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Vortex motion in a rotating barotropic fluid layer

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Abstract. To study vortex motion and the mechanisms of geostrophic adjustment (i.e. the equilibrium between pressure gradient and Coriolis force, which leads to the weakening of inertio-gravity waves) in large scale geophysical flows, we simulate the dynamics of a shallow-water layer in uniform rotation, without any forcing other than the initial injection of energy and potential enstrophy. Such a flow generates inertio-gravity waves which interact with the rotational eddies. We found that both inertio-gravity waves and rotation reduce the non-linear interactions between vortices, namely the condensation of the vorticity field into isolated coherent vortices, corresponding to the inverse rotational energy cascade, and the associated production of vorticity filaments, due to the direct potential enstrophy cascade. Rotation also inhibits the direct inertio-gravitational energy cascade for scales larger than the Rossby deformation radius. Therefore, if inertio-gravity waves are initially excited at large enough scales, they will remain trapped there due to rotation and there will be no geostrophic adjustment. On the contrary, if inertio-gravity waves are only present at scales smaller than the Rossby deformation radius, which are insensitive to the effect of rotation, they will non-linearly interact and cascade towards the dissipative scales, leaving the flow in geostrophic equilibrium.

1. Introduction

The terrestrial atmosphere is a rotating stratified fluid layer in quasi-geostrophic equilibrium. The linear theory of geostrophic adjustment (Cahn 1945, Obukhov 1949), which explains the prevalence of vortex motion by the dispersion of inertio-gravity waves over an infinite domain, does not work on a finite or periodic domain like the sphere. Sadourny (1975) then proposed a non-linear mechanism, where inertio-gravity waves cascade towards small scales to be dissipated there, while vortices present an inverse rotational energy cascade towards large scale as it is the case for two-dimensional incompressible turbulence (Kraichnan 1967). In this paper we will focus on the interactions between inertio-gravity waves and vortices in a barotropic fluid layer, i.e. for which pressure only depends on the surface height, submitted to a uniform rotation $\Omega = 2f$, f being the Coriolis parameter. A more realistic baroclinic flow, for which pressure also depends on temperature, can always be decomposed into a set of vertical normal modes, each of them behaving as a barotropic layer of a given equivalent height.

2. Model

The dynamics of such a flow is described by the shallow-water (or Saint-Venant's) equations, for which the free-surface height h, and the related geopotential $\phi = gh$ (g, gravity), acts as a two-dimensional variable density and generates divergence δ in addition to vorticity ξ in the velocity field $\boldsymbol{V}$:

$$\partial \boldsymbol{V}/\partial t + (\boldsymbol{V}\cdot\nabla)\boldsymbol{V} + \boldsymbol{n}\times f\boldsymbol{V} + \nabla\phi = \boldsymbol{0}, \qquad \partial\phi/\partial t + \nabla\cdot(\boldsymbol{V}\phi) = 0.$$

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In this inviscid limit, these equations have two Eulerian invariants, energy $\langle \phi^2 + \phi V^2 \rangle$ and potential enstrophy $\langle (\xi + f)^2/\phi \rangle$, and a Lagrangian invariant, potential vorticity $\eta = (\xi + f)/\phi$, all non-quadratic, which gives rise to specific numerical difficulties (Farge and Lacarra 1986).

The space integration is performed with a pseudo-spectral technique, associated to a leapfrog scheme for the time integration. In order to get a wider inertial range, dissipation is an iterated Laplacian $\nu(\nabla^2)^8$ (ν, eddy kinematic viscosity), which is exactly computed for each time step. Our choice to dissipate the free-surface geopotential comes from the fact that, for two-dimensional flows, turbulent viscosity should model altogether the return to three-dimensionality and the dissipative properties of sub-grid scales. Therefore, the mixing by velocity gradients must diffuse all perturbations, including those of the free-surface height.

3. Vortex-wave separation

How to distinguish between vortex and waves when non-linearities become important? Moreover, invariants being non-quadratic, Parseval's theorem does not hold and there is no clear-cut scale separation to analyze the results in terms of energy spectra. Actually, we only consider relatively weak geopotential fluctuations (5%), like on the Earth, in order to recover quadratic invariants in the zero-perturbation limit.

