Influence of no-slip boundary conditions on coherent structures in 2D turbulence

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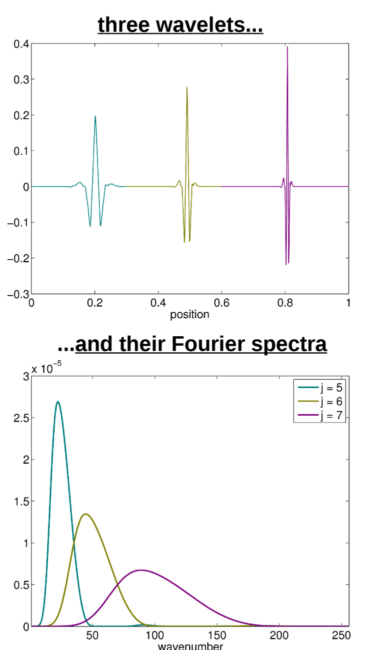
INTRODUCTION

- In wall-bounded turbulence, we expect **many elementary events of dipole-wall type** to occur,
- you may find a study of this elementary event in the companion poster by the same authors,
- here, we work in the same setting, except that we switch to a random initial condition, chosen so that the flow is in the **fully developed turbulence** regime,
- we expect **a lot of dissipative structures** to be produced at the wall and then advected by the flow,
- here, we examine the influence of these structures on **vorticity statistics** using **wavelet analysis**,
- in addition to the **wall-bounded** case, we also study a **wall-less** case to serve as a reference.

WAVELET REPRESENTATION

- We work with the periodized Coiflet 12 orthogonal wavelet basis on $L^2(\mathbb{R}/\mathbb{Z})$



- The vorticity field has the following multiscale expansion (where $\overline{\omega}$ is the mean value of ω):

$$\omega = \overline{\omega} + \sum_{\lambda \in \Lambda} \langle \omega \mid \psi_\lambda \rangle \psi_\lambda = \overline{\omega} + \sum_{\lambda \in \Lambda} \tilde{\omega}_\lambda \psi_\lambda$$

$$\Lambda = \left\{(j, i_x, i_y, \mu) \mid j \in \mathbb{N}, (i_x, i_y) \in \{0, \dots, 2^j - 1\}^2, \mu \in \{1, 2, 3\}\right\}$$

$$\lambda = (\underset{\text{scale}}{j}, \underbrace{i_x, i_y}_{\text{position}}, \underset{\text{direction}}{\mu})$$

- By convention, large values of j correspond to fine scales.
- Fundamental statistical properties of the flow are encoded in its **scalewise histograms**, obtained by counting wavelet coefficients inside equally sized bins. Below we shall consider only the horizontal direction (μ=1).

COHERENT VORTICITY EXTRACTION

- We define by « **coherent** » everything that is not **noise**[2],
- By « **noise** », we mean anything that can be modelled by a **random process**,
- To extract coherent vorticity is equivalent to **denoising** the vorticity field,
- As a first guess, we make the hypothesis that the noise is **additive and Gaussian.**

$$\omega = \omega_C + \omega_I = \text{coherent} + \text{incoherent} = \text{non-Gaussian} + \text{quasi-Gaussian}$$

- Previous definitions of coherent vorticity extraction assumed that the noise was decorrelated (white), but here we allow **any kind of correlation** for the noise.
- The wavelet representation is well suited for denoising, since Gaussian contributions correspond to the smallest wavelet coefficients at their respective scale and direction:

$$|\tilde{\omega}_{\mu,j,\mathbf{i}}| \leq T_{\mu,j} \quad \rightarrow \textbf{incoherent}$$

$$|\tilde{\omega}_{\mu,j,\mathbf{i}}| > T_{\mu,j} \quad \rightarrow \textbf{coherent}$$

- at large scales (j < 6), thresholds are arbitrarily set to zero, since they cannot be estimated statistically due to lack of sampling,
- at all other scales, thresholds are determined from the wavelet coefficients by an iterative algorithm[1]. In the limit, the thresholds equal 3 times the standard deviation of the incoherent wavelet coefficients.

