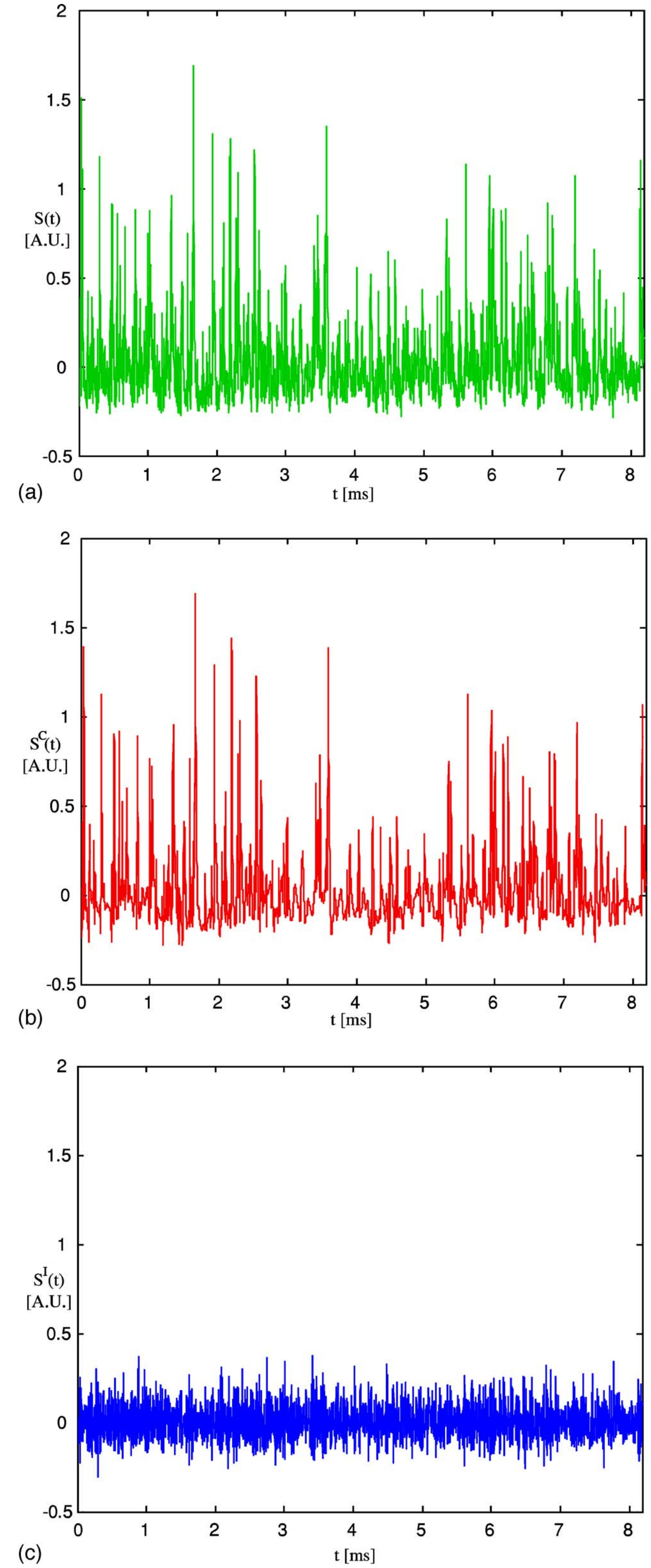

FIG. 4. Signal $S(t)$ of duration 8.192 ms, corresponding to the saturation current fluctuations measured at 1 MHz in the SOL of the tokamak Tore Supra, Cadarache. (Top) Total signal S, (middle) coherent part S^C, and (bottom) incoherent part S^I.

Figure 6 shows the PDFs in semi-logarithamic coordinates for the total, coherent and incoherent contributions, estimated using histograms with 50 bins and integrals normalized to one. The PDFs of the total signal and the coherent

