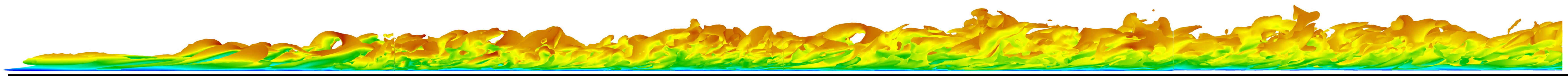

Coherent Vorticity Extraction in Turbulent Boundary Layers Using Orthogonal Wavelets

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Boundary layer flow, coherent structures and wavelets

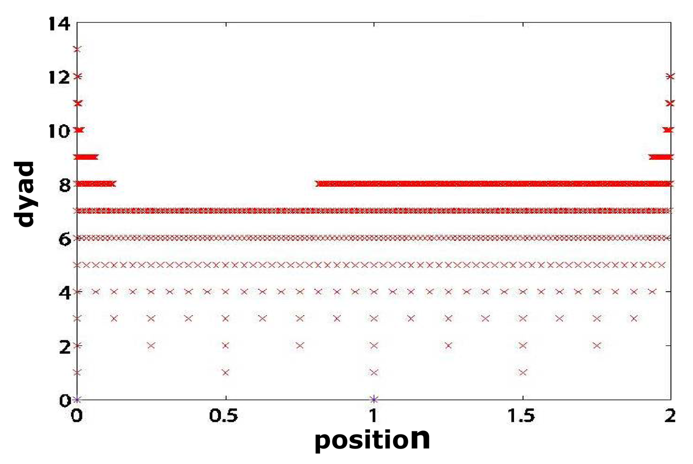
- **Motivation:** Complex, multiscale, random fields of turbulent motion can be split into coherent organized motion and an incoherent background flow.
- **There is no widely accepted definition of coherent structures!**
- **Aim:** Extract coherent vorticity structures from a zero-pressure-gradient turbulent boundary layer flow by orthogonal wavelets.

Flow configuration and parameters

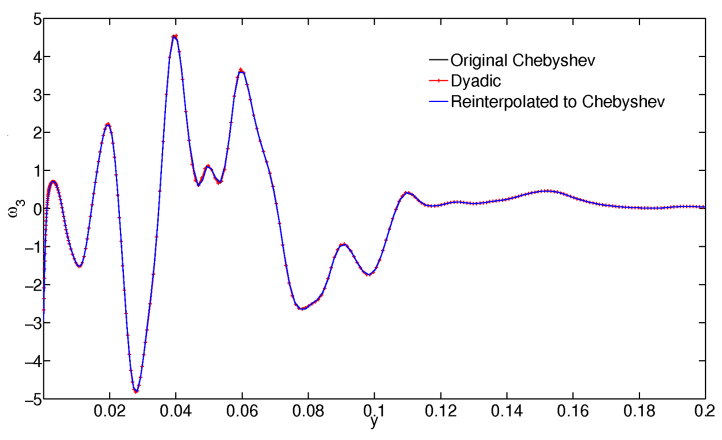
- DNS of ZPG turbulent boundary layer flow was performed with the following parameters[1]:
  - Grid: $N_x \times N_y \times N_z = 2048 \times 513 \times 256$
  - Box: $L_x \times L_y \times L_z = 1000\,\delta^*|_{x=0} \times 30\,\delta^*|_{x=0} \times 34\,\delta^*|_{x=0}$
  - Reynolds number: $Re_\theta \approx 1470, \quad Re_\theta = \frac{u_\infty \theta}{\nu}$
  - Resolution: $\Delta x^+ \times \Delta y^+ \times \Delta z^+ = 12.8 \times (0.018\text{-}5) \times 3.5$

Orthogonal wavelet decomposition

- The multiscale representation using wavelet is useful in understanding the physics of turbulent flows as locality in both space and scale is preserved.
- From the velocity field $\mathbf{u} = (u_1, u_2, u_3)$ we compute the vorticity field
$$\omega = (\omega_1, \omega_2, \omega_3) = \nabla \times \mathbf{u}$$
- The vorticity is given on the discrete grid points: (x_i, y_n, z_k)
  - where $i = 1, ..., N_x, \quad n = 1, ..., N_y, \quad k = 1, ..., N_z$ which equidistant in x and z directions and correspond to a Chebyshev grid in y direction.
- The data, in wall-normal direction are first interpolated onto an adapted dyadic grid.



