

Vortex motion in a rotating barotropic fluid layer

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Abstract. To study vortex motion and the mechanisms of geostrophic adjustment (i.e. the equilibrium between pressure gradient and Coriolis force, which leads to the weakening of inertio-gravity waves) in large scale geophysical flows, we simulate the dynamics of a shallow-water layer in uniform rotation, without any forcing other than the initial injection of energy and potential enstrophy. Such a flow generates inertio-gravity waves which interact with the rotational eddies. We found that both inertio-gravity waves and rotation reduce the non-linear interactions between vortices, namely the condensation of the vorticity field into isolated coherent vortices, corresponding to the inverse rotational energy cascade, and the associated production of vorticity filaments, due to the direct potential enstrophy cascade. Rotation also inhibits the direct inertio-gravitational energy cascade for scales larger than the Rossby deformation radius. Therefore, if inertio-gravity waves are initially excited at large enough scales, they will remain trapped there due to rotation and there will be no geostrophic adjustment. On the contrary, if inertio-gravity waves are only present at scales smaller than the Rossby deformation radius, which are insensitive to the effect of rotation, they will non-linearly interact and cascade towards the dissipative scales, leaving the flow in geostrophic equilibrium.

1. Introduction

The terrestrial atmosphere is a rotating stratified fluid layer in quasi-geostrophic equilibrium. The linear theory of geostrophic adjustment (Cahn 1945, Obukhov 1949), which explains the prevalence of vortex motion by the dispersion of inertio-gravity waves over an infinite domain, does not work on a finite or periodic domain like the sphere. Sadourny (1975) then proposed a non-linear mechanism, where inertio-gravity waves cascade towards small scales to be dissipated there, while vortices present an inverse rotational energy cascade towards large scale as it is the case for two-dimensional incompressible turbulence (Kraichnan 1967). In this paper we will focus on the interactions between inertio-gravity waves and vortices in a barotropic fluid layer, i.e. for which pressure only depends on the surface height, submitted to a uniform rotation $\Omega = 2f$, f being the Coriolis parameter. A more realistic baroclinic flow, for which pressure also depends on temperature, can always be decomposed into a set of vertical normal modes, each of them behaving as a barotropic layer of a given equivalent height.

2. Model

The dynamics of such a flow is described by the shallow-water (or Saint-Venant's) equations, for which the free-surface height h , and the related geopotential $\phi = gh$ (g , gravity), acts as a two-dimensional variable density and generates divergence δ in addition to vorticity ξ in the velocity field V :

$$\partial V / \partial t + (V \cdot \nabla) V + n \times fV + \nabla \phi = 0, \quad \partial \phi / \partial t + \nabla \cdot (V \phi) = 0.$$

In this inviscid limit, these equations have two Eulerian invariants, energy $\langle \phi^2 + \phi V^2 \rangle$ and potential enstrophy $\langle (\xi + f)^2 / \phi \rangle$, and a Lagrangian invariant, potential vorticity $\eta = (\xi + f) / \phi$, all non-quadratic, which gives rise to specific numerical difficulties (Farge and Lacarra 1986).

The space integration is performed with a pseudo-spectral technique, associated to a leapfrog scheme for the time integration. In order to get a wider inertial range, dissipation is an iterated Laplacian $\nu(\nabla^2)^8$ (ν , eddy kinematic viscosity), which is exactly computed for each time step. Our choice to dissipate the free-surface geopotential comes from the fact that, for two-dimensional flows, turbulent viscosity should model altogether the return to three-dimensionality and the dissipative properties of sub-grid scales. Therefore, the mixing by velocity gradients must diffuse all perturbations, including those of the free-surface height.

3. Vortex-wave separation

How to distinguish between vortex and waves when non-linearities become important? Moreover, invariants being non-quadratic, Parseval's theorem does not hold and there is no clear-cut scale separation to analyze the results in terms of energy spectra. Actually, we only consider relatively weak geopotential fluctuations (5%), like on the Earth, in order to recover quadratic invariants in the zero-perturbation limit.