The problem can then be separated into two sets of normal eigenmodes:
- potential vortices, which do not propagate (frequency $\omega_V = 0$), are non-divergent (velocity potential $\chi_V = \nabla^{-2}\delta_V = 0$), correspond to geostrophic equilibrium ($\psi_V = \nabla^{-2}\xi = \phi_V/f$) and contain all the potential vorticity of the flow,
- inertio-gravity waves, which propagate with sound speed ($c = \langle \phi \rangle^{1/2}$) and are dispersive in presence of rotation.

This decomposition may be interpreted in terms of slow manifold, for the potentio-vortical mode, and fast manifold, for the inertio-gravitational mode. We can also perform an Helmholtz decomposition of the two-dimensional velocity field into a rotational incompressible part and a divergent 'compressible' part, total energy being then the sum of potential, rotational and divergent energies. But this separation does not have the orthogonal structure of the first one, which, although based on an approximation, appears more physical. We will present energy spectra using both decompositions.

Plates 1(A,B). Flow initially dominated by potential vortices.
Plates 2(A,B). Flow initially dominated by inertio-gravity waves.

Plates 1,2.
(α) From left to right:
- energy spectra, at initial time t_0 (thin lines) and at final time t_{max} (thick lines), of potentio-vortical ——— and inertio-gravitational - - - - - - energies, ▲ indicating the Rossby deformation wavenumber;
- time evolution, from t_0 to t_{max} potential enstrophy ———, total ———, potentio-vortical ——— and inertio-gravitational - - - - - - energies.
- time evolution, from t_0 to t_{max}, of rotational———, divergent - - - - - - and potential –·–·–·– energies.

(β) Structure of the flow in physical space at t_{max}. From left to right:
vorticity (three-dimensional representation), vorticity (two-dimensional representation), potential vorticity, free-surface geopotential and stream function fields *.

(γ) Time evolution of the vorticity field; the time scales are not necessarily the same, because they are chosen in order to follow different characteristic structures (isolated coherent vortices, solitary dipoles or nonisolated coherent vortices).

* The geostrophic equilibrium is reached if, in physical space, the extrema of the stream function and the geopotential fields are correlated, because in this case we have $\psi_V = \phi_V/f$.