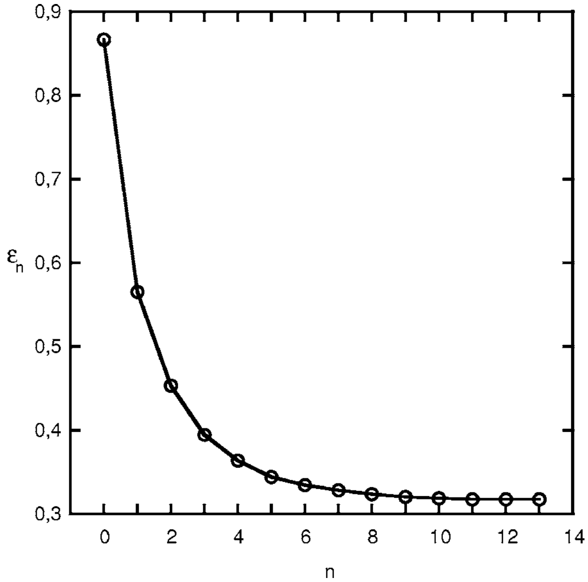

FIG. 5. Threshold value ϵ_n vs iteration number n.

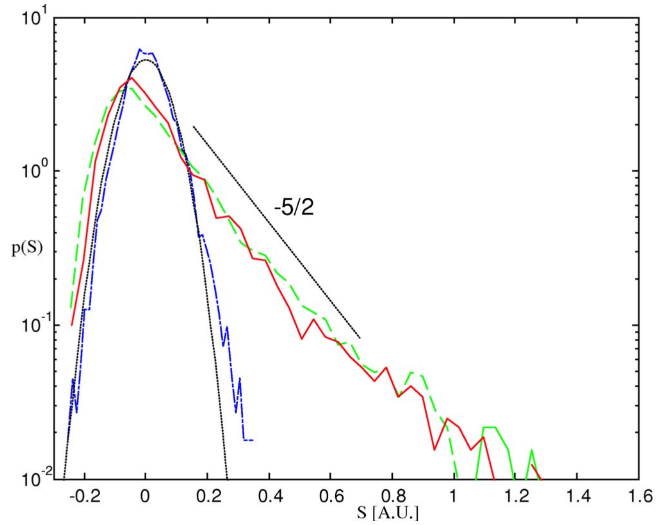

FIG. 6. Probability density function $p(S)$, estimated using histogram with 50 bins. PDF of the total signal S (green dashed line), of the coherent component S^C (red solid line), and of the incoherent component S^I (blue dotted-dashed line), together with a Gaussian fit with variance σ_I^2 (black dotted line).

contribution are skewed and present the same behavior: positive values have exponential tails with $p(S) \propto \exp(-5/2S)$, whereas negative values yield a Gaussian behavior (Fig. 6). In contrast, the PDF of the incoherent component is almost symmetric, with skewness 0.38, instead of 2.56 and 2.84 for the total and coherent part, respectively. It has a quasi-Gaussian shape with flatness 4.03, instead of 12.00 and 14.22, respectively (Fig. 6).

C. Fourier spectrum and modified periodogram

To study the spectral distribution of the density variance for the different components, we consider the Fourier spectrum

$$E(\omega) = \frac{1}{2}|\hat{S}(\omega)|^2, \tag{9}$$

where $\hat{S}(\omega)$ denotes the Fourier transform as defined in Eq. (3). As estimator for the spectrum we take the periodogram, which is a discrete version of Eq. (9), although it is known to be an inconsistent estimator due to the presence of oscillations.[28] To obtain a consistent estimator we also compute the modified periodogram, by first tapering the data with a raised cosine window (affecting 40 data points at each boundary), and then convolving the periodogram with a Gaussian window (with standard deviation of 40 data points). Figure 7 shows the periodogram and the modified periodogram for S, S^C, and S^I, which confirms that the latter yields a stabilized estimator of the spectrum, presenting no more spurious oscillations.

TABLE I. Statistical properties of the signal $S(t)$ from the tokamak Tore Supra, Cadarache, for the signal and its coherent and incoherent components using the Coifman 12 orthogonal wavelet.

