Wavelet based density estimation for noise reduction in plasma simulation using particles

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Context and goals of the present study

- Particle methods have been advocated for numerical simulation of fusion-relevant plasmas but are subject to the onset of statistical noise.
- Understanding this noise and dealing with it is an important practical issue but also a theoretical challenge of non-equilibrium thermodynamics.
- Based on recent developments in statistics and data analysis, we introduce a new approach to better reconstruct plasma particle distribution functions.
- As a first step towards implementing our method in a full simulation, we study its effect on particle data obtained via a classical simulation scheme[1].
- We make systematic comparisons with a proper orthogonal decomposition (POD) method introduced in Ref. 2.

From finite size particles…

- Reconstruction of the particle distribution function by assuming that each particle represents a cloud of charge spread in a small volume.
- Closely related to the **kernel density estimation** method, ubiquitous in statistics.
- Imposes a **single smoothing scale**, of the order of the plasma Debye length.

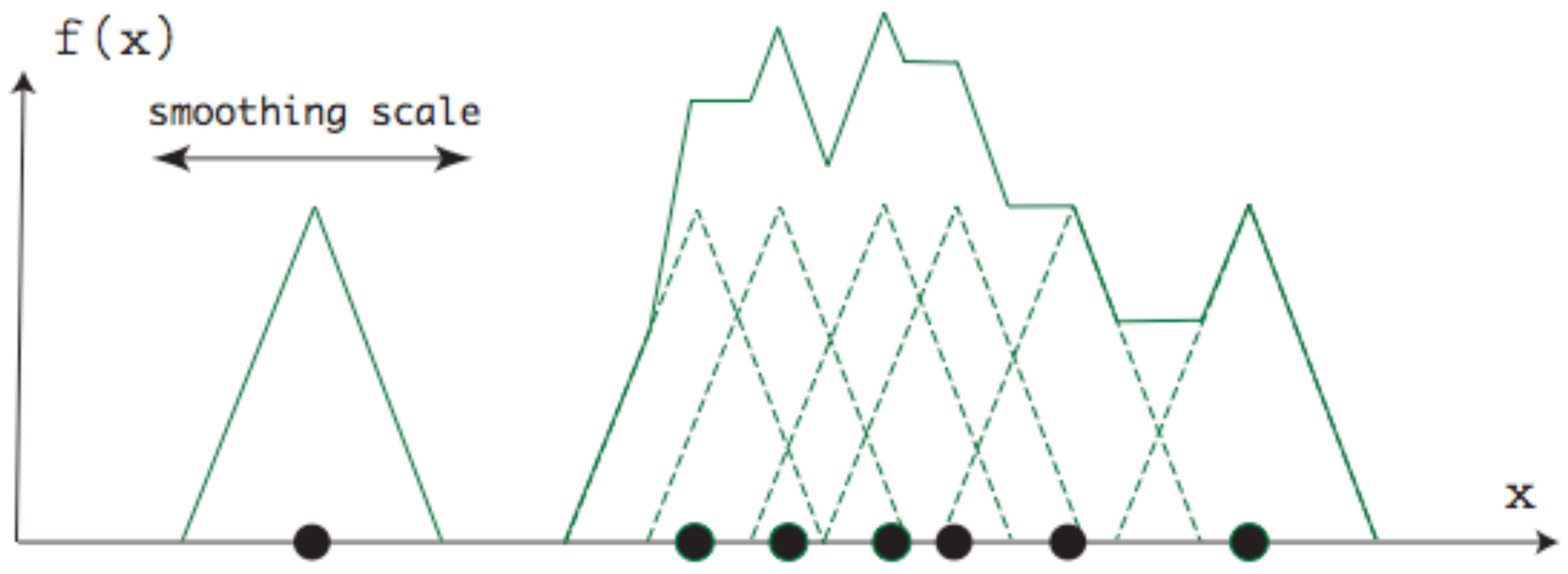
…to wavelets.