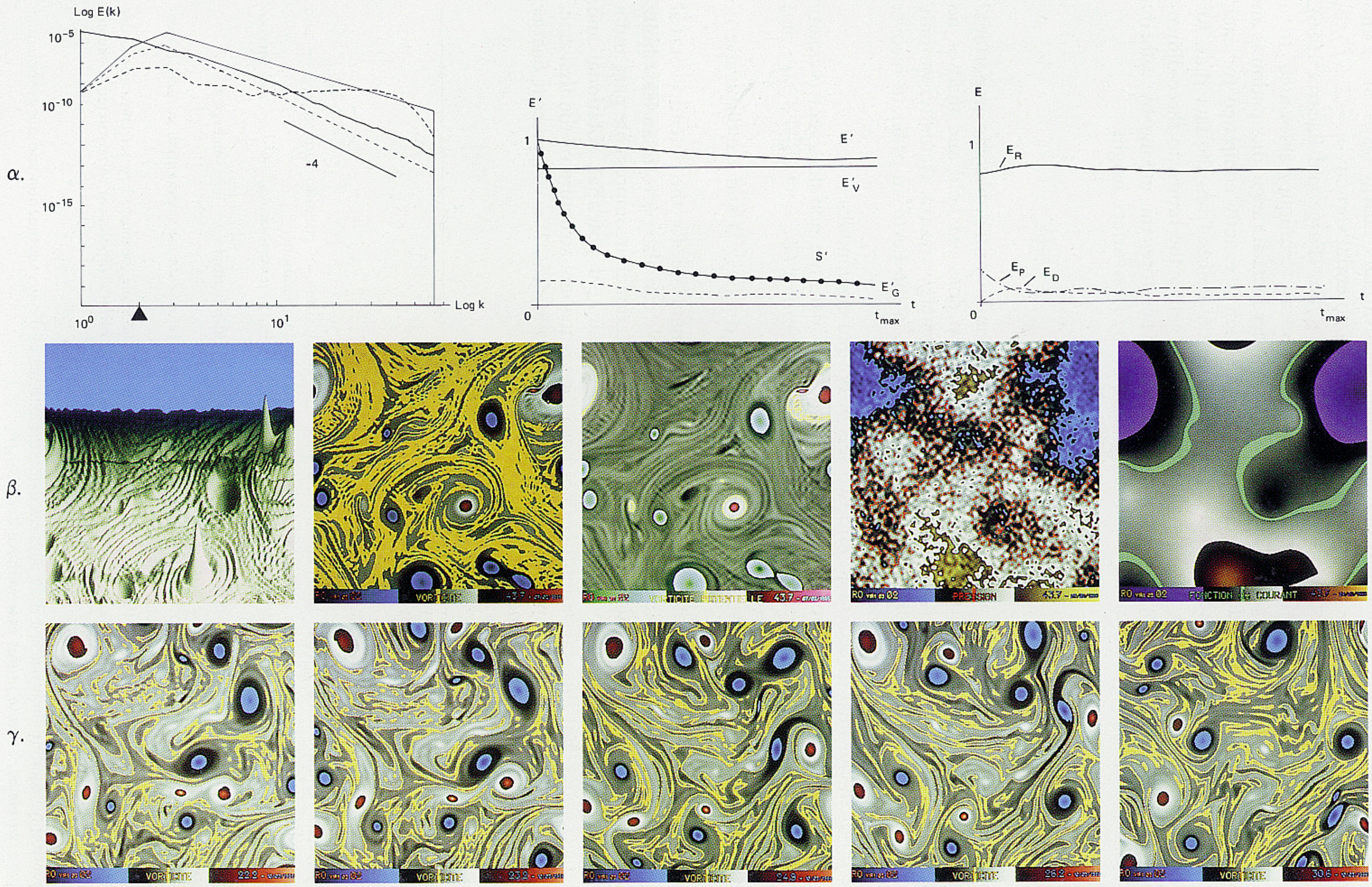

Plate 1A. Rotation rate $f = 10^{-4}$ s^{-1}.