Properties	Total S	Coherent S^C	Incoherent S^I
Number of coefficients	8192	479	7713
Percent of coefficients	100	5.8	94.2
Minimum value	−0.284	−0.282	−0.307
Maximum value	1.689	1.686	0.374
Mean value	0.019	0.019	$<10^{-11}$
Variance σ^2	0.0417	0.0361	0.0056
Percent of variance	100	86.6	13.4
Skewness	2.564	2.842	0.383
Flatness	12.001	14.224	4.026

D. Wavelet spectrum

The wavelet decomposition, given in Eq. (1), yields the distribution of the variance of the signal scale per scale, which is called scalogram.[19] It is defined as

$$\widetilde{E}_j = \frac{1}{2}\sum_{i=0}^{2^j-1} |\widetilde{S}_{ji}|^2. \tag{10}$$

Parseval's theorem implies that $E=\Sigma_{j\geqslant 0}\widetilde{E}_j$. Using the relation $\omega_j=\omega_\psi 2^j$ between the scale index j and the frequency ω, the wavelet spectrum can be defined as $\widetilde{E}(\omega)=\widetilde{E}_j \cdot 2^{-j}$, with ω_ψ being the centroid frequency of the mother wavelet whose value is $\omega_\psi=1.3$ for the Coifman 12 wavelet used here. It corresponds to a smoothed version of the Fourier spectrum (9), the smoothing kernel being the square of the Fourier transform of the wavelet, since

$$\widetilde{E}(\omega) = \frac{1}{\omega_\psi}\int_0^{+\infty} E(\omega')\left|\hat{\psi}\left(\frac{\omega_\psi \omega'}{\omega}\right)\right|^2 d\omega'. \tag{11}$$

Note that, as frequency increases, i.e., when one goes to small scale, the smoothing interval becomes larger, which explains why the wavelet spectrum is a well-conditioned statistical estimator. The advantage of the wavelet spectrum in comparison to the modified periodogram is that the smooth-

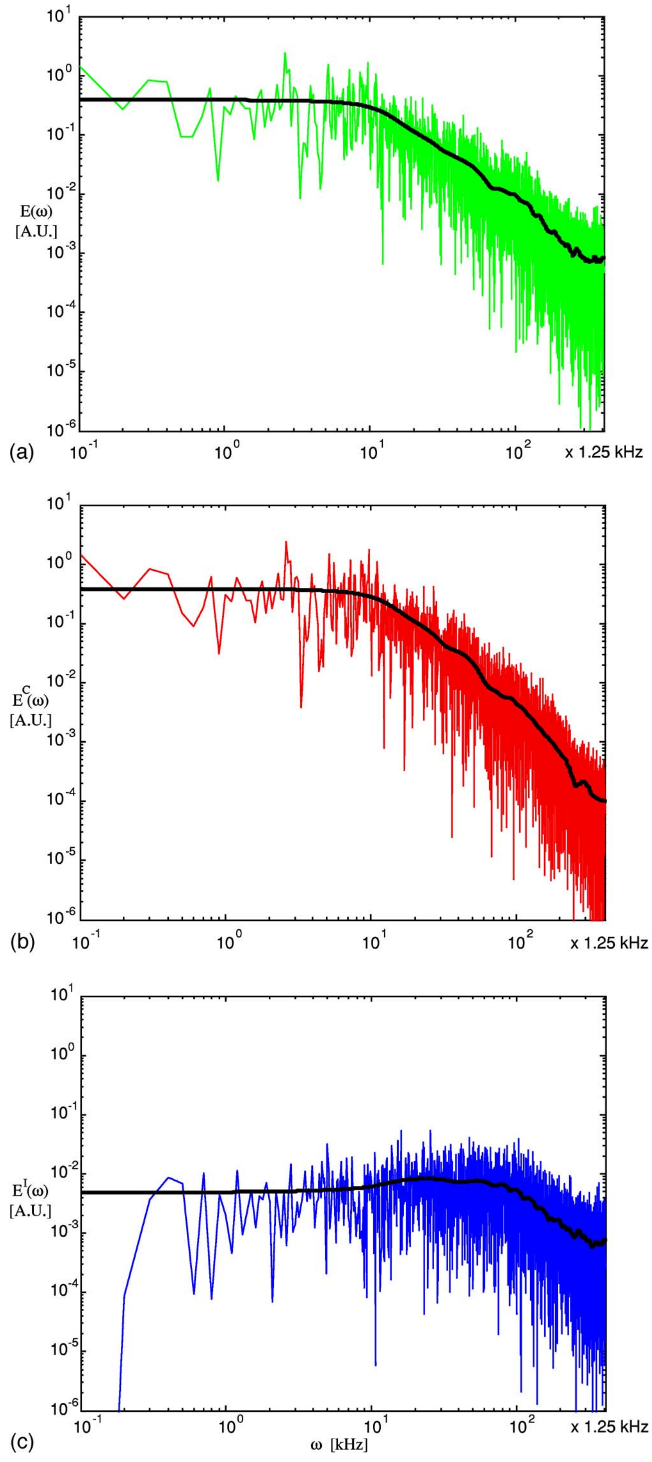

FIG. 7. Fourier spectrum $E(\omega)$. (Top) Spectrum of the total signal $S(t)$, (middle) coherent component $S^C(t)$, and (bottom) incoherent component $S^I(t)$. Note that the periodogram is plotted in green, red, and blue for the total, coherent, and incoherent signals, respectively. Superimposed are the modified periodograms (black thick line).