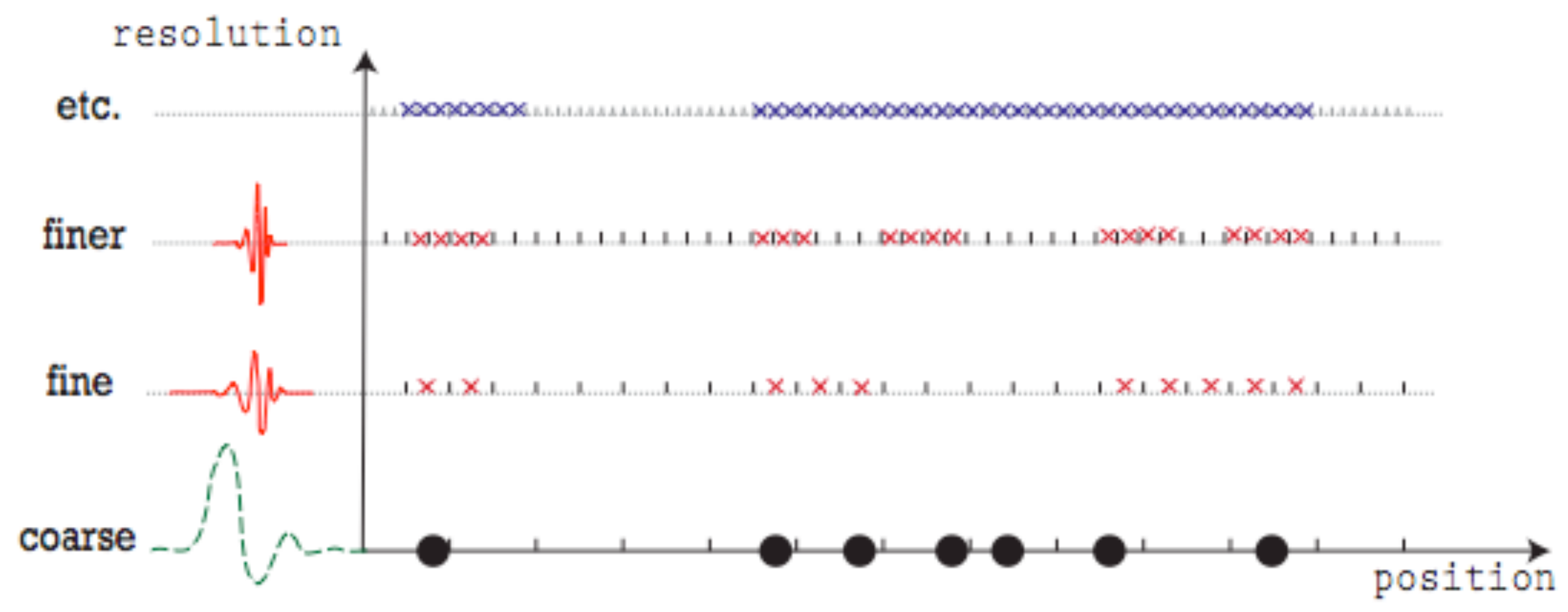
- The wavelet transform offers a **multiscale** representation of the particle distribution function, while also remembering the location of its features in phase space.
- One can reconstruct the Dirac delta function corresponding to each particle from its wavelet coefficients. Denoising consists in **discarding** some of these coefficients.
- We use **orthogonal** Daubechies wavelets with 6 vanishing moments.

Schematic views of the particle distribution function reconstruction procedures based on FSPs (left) and wavelets (right).