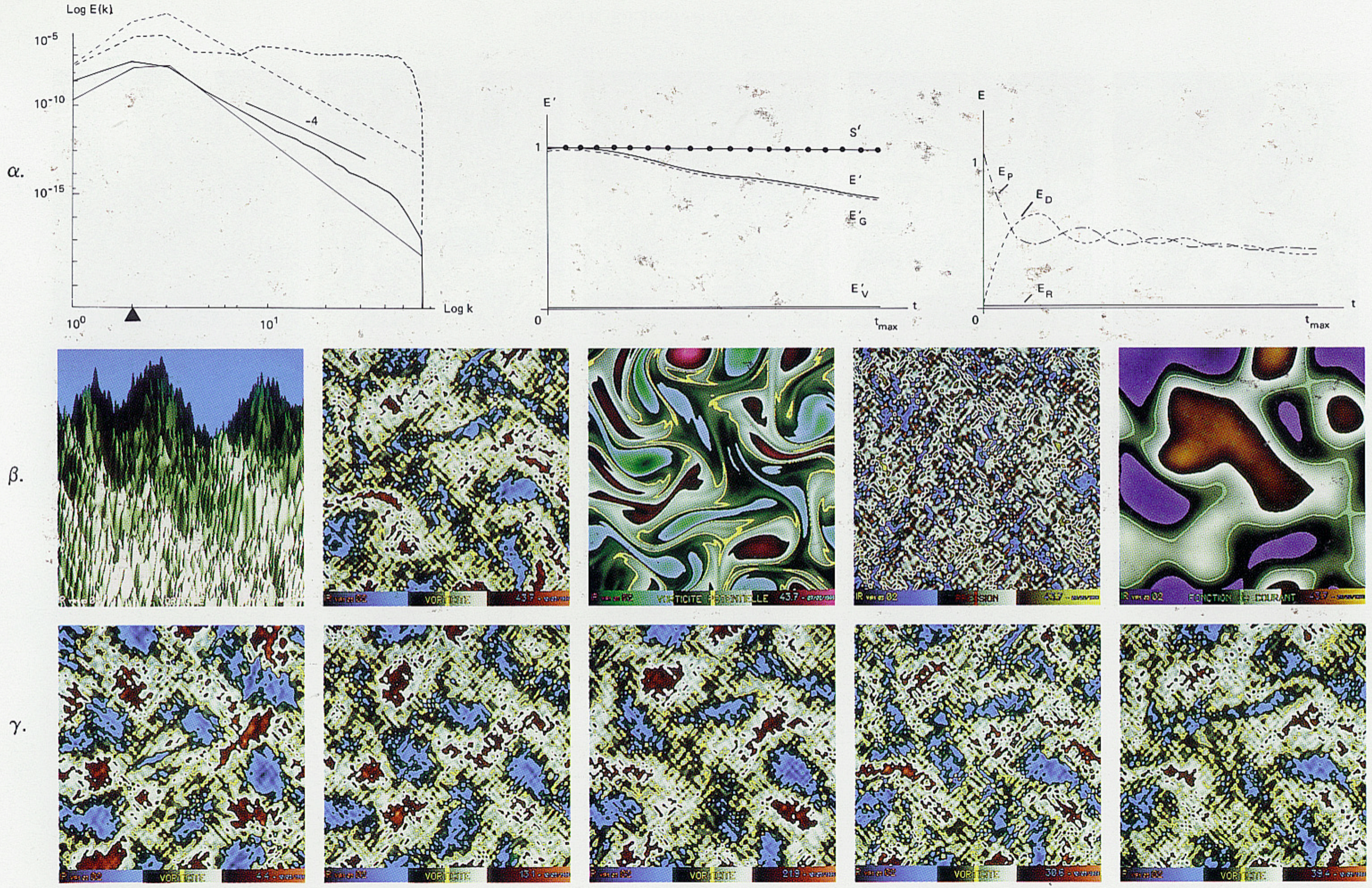

Plate 2A. Rotation rate $f = 10^{-4}\ s^{-1}$.