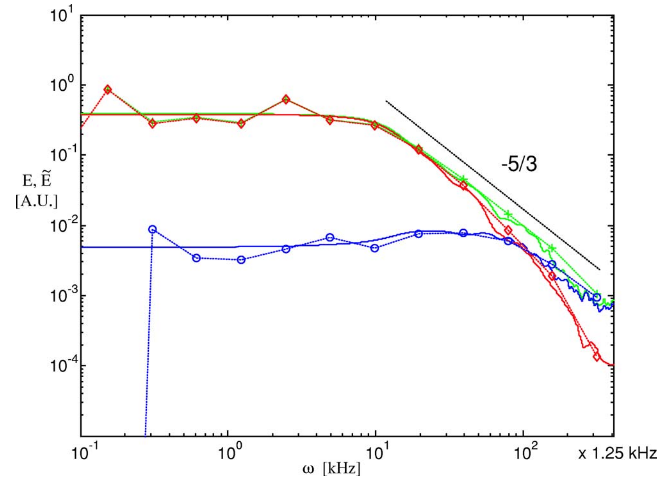

FIG. 8. Wavelet spectra $\widetilde{E}(\omega_j)$ (lines with symbols) and modified periodograms $E(\omega)$ (lines) of the total signal S (green and +), coherent signal S^C (red and ◇) and incoherent signal S^I (blue and ○).

ing window is automatically adjusted by the wavelet representation, since wavelets correspond to filters with constant relative bandwidth $\Delta\omega/\omega$.[19]

In Fig. 8 wavelet spectra, together with modified periodograms, are displayed. We observe that the signal and its coherent component present a similar scaling in $\omega^{-5/3}$, which characterizes long-range correlation since the spectral slope is negative. As proposed in Ref. 10, this may be interpreted as an inverse energy cascade, similar to what is encountered in 2D fluid turbulence. In contrast, the incoherent component has a different scaling, with a flat spectrum up to frequency $\omega = 120$ kHz, corresponding to decorrelation. For higher frequencies we observe a ω^{-1} scaling, which may be due to experimental noise, since it presents the same scaling at high frequencies, although its amplitude remains smaller than the incoherent fluctuations. Figure 8 also shows that the wavelet spectrum almost coincides with the modified periodogram, and that, the higher the frequency, the better the stabilization obtained using wavelets.

Note that the scalogram and the wavelet spectrum are optimal to characterize scaling laws, as long as the analyzing wavelet has at least M vanishing moments, with $M > (\beta - 1)/2$, to detect power laws in $\omega^{-\beta}$, see, e.g., Refs. 21 and 29.

E. Intermittency

Intermittency characterizes the fact that the time support of the fluctuations decreases with scale.[30,31] It therefore quantifies how bursty a signal is. Townsend[32] has proposed the "intermittency factor" as the ratio between the time supports of active and quiescent regions. But the main deficiency is that intermittency factors depend on the choice of the threshold below which the variation is considered to be inactive.[33] As we have already mentioned, one of the drawbacks of such a clipping method is that the active bursts, and the corresponding intermittency factor, depend on the choice of the threshold, which can be avoided by using the wavelet representation.

Biskamp stated in[30] that "the spottiness of the dissipative eddies is a special feature of what is now believed to be a general property of fully developed turbulence that with decreasing scale turbulent fluctuations become less and less space-filling, i.e., are concentrated in regions of smaller and smaller volume but increasingly complicated shape. This phenomenon is called intermittency, which is a central topic