The problem can then be separated into two sets of normal eigenmodes:

- potential vortices, which do not propagate (frequency $\omega_v = 0$), are non-divergent (velocity potential $\chi_v = \nabla^{-2} \delta_v = 0$), correspond to geostrophic equilibrium ($\psi_v = \nabla^{-2} \xi = \phi_v / f$) and contain all the potential vorticity of the flow,
- inertio-gravity waves, which propagate with sound speed ($c = \langle \phi \rangle^{1/2}$) and are dispersive in presence of rotation.

This decomposition may be interpreted in terms of slow manifold, for the potention-vortical mode, and fast manifold, for the inertio-gravitational mode. We can also perform an Helmholtz decomposition of the two-dimensional velocity field into a rotational incompressible part and a divergent 'compressible' part, total energy being then the sum of potential, rotational and divergent energies. But this separation does not have the orthogonal structure of the first one, which, although based on an approximation, appears more physical. We will present energy spectra using both decompositions.

Plates 1(A,B). Flow initially dominated by potential vortices.

Plates 2(A,B). Flow initially dominated by inertio-gravity waves.

Plates 1,2.

(α) From left to right:

- energy spectra, at initial time t_0 (thin lines) and at final time t_{\max} (thick lines), of potention-vortical ——— and inertio-gravitational - - - - - energies, \blacktriangle indicating the Rossby deformation wavenumber;

- time evolution, from t_0 to t_{\max} potential enstrophy ———, total ———, potention-vortical ——— and inertio-gravitational - - - - - energies.

- time evolution, from t_0 to t_{\max} , of rotational ———, divergent - - - - - and potential - - - - - energies.

(β) Structure of the flow in physical space at t_{\max} . From left to right:

vorticity (three-dimensional representation), vorticity (two-dimensional representation), potential vorticity, free-surface geopotential and stream function fields*.

(γ) Time evolution of the vorticity field; the time scales are not necessarily the same, because they are chosen in order to follow different characteristic structures (isolated coherent vortices, solitary dipoles or nonisolated coherent vortices).

* The geostrophic equilibrium is reached if, in physical space, the extrema of the stream function and the geopotential fields are correlated, because in this case we have $\psi_v = \phi_v / f$.