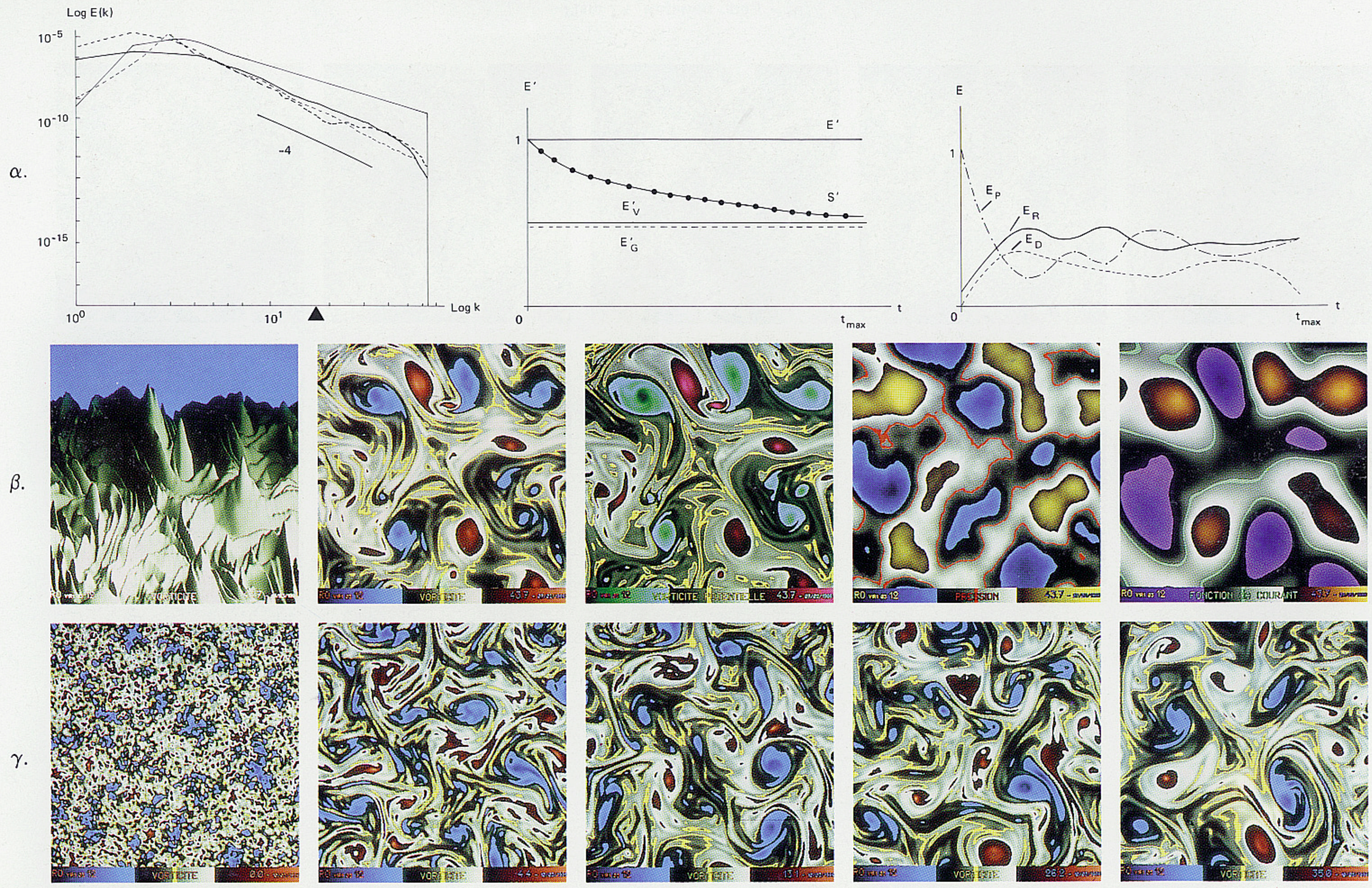

Plate 1B. Rotation rate $f = 6\times10^{-4}\ s^{-1}$.

α.

β.

γ.