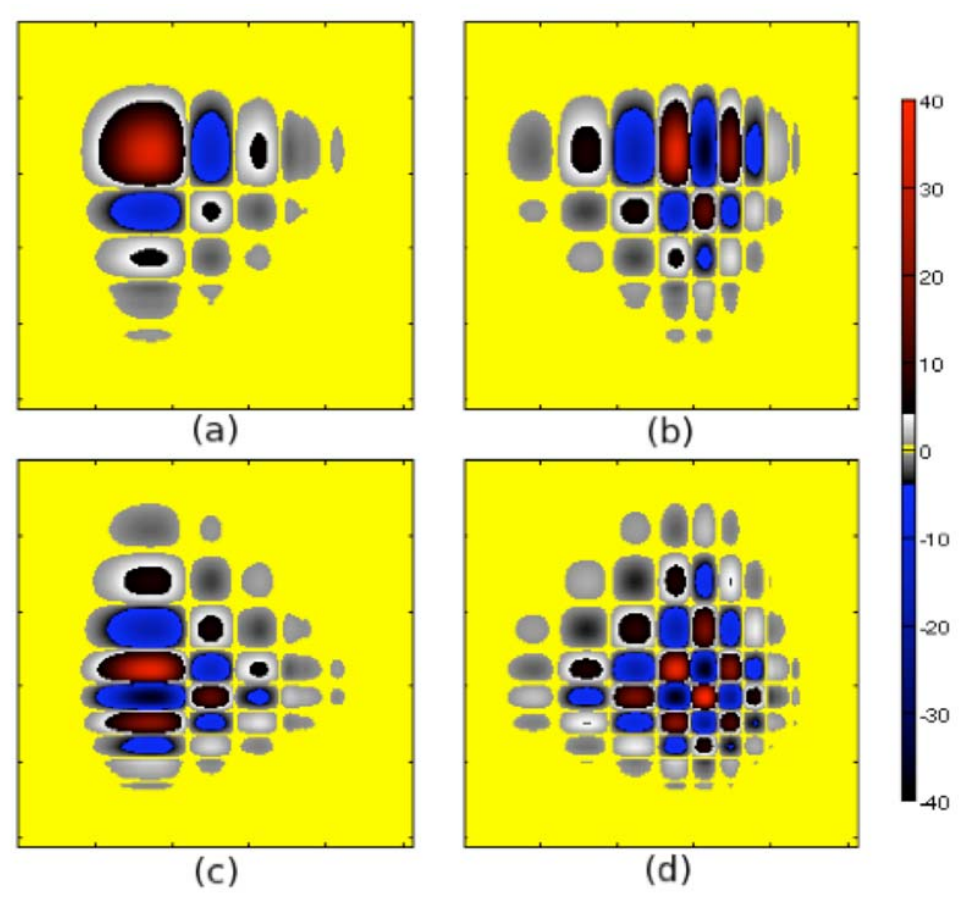
The particles are represented as black circles in a 1D phase space.

Strong wavelet coefficients, are indicated with crosses, red if they are kept, and blue if they are discarded.





And in 2D :



A classical toy model

- the 1D Vlasov-Poisson equation for electrons in a neutralizing background.
- Particle-in-Cell scheme with classical triangular charge assignment function. (Ref 1.)
- N_p simulated particles

Wavelet based density estimation (WBDE)

1. Construct a histogram f^H on a grid of size $(N_g)^d$ and compute its wavelet coefficients $\tilde{f}_\lambda^H$
2. Process wavelet coefficients as follows (adapted from Ref. 3):

coarse scales	L	intermediate scales	J	fine scales
keep all	$2^{dL} := N_p^{\frac{1}{3}}$	keep only coefficients satistying: $\left\|\tilde{f}_\lambda^H\right\| \geq \sqrt{\frac{j}{4N_p}}$	$2^{dJ} := \frac{N_p}{\log_2 N_p}$	discard all

3. Reconstruct the denoised distribution function

POD method

Construct a histogram and consider it as a matrix M, then compute the SVD of M

$$M = {}^tUSV$$

And set all but a few singular values to zero. (Ref. 2)