Adapted dyadic grid by the position of the corresponding wavelets.



Interpolation of vorticity in wall-normal direction.

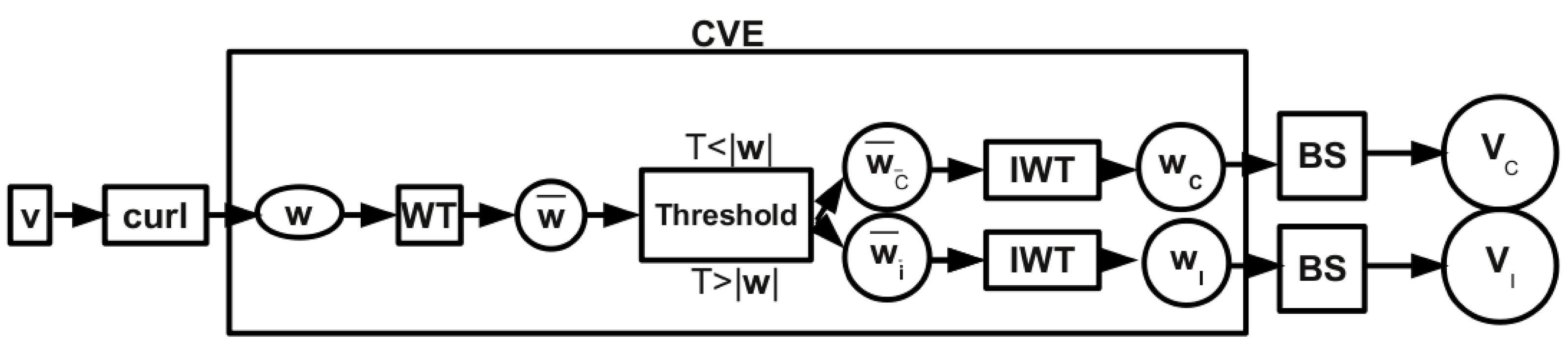
- The vorticity vector is then decomposed into a two-dimensional (2D) orthogonal wavelet series in the x-z direction using a 2D multiresolution analysis (Coiflet 12 wavelets);
- A wavelet decomposition is applied on the adaptive dyadic grid in wall-normal direction using Daubechies 4 wavelets.

Coherent vorticity extracion (CVE)

- CVE proposes minimum hypothesis[2,3]:

Coherent structures are not noise

Extracting coherent structures = removing noise

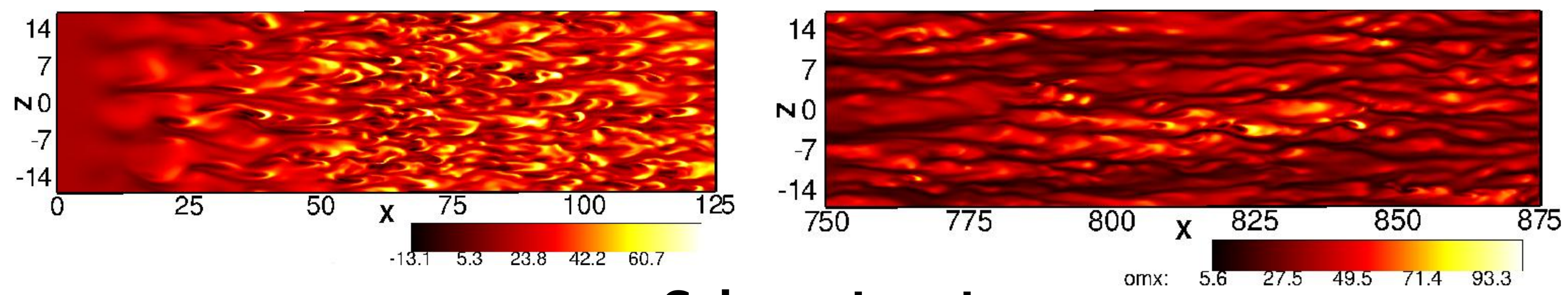


- **The underlying idea:** to perform denoising of vorticity in wavelet coefficient space. Thresholding the wavelet coefficients then detrermines which coefficients belong to the coherent and to the incoherent cintributions. The latter is assumed to be noise-like.
- Thresholding depends only on the enstrophy and the resolution:
$$T = \sqrt{4Z \ln N}, \quad Z = \frac{1}{2}\langle \omega \cdot \omega \rangle_{xyz}$$
- Finally we obtain coherent and incoherent parts of vorticity and corresponding enstrophy
$$\omega = \omega_c + \omega_i, \quad Z = Z_c + Z_i$$

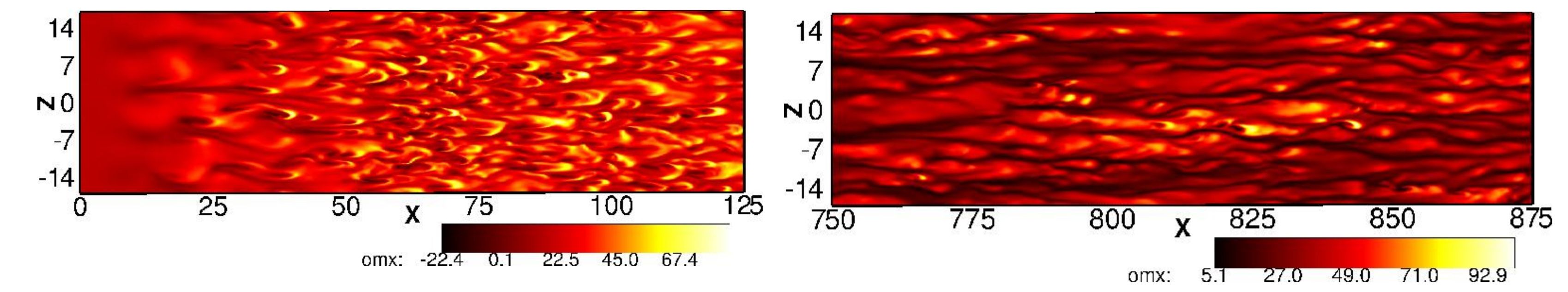
Numerical results

- Visualisations of the component of vorticity (ω_x): total, coherent and incoherent parts are shown on the plots:

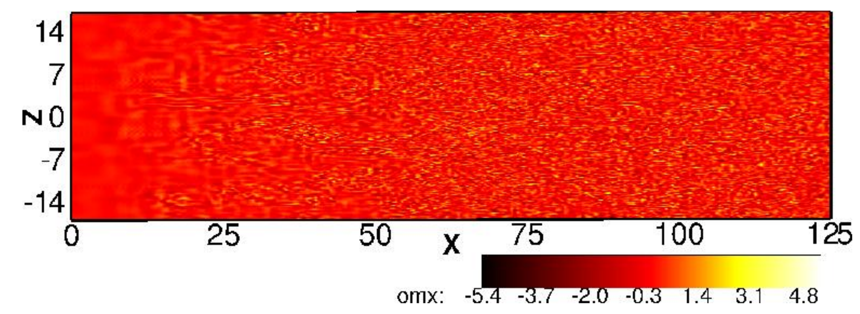
Total



Coherent part

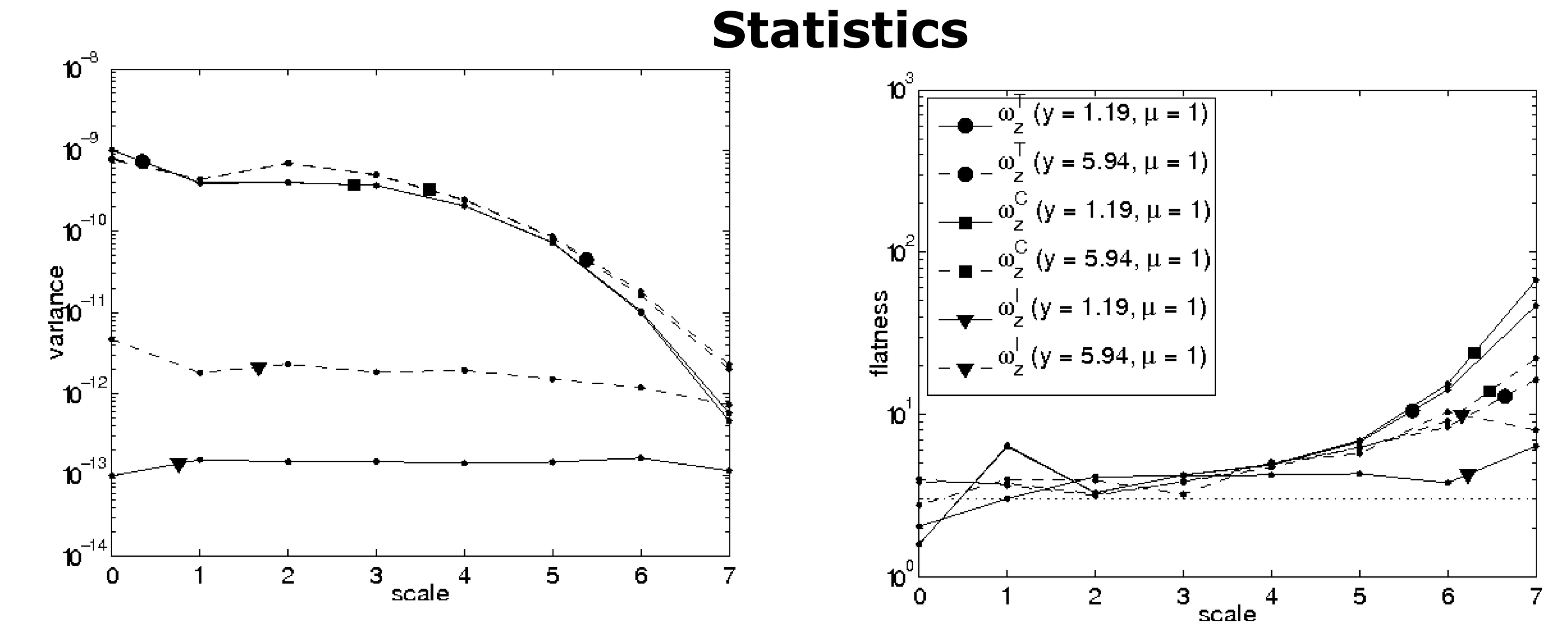


Incoherent part

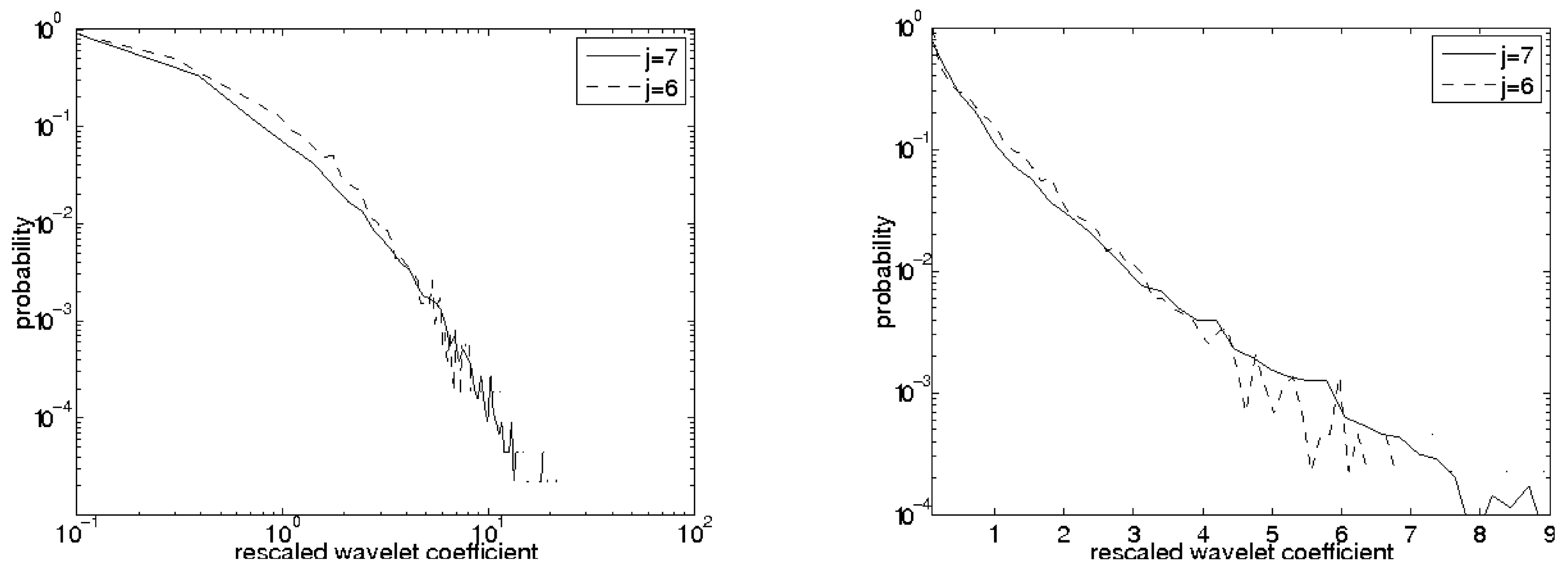


The coherent vortices present in the total field are well preserved in the coherent field using only 0.84% of the total number of wavelet coefficients, which retain 99.61% of the total enstrophy of the flow. In contrast, the incoherent part has weaker amplitude and is almost structurless.

Statistics



Scale-dependent second-order moments (left) and scale dependent flatness (right) for ω_z, at $y^+ = 34$ (solid line) and $y^+ = 170$ (dashed line).



PDFs of the wavelet coefficients (at two different scales) for ω_z at two different wall distances: $y^+ = 34$ (left) and $y^+ = 170$ (right).

The variance illustrates good agreement between the total and coherent vorticity. The variance of the incoherent part only weakly depends on scale, which indicates an equipartition of enstrophy, that confirms that the incoherent part is close to white noise (the flatness exhibits values around three). The PDFs of wavelet coefficients near the wall show algebraic decay with a slope -2 which is close to a Cauchy distribution and corresponds to strong intermittency. Away from the wall the PDFs become exponential. We also observe that the PDFs do not differ much for different scales considered here.

Conclusion: A zero-pressure gradient three-dimensional turbulent boundary layer was studied by means of high-resolution DNS. A new adaptive three-dimensional wavelet transform was developed which accounts for the flow anisotropy by using different scales in the wall-normal and wall-parallel directions. CVE was applied and the obtained results showed that fewer wavelet coefficients (<1%) are sufficient to retain the coherent flow structures, while the large majority of coefficients corresponds to the incoherent background flow which is unstructured and noise-like.

1. Khujadze, M. Oberlack, 2004, Theor. And Comput. Fluid Dyn. **18**, 391,
2. Farge, M., 1992, Annu. Rev. Fluid Mech. **24**, 395,
3. Farge, M., Pellegrino, Schneider, 2001, Phys. Rev. Lett. **87**(8), 45011.