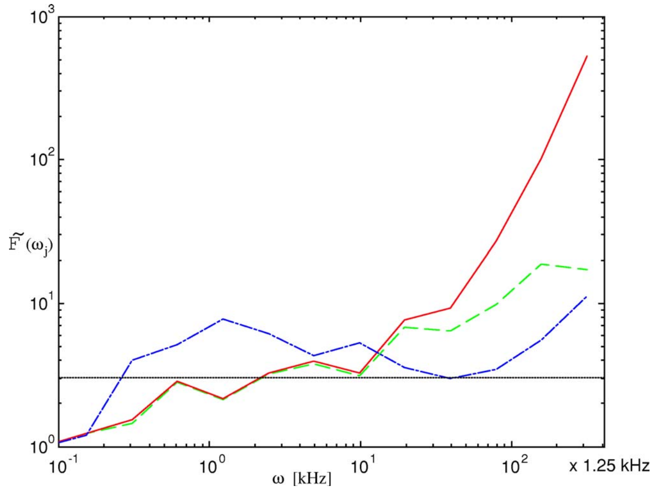

FIG. 9. Flatness of the band-pass filtered signal $\widetilde{\mathcal{F}}$ vs frequency ω_j for the total signal S (green dashed line), coherent signal S^C (red solid line), and incoherent signal S^I (blue dotted-dashed line). The horizontal dotted line $\widetilde{\mathcal{F}}(\omega_j)=3$ corresponds to the flatness of a Gaussian process.

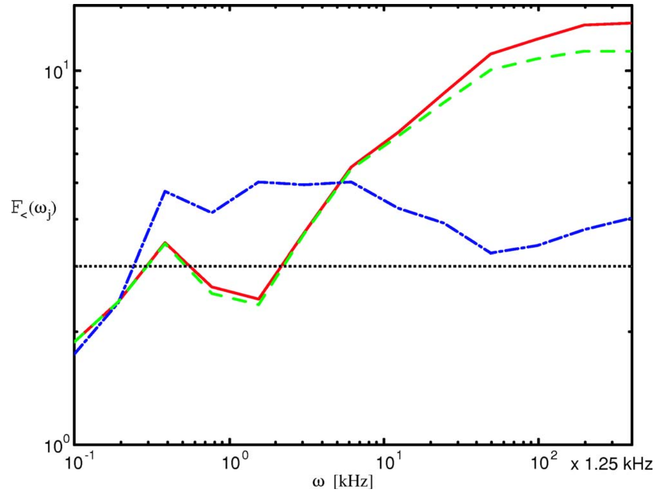

FIG. 10. Flatness of the low-pass filtered signal $\mathcal{F}_<$ vs frequency ω_j for the total signal S (green dashed line), coherent signal S_C (red solid line), and incoherent signal S^I (blue dotted-dashed line). The horizontal dotted line $\mathcal{F}_<(\omega_j)=3$ corresponds to the flatness of a Gaussian process.

in actual turbulence research". Frisch explained in Ref. 31 that intermittency can be quantified by computing the variation of the flatness when scale decreases: if flatness remains constant the signal is non-intermittent, if it increases when scale decreases it is intermittent. We use the same definition of intermittency and compute the scale dependent flatness from the higher order moments of the wavelet coefficients $\widetilde{S}_{ji}$, as introduced in Refs. 21 and 29. By summing up the pth power of the wavelet coefficients over all positions i, one obtains the pth order moments

$$\widetilde{\mathcal{M}}_j^p = \frac{1}{2^j}\sum_{i=0}^{2^j-1} (\widetilde{S}_{ji})^p. \qquad (12)$$

The scale dependent flatness is then defined as

$$\widetilde{\mathcal{F}}_j = \frac{\widetilde{\mathcal{M}}_j^4}{(\widetilde{\mathcal{M}}_j^2)^2}. \qquad (13)$$

The relation between scale and frequency allows one to express the flatness as a function of the frequency ω_j, similarly to the wavelet spectrum. Note that Gaussian white noise, which is by definition non-intermittent, would yield a flatness equal to three for all frequencies.

To characterize the intermittency of the signal and its different contributions we plot in Fig. 9 the flatness $\widetilde{\mathcal{F}}_j$ versus the frequency ω_j. We observe that the flatness of the coherent contribution increases faster for high frequencies than that of the total signal. This proves that the coherent contribution is more intermittent than the signal itself, which is obvious since it only retains the bursts. In contrast, the flatness of the incoherent contribution decreases to the value $\widetilde{\mathcal{F}}_j=3$, up to frequency $\omega=120$ kHz, which gives evidence for its non-intermittent behavior. The wavelet based flatness corresponds to the flatness of the band-pass filtered signal, as typically used in the fluid turbulence community.[31] Note that the signal reconstructed from its wavelet coefficients at a given scale j corresponds to the band-pass filtered signal around the frequency $\omega_j=\omega_\psi 2^j$.

