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Coherent enstrophy production and turbulent dissipation in two-dimensional turbulence, with and without walls, in the vanishing viscosity limit

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Abstract. *The viscosity dependence of two-dimensional turbulence is studied by means of direct numerical simulation. We consider either periodic or no-slip boundary conditions. We compare the Navier-Stokes solutions to those of the regularized Euler equations. The regularization is perfomred at each time step by applying the wavelet-based CVS filter which splits turbulent fluctuations into coherent and incoherent contributions. We show that for Re* $> 10^5$ *the dissipation of coherent enstrophy tends to become independent of Re, while dissipation of incoherent enstrophy grows logarithmically with Re. In the wall bounded case we observe an additional production of enstrophy at the wall. As a result coherent enstrophy diverges when Reynolds tends to infinity, however its time derivative seems to remain bounded.*

Keywords*:* two-dimensional turbulence, vanishing viscosity limit, no-slip walls.

In the fully-developed turbulent regime one observes that dissipation becomes independent on the molecular viscosity of the fluid for three-dimensional incompressible flows when Reynolds number is larger than 10^5. This has been confirmed by numerical experiments [1]. Here, we will study if incompressible two-dimensional turbulent flows may exhibit a similar behaviour in the vanishing viscosity limit.

For this, we apply the coherent vorticity simulation (CVS) filter, introduced in [2], to incompressible decaying two-dimensional turbulence, in periodic and wall-bounded domains, for Reynolds numbers varying from 10^3 to 10^7. CVS expands the vorticity field into an orthogonal wavelet basis and splits the flow into two orthogonal contributions: a coherent and an incoherent flow. The coherent vorticity field and the induced coherent velocity field are reconstructed from the largest wavelet coefficients, which correspond to the coherent vortices

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and are the only components advanced in time. In previous work [3], we have shown that applying the CVS filter at each time step to the inviscid Burgers equation models dissipation.

We examine the viscosity dependence of the solutions of the two-dimensional Navier-Stokes equations and we compare them to those of the two-dimensional Euler equations regularized by the CVS filter at each time step. The solutions of both equations are computed with a parallelized fully-dealiased pseudo-spectral code (written in C++), using a fourth-order Runge-Kutta time scheme and up to 8192^2 grid points, on the IBM BlueGene/P of IDRIS-CNRS with up to 1024 processors. For the wall-bounded case we consider a circular domain and use a volume penalization method to impose no-slip boundary conditions, as in [4].

In the periodic case (Fig. 1, left), we observe that the enstrophy dissipation vanishes like $(\ln Re)^{-1}$ in the inviscid limit, which confirms previous results [5]. In contrast, dissipation of coherent enstrophy does not vanish in the same limit and tends to become independent of Re.

For the wall-bounded case (Fig. 1, right), we observe an additional production of enstrophy at the wall. As a result, coherent enstrophy diverges when $Re \to \infty$, but its time derivative seems to remain bounded independently of Re. This may indicate that a balance has been established between coherent enstrophy production at the wall and coherent enstrophy dissipation.

In conclusion, the above results for two-dimensional turbulence, investigated for Reynolds numbers up to 10^7, suggest that the dissipation of coherent enstrophy becomes constant when $Re > 10^5$. We propose to define this as the onset of the fully-developed turbulent regime where viscous dissipation, due to the fluid's molecular viscosity, becomes negligible in front of the turbulent dissipation due to the flow nonlinear dynamics of Euler equations.

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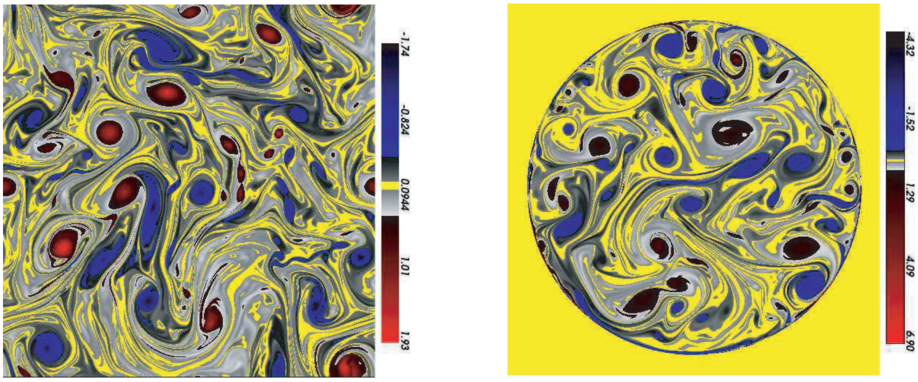

Figure 1: Vorticity field at $Re = 10^4$, $t \simeq 60$ turnover times. Left: periodic domain. Right: circular domain.

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