$$\begin{cases} \Theta_0 = +\infty \\ \Theta_{n+1} = 3\sigma_{j,\mu}(\Theta_n) = 3\left(\dfrac{\sum_i |\tilde{\omega}_{j,\mu,i}|^2 I(|\tilde{\omega}_{j,\mu,i}| < \Theta_n)}{\sum_i I(|\tilde{\omega}_{j,\mu,i}| < \Theta_n)}\right)^{\frac{1}{2}} \end{cases}$$

- we focus on two quantities:
  1. the **global compression rate**

$$\rho = \frac{\mathcal{N}}{\mathcal{N}_C}$$

where $\mathcal{N}$ is the total number of wavelet coefficients, $\mathcal{N}_C$ is the number of coherent wavelet coefficients,

  2. the **scalewise compression rate**

$$\rho_j = \frac{\mathcal{N}_j}{\mathcal{N}_{C,j}}$$

where $\mathcal{N}_j$ is the total number of wavelet coefficients **at scale j**, $\mathcal{N}_{C,j}$ is the number of coherent wavelet coefficients **at scale j**,

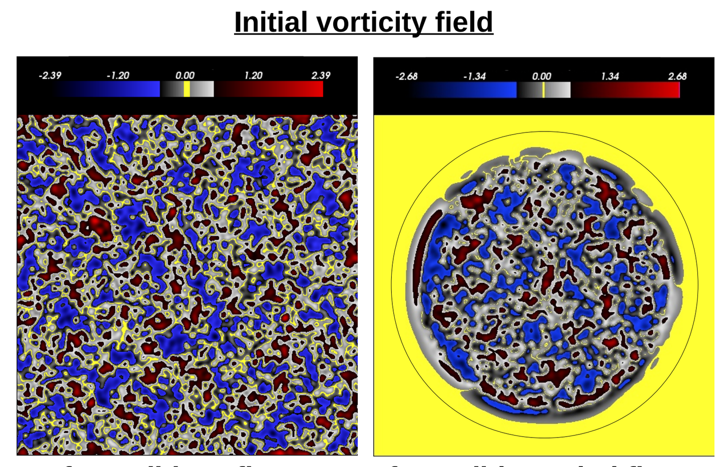
SETUP

To study wall-less flows, we take as initial vorticity one realization of a Gaussian process with a power spectrum peaked at k=6 and with spectral slope -1 up to k=48, and **vanishing for k>48**.
To study wall-bounded flows, we take as fluid domain the disk of radius 0.45 centered in (0.5,0.5).
We obtain an initial vorticity field satisfying the relevant boundary conditions as follows:

1. start from the same random vorticity field that is used for wall-less flows,
2. compute the stream function by solving the Poisson equation on the torus,
3. smoothly set the stream function to zero outside the fluid domain by multiplying it with a large scale, Reynolds independent mask function,
4. compute the resulting vorticity field.

The **same initial data** are used for all subsequent experiments.
The initial energy is 2.10^{-3} in the wall-less case and $0.88\ 10^{-3}$ in the wall-bounded case.