For comparison we also show in Fig. 10 the flatness of the low-pass filtered signal, for dyadically increasing cutoff frequencies $\omega_C=\omega_\psi 2^{J_C}$. Therefore, we reconstruct the signal in physical space on N grid points using only the wavelet coefficients up to a given scale J_C, corresponding to the filter cutoff. The wavelet coefficients for scales $j \geqslant J_C$ are set to zero and the low-pass filtered signal is computed by inverse wavelet transform using Eq. (1).

Similarly to Fig. 9, we observe in Fig. 10 that the flatness of the total and coherent signal increases with frequency for $\omega > 3$ kHz. Considering the signal filtered at 20 kHz we observe that its flatness is just above 7, however the signal contains only large bursts, since all smaller scale details have been filtered out. This shows that the signal is already intermittent at medium scales. For the small scales, i.e., for $\omega \geqslant 20$ kHz, the flatness of the total and the coherent signals is above 10. This shows that adding small scale details to the large scale bursts increases the flatness, and hence the signal's intermittency, as quantified by its flatness.

The flatness $\mathcal{F}_<$ of the low-pass filtered signal, considered for increasing cutoff frequencies, quantifies the intermittency of the signal reconstructed up to the corresponding cutoffs, whereas the flatness $\widetilde{\mathcal{F}}$ of the band-pass filtered signal, considered for bands of increasing frequency, yields incremental information on the flatness of the signal scale by scale. The latter quantity can be compared with the wavelet spectrum which gives the energy distribution scale by scale, whereas the former gives some cumulative information, since information on the flatness of the lower frequency contributions of the signal is included in the flatness of the higher frequency contributions. Hence, both quantities do not yield the same values if the PDF of the signal varies with scale.

IV. CONCLUSION

We presented a wavelet-based recursive method to extract coherent bursts out of turbulent signals. The algorithm decomposes the signal into an orthogonal wavelet basis and reconstructs the coherent contribution from the wavelet coefficients whose modulus is larger than a given threshold. The threshold value is recursively determined without any adjustable parameter. Moreover, we have shown that this algorithm is fast, since it has only linear complexity.

Compared to classical extraction methods, which are based, either on thresholding in physical space ("clipping"), or on conditional averaging, working in wavelet space presents the following advantages:

(i) there is no need to suppose the signal to be statistically stationary in time,
(ii) the wavelet decomposition preserves the spectral properties of the signal, and thus respects its scaling as long as the analyzing wavelet is smooth enough (which depends on the number of vanishing moments it has),
(iii) the wavelet-based extraction method does not require any prior about the shape or the intensity of the bursts to be extracted; the only prior is to assume the noise to be Gaussian and white.

We have applied this recursive wavelet algorithm to ion saturation current measured in the SOL of the tokamak Tore Supra in Cadarache. We have thus extracted the coherent bursts from an incoherent background noise. The former contain most of the density variance and are correlated, with non-Gaussian statistics, whereas the latter is almost decorrelated and quasi-Gaussian. We have also observed that the non-Gaussianity of the PDF of the coherent component increases with the frequency, which confirms that the bursts are highly intermittent. In contrast, the incoherent component remains quasi-Gaussian up to high frequencies, which confirms the non-intermittency of the background noise. By analogy with previous studies we have made in the context of 2D fluid turbulence,[34] we conjecture that the coherent bursts are due to organized structures produced by nonlinear interactions and responsible for turbulent transport. On the other hand, the incoherent background corresponds to the turbulent fluctuations which only contribute to turbulent diffusion. Moreover, the variance of the incoherent fluctuations yields a good estimation of the turbulence level.

In Ref. 35 we applied this extraction method to both plasma velocity and density signals, measured at different poloidal positions, to study turbulent fluxes and thus characterize the transport properties of the coherent bursts. These results will be subject of a forthcoming article. We also have already extended this extraction method to treat 2D and 3D, scalar and vector, fields,[15–17] and we plan to apply it to spatio-temporal signals and to images of plasma density fluctuations obtained by fast framing cameras. Our aim is to improve the characterization of coherent bursts.

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