First test case: bump-on-tail instability

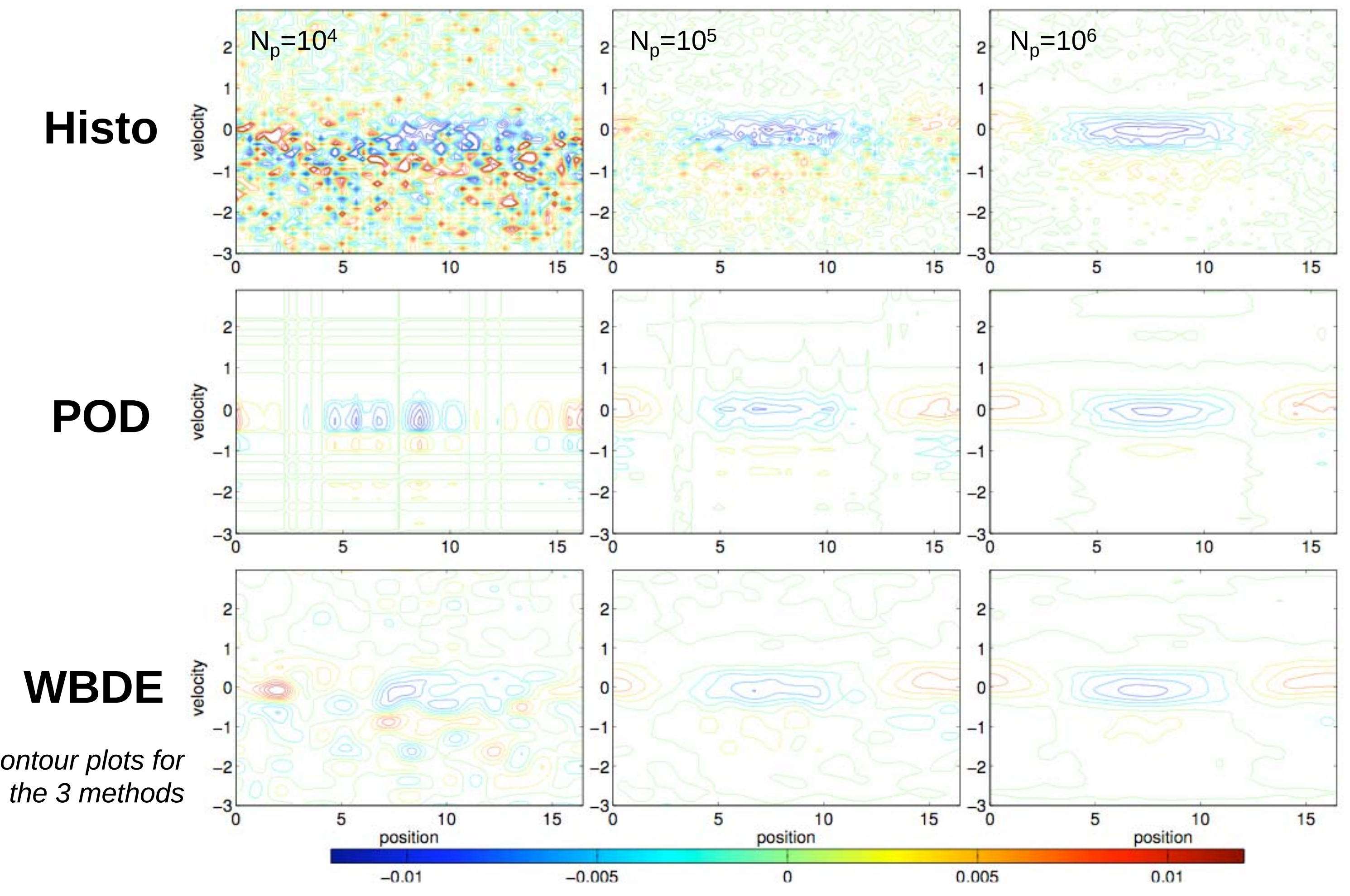
Initial condition: continuous distribution with slightly unstable tail. Random sampling of particle positions.

Remarks: A k=1 mode grows exponentially. Denoising is applied after nonlinear saturation. The initial condition is substracted for visualization. Respectively 1, 2 and 3 modes were used for the POD reconstructions (from left to right).