Initial vorticity field



for wall-less flows — for wall-bounded flows

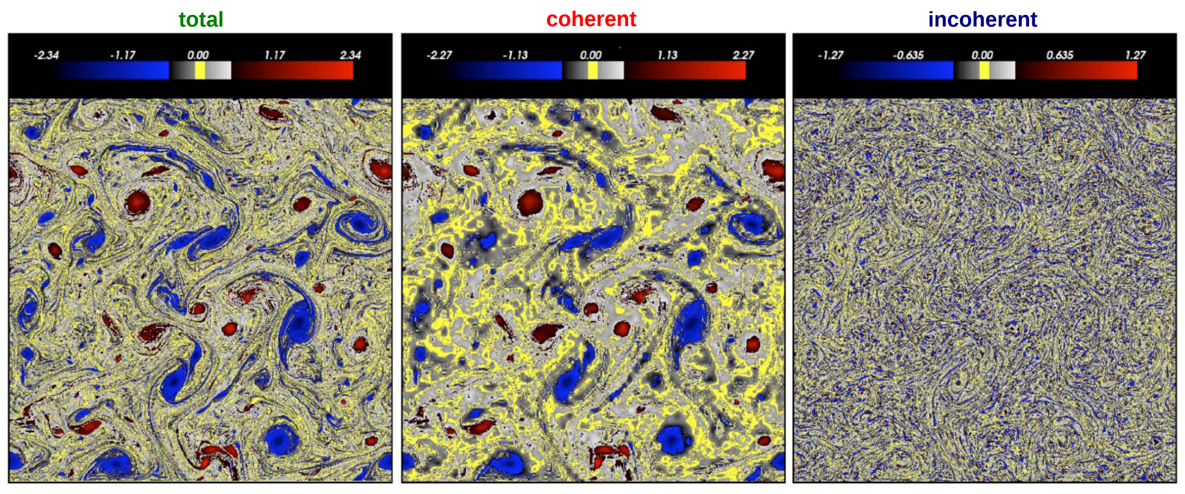
Summary of numerical computations reported here

	Wall-less flows				Wall-bounded flows			
Re x 10^{-3}	64	255	1025	4100	2	8	33	130
N	1024	2048	4096	8192	1024	2048	4096	8192
η x 10^{3}	-	-	-	-	1.	0.5	0.25	0.5
CPUs	4	4	4	8	4	4	4	8

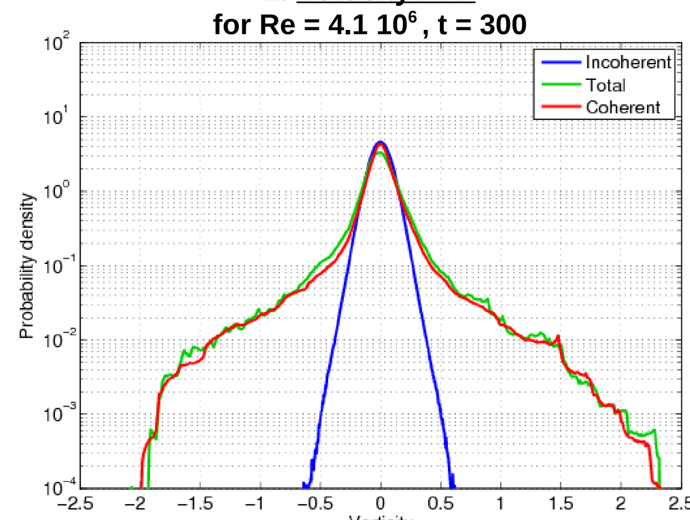
RESULTS

Wall-less flow

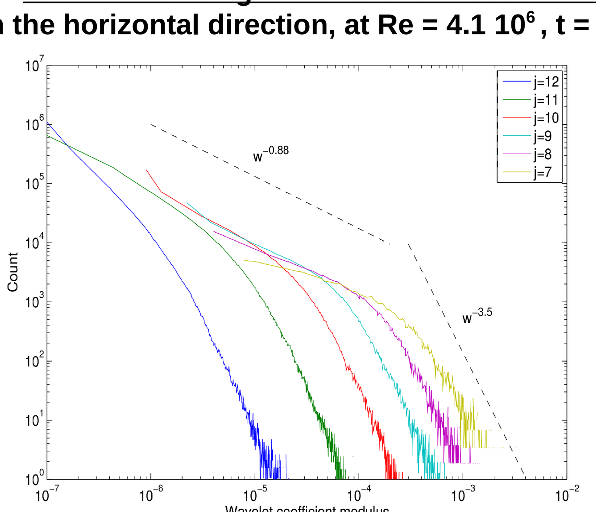
1. Snapshots of vorticity field for Re = 4.1 10^6, at t = 50