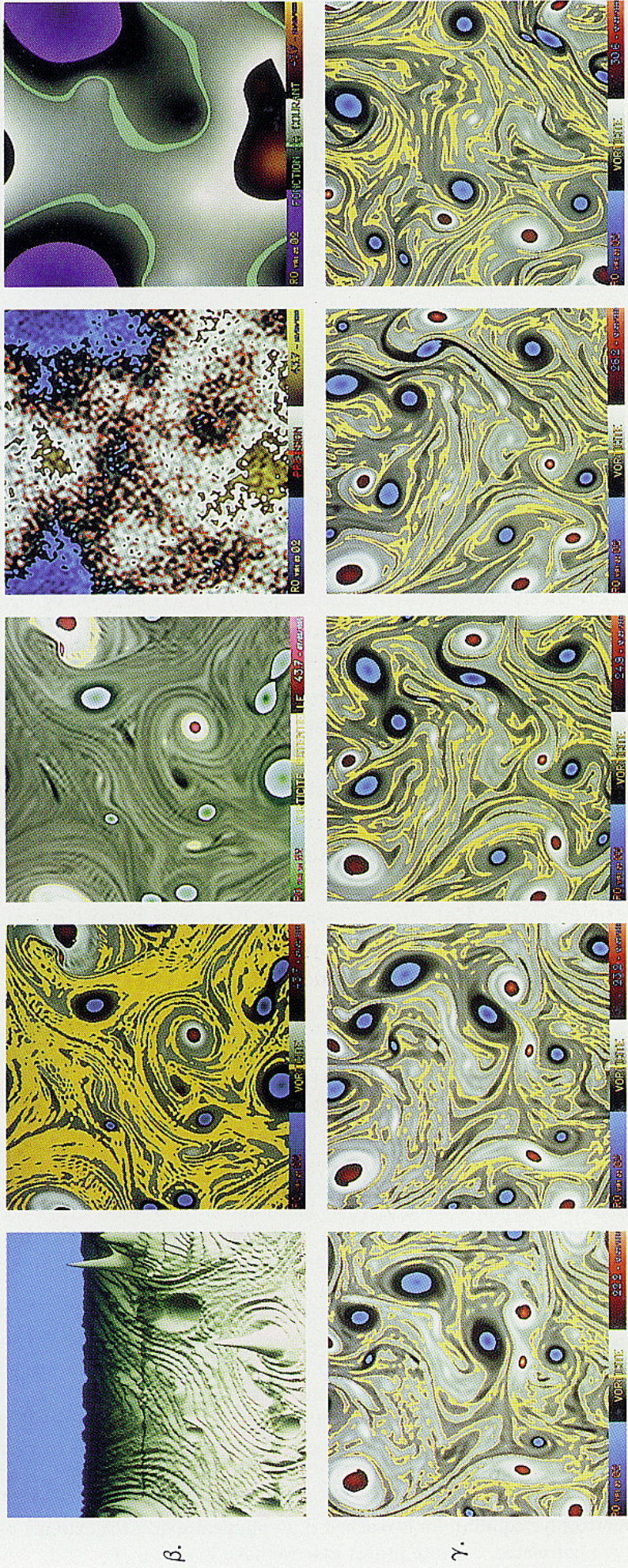
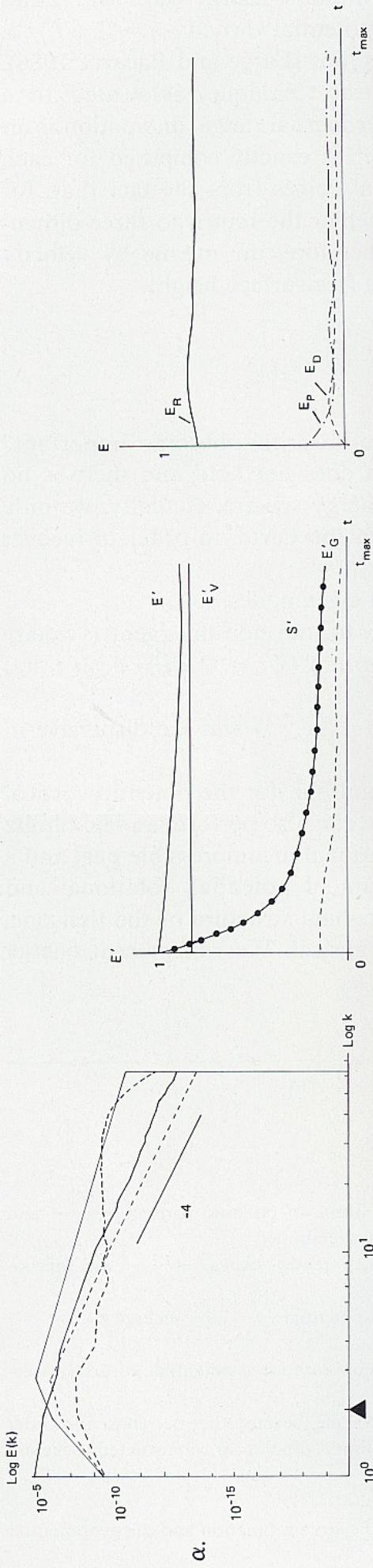


Plate 1A. Rotation rate $f = 10^{-4} \text{ s}^{-1}$.