Relative mean squared error

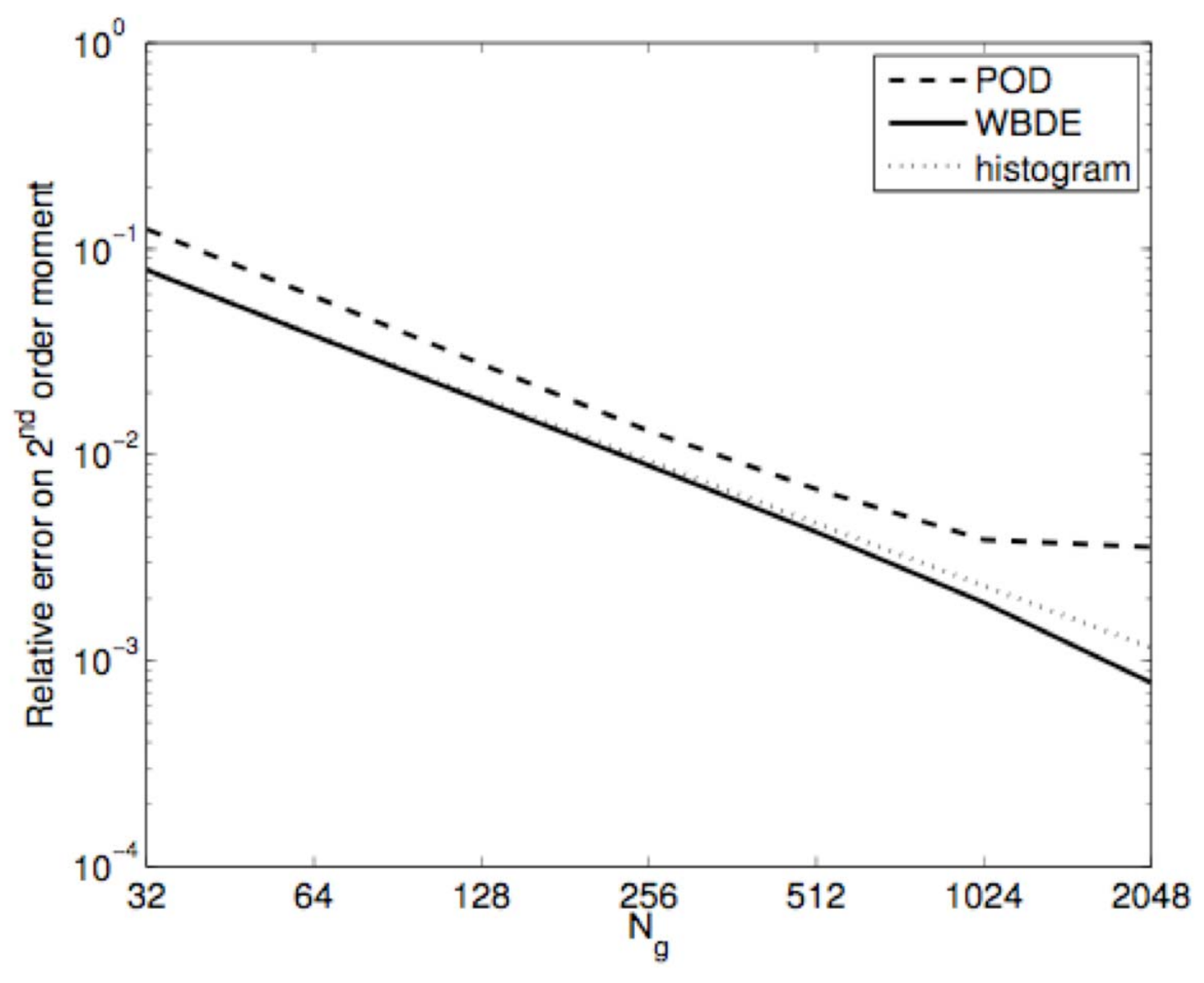
	$N_p = 10^4$	$N_p = 10^5$
f_H	0.443	0.140
f_P	0.163	0.090
f_W	0.173	0.086

Results: similar performance of WBDE and POD, both qualitatively (cf. right) and quantitatively (cf. left).