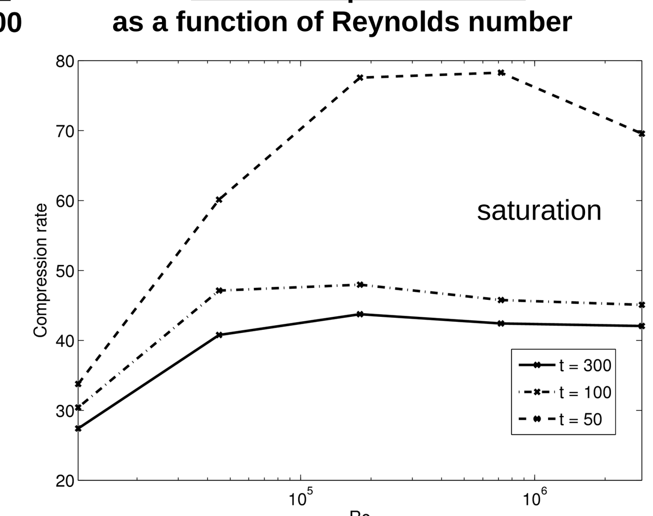
2. Vorticity PDF for Re = 4.1 10^6, t = 300



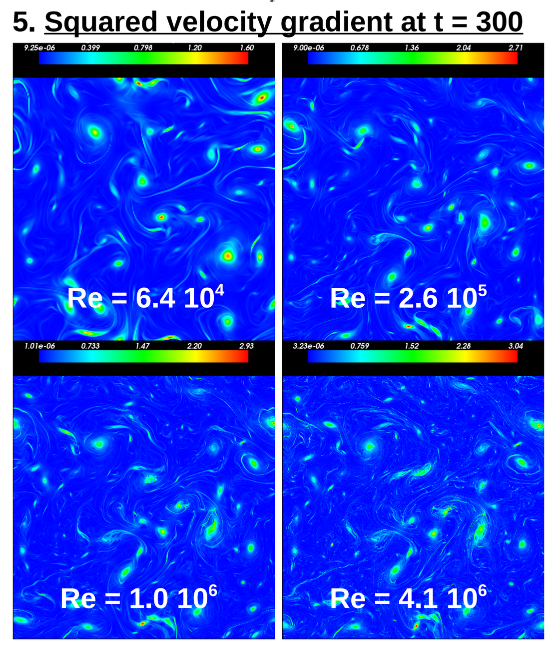
3. Scalewise histograms of wavelet coefficients in the horizontal direction, at Re = 4.1 10^6, t = 300