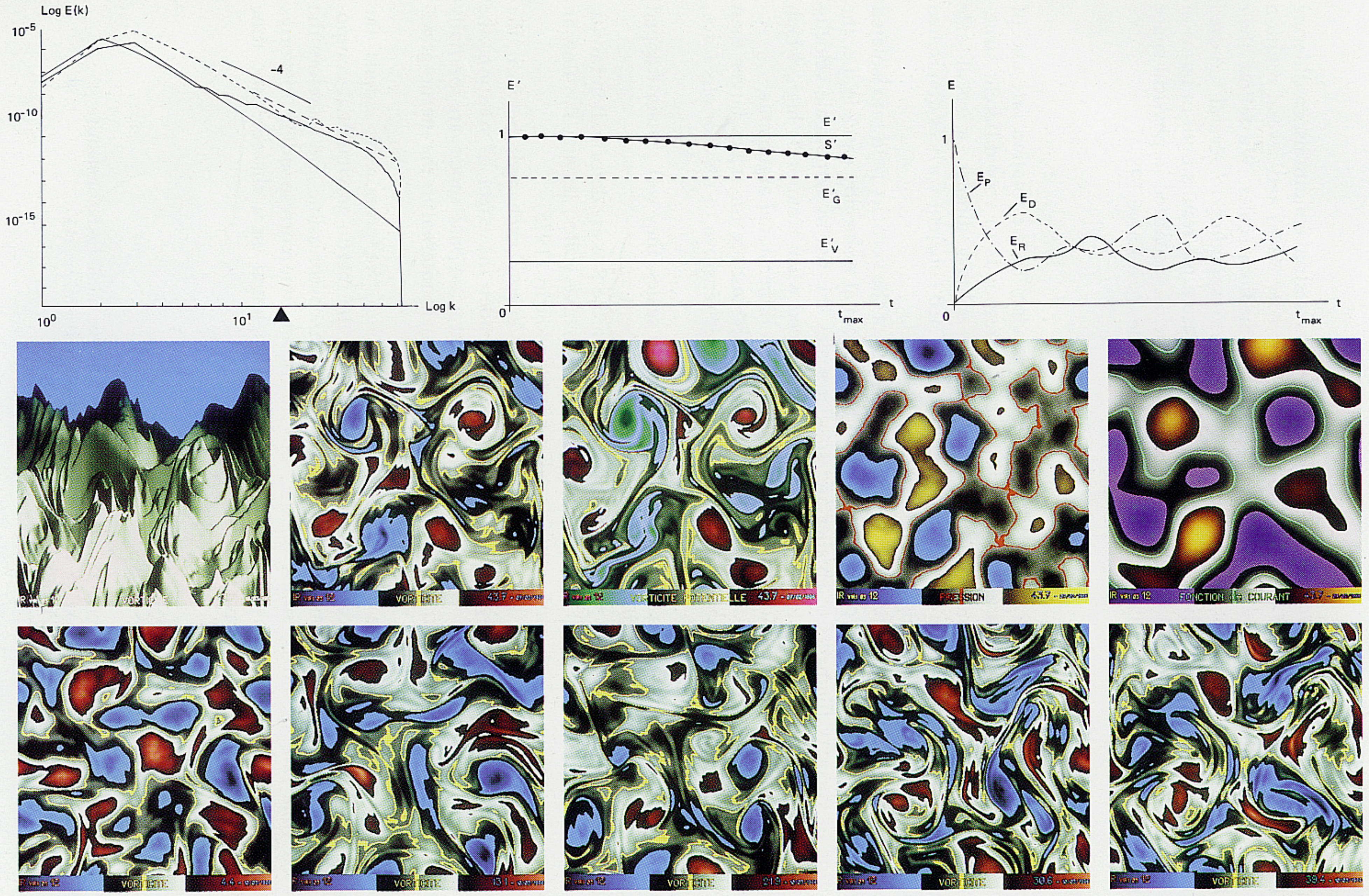

Plate 2B. Rotation rate $f = 6\times10^{-4}\ s^{-1}$.

4. Numerical experiments

We simulate the dynamics of an atmospheric layer close to terrestrial conditions, considering several initial flows presenting different amounts of inertio-gravity waves and several rotation rates; they are random realizations with energy peaked at medium scales. We follow their evolution during 44 'terrestrial' days (referred to as t_{max}), corresponding to several hundred eddy turn-over times, without adding any forcing term. We present the results both in terms of energy spectra and morphological structures of the relevant fields, visualized in physical space using high-resolution raster display with normalized color scales (see Farge 1987).

5. Conclusion

The rotational energy has always a k^{-4} spectrum, independent of the rotation rate and of the intensity of inertio-gravity waves, but this behaviour corresponds to very different structures in physical space (coherent vortices and modon-like solitary dipoles or small-scale inertio-gravity waves for moderate rotation rates, and non-isolated coherent vortices dominated by large-scale inertial waves for high rotation rates).

The inverse rotational energy cascade, i.e. the condensation of the vorticity field into isolated coherent vortices through vortex pairings, and the direct potential enstrophy cascade, i.e. the production of vorticity filaments of smaller and smaller scales sheared by large-scale velocity gradients until the dissipative scale is reached, are both reduced when rotation is high, which confirms a prediction of Farge and Sadourny (1986) based on dimensional arguments.

Rotation also inhibits the direct inertio-gravitational energy cascade for scales larger than the Rossby deformation radius, above which the flow becomes sensitive to the Coriolis force, therefore:
- if inertio-gravity waves are initially excited at large enough scales, they will remain trapped there due to rotation and there will be no geostrophic adjustment,
- if inertio-gravity waves are only present at scales smaller than the Rossby deformation radius, they will nonlinearly interact and cascade towards the dissipative scales, leaving the flow in geostrophic equilibrium, which confirms the prediction of Sadourny (1975).

In the range of parameters studied here, there are no energy exchanges between the potentio-vortical and the inertio-gravitational modes, but, under the effect of rotation, the potential and divergent energies contained in the inertio-gravitational mode are partially converted into divergent energy, transfers which therefore remain internal to the inertio-gravitational modes.

In conclusion, to parametrize the effect of sub-grid scales, we propose to adjust the dissipation rates independently for the potentio-vortical and the inertio-gravitational components. The potentio-vortical part behaves as if the flow were incompressible and therefore the dissipation time rate should be tuned on the eddy turn-over time corresponding to vortex motion, while for the inertio-gravitational part it should be tuned on a characteristic time corresponding to the divergent wave motions.

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