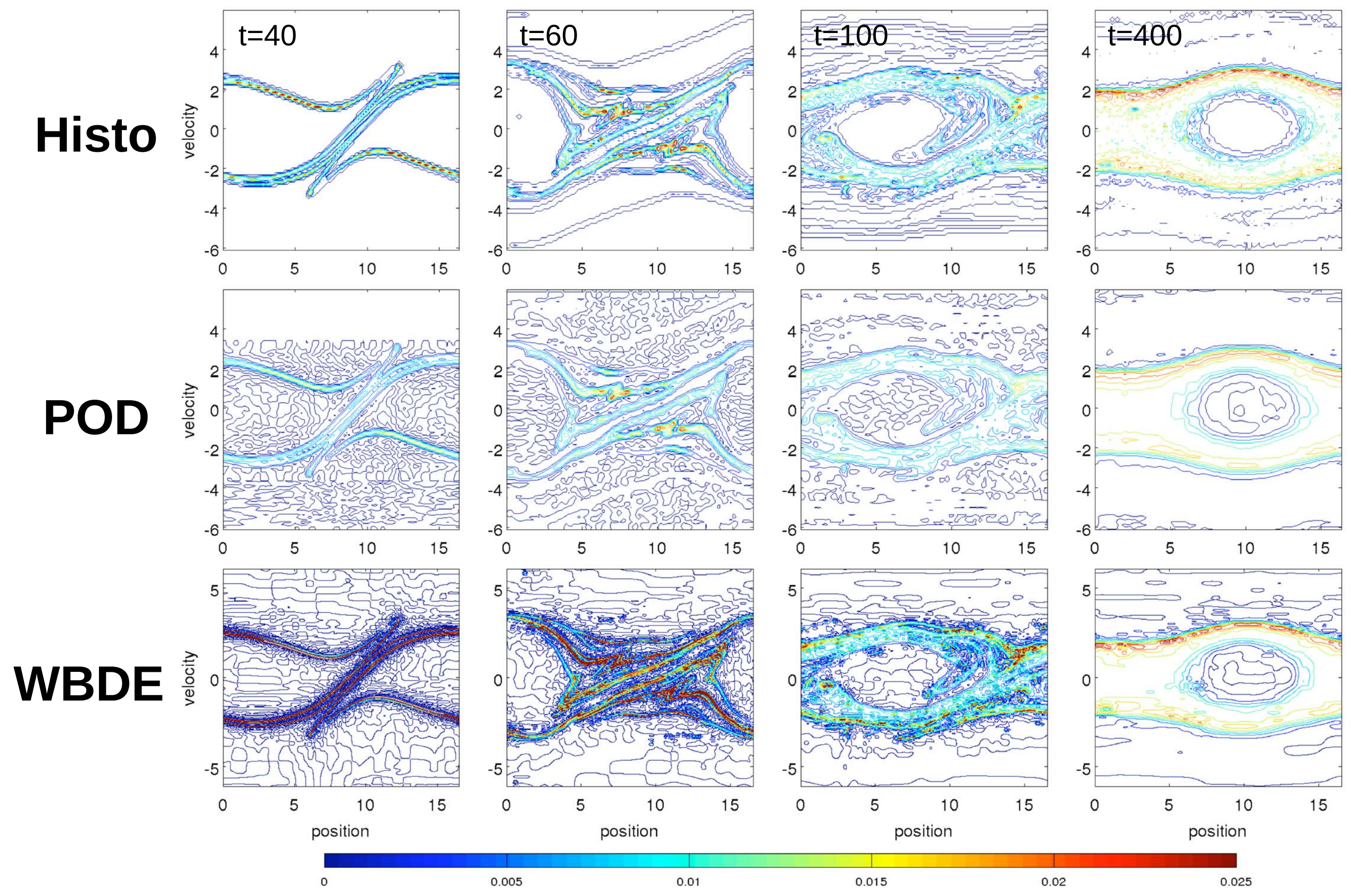


PDF contour plots for the 3 methods

Relative error on the second order moment as a function of grid resolution



Wavelet truncation preserves moments, but making a histogram introduces an error proportional to its grid spacing.



Second test case: two streams instability

Initial condition: two cold interpenetrating electron beams, with imposed particle velocities and positions (quiet start).

Remarks: particle trajectories start to loose memory of the initial condition and self-consistent noise builds up in the system, hence tough test for denoising methods. Moreover, WBDE assumes that the particles are statistically independent, which is not the case here at least in the beginning. Respectively 28, 27, 18 and 5 POD modes were used for the 4 time instants.

Results: WBDE preserves most of the relevant information even in the absence of noise, but manages to remove noise when it shows up.

Conclusions

The advantages of our proposed denoising method are:

- preservation of sharp features,
- low computational cost,
- applies in any phase space dimension,
- order 1 conservation of moments.

As a first test, it was applied to post-process simulations of a 1D Vlasov-Poisson plasma.

Ongoing investigations consider higher dimensionality and apply the denoising in real time during a simulation.

References :

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