4. Global compression rate as a function of Reynolds number

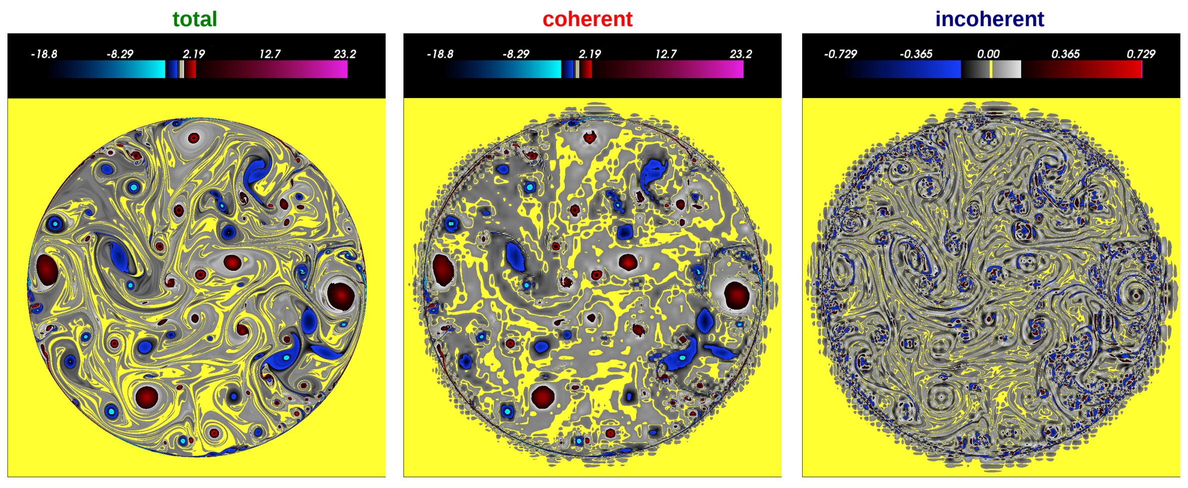


5. Squared velocity gradient at t = 300

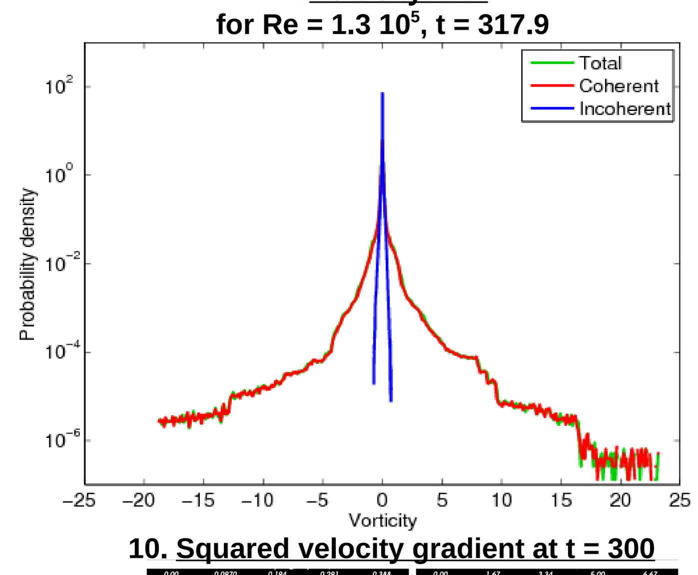


Wall-bounded flow

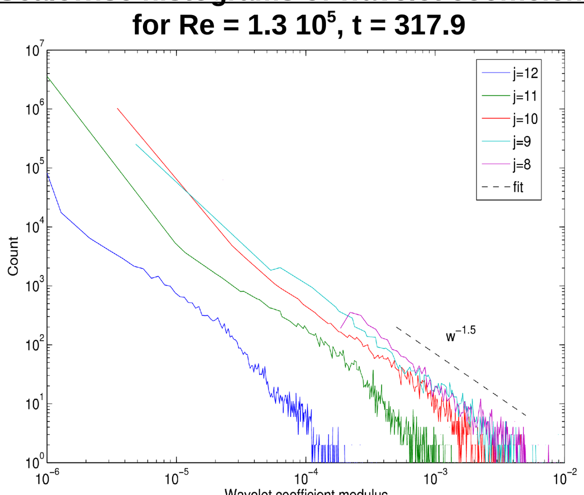
6. Snapshots of vorticity field for Re = 1.3 10^5 at t = 317.9



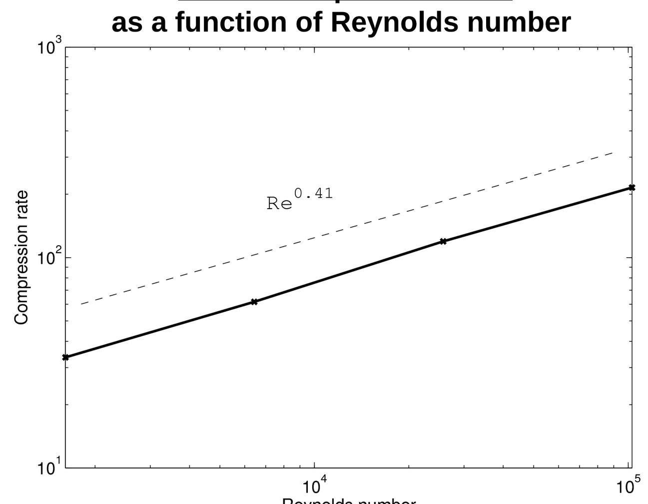
7. Vorticity PDF for Re = 1.3 10^5, t = 317.9