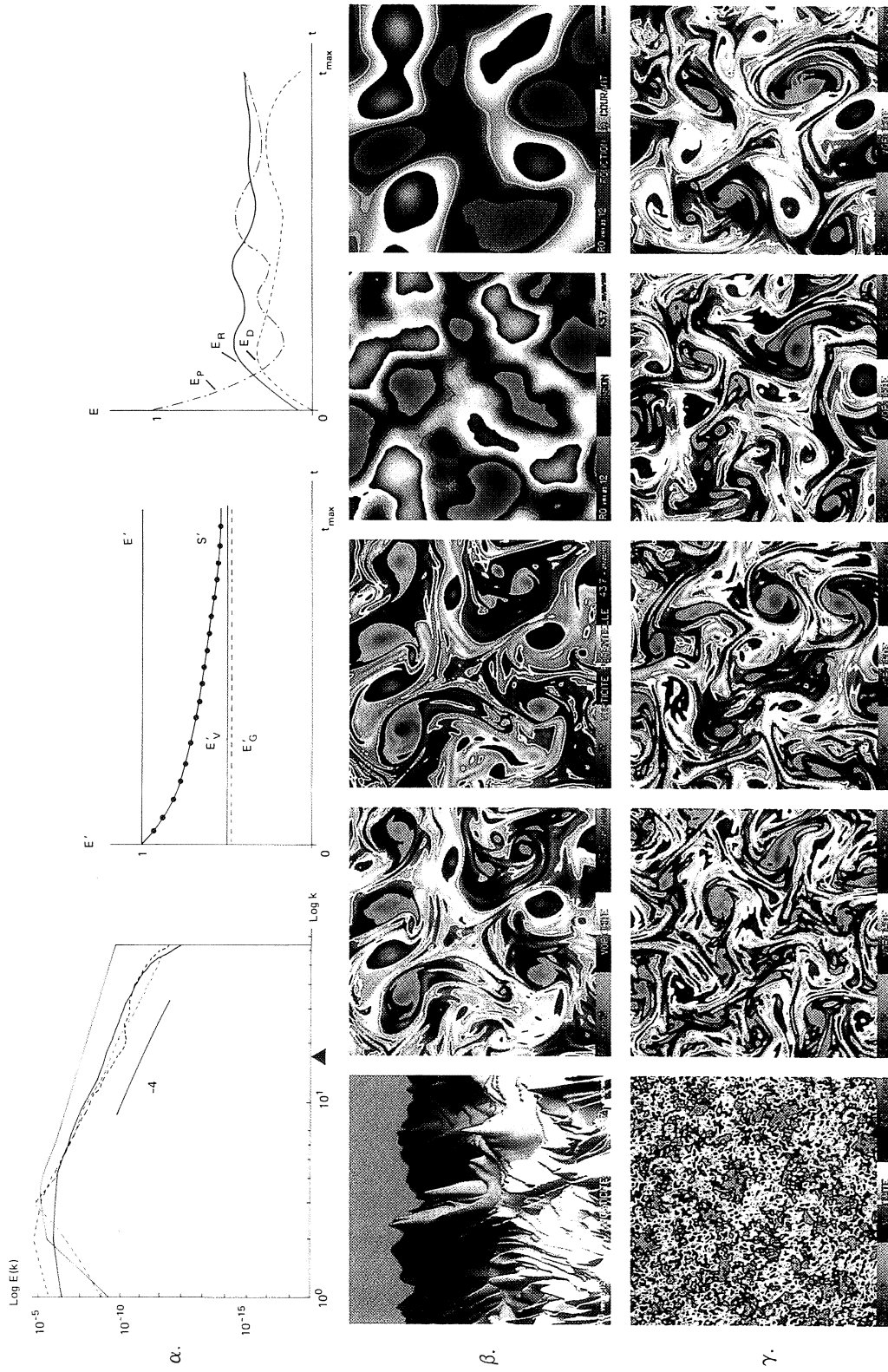


Plate 1B. Rotation rate $f = 6 \times 10^{-4} \text{ s}^{-1}$.