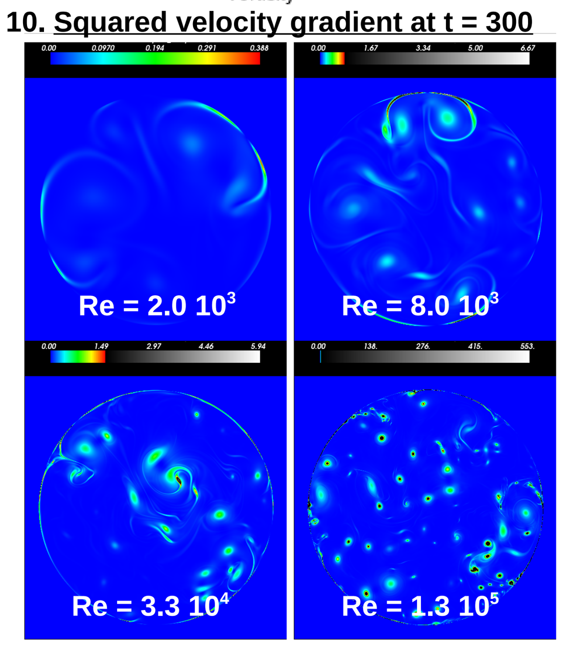
8. Scalewise histograms of wavelet coefficients for Re = 1.3 10^5, t = 317.9



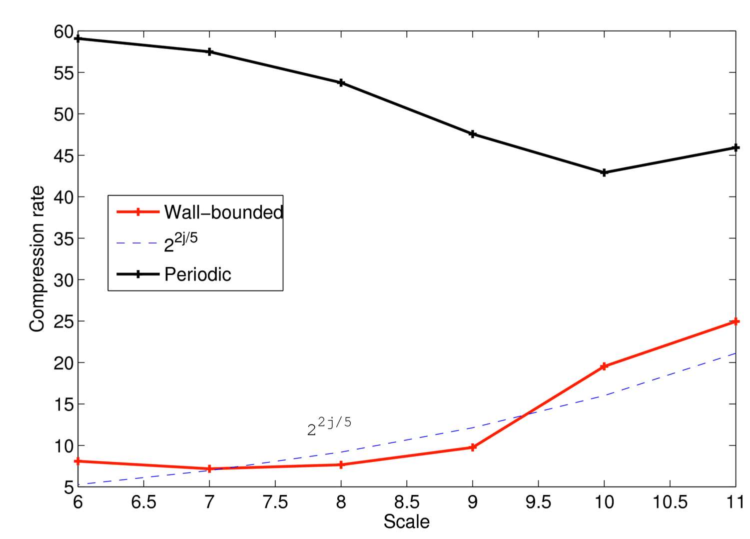
9. Global compression rate as a function of Reynolds number



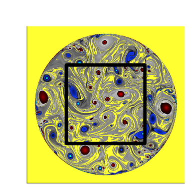
10. Squared velocity gradient at t = 300



- By looking at the coherent and incoherent parts in physical space (Figs. 1, 6), we see that the separation is consistent with our intuition about coherent structures and of noise.
- The probability density function (PDF) of coherent vorticity closely matches the PDF of total vorticity (Fig. 2, 7), both for the wall-less flow and for the wall-bounded flow.
- The PDF of the incoherent part is concentrated around zero, but its tails are better matched by an exponential distribution than by a Gaussian distribution.
- Plotting the histograms of wavelet coefficients (Figs. 3, 8) reveals striking differences between the wall-bounded and wall-less cases. When walls are present, the tails of the histograms are much heavier (with a -1.5 power law decay corresponding to a probability density whose mathematical expectation is not even defined).
- Compression rate keeps increasing with Reynolds number for the wall-bounded case, whereas it saturates for the wall-less case (Figs. 4,9).
- We interpret this as a signature of **inertial range intermittency** in the wall-bounded case, as can be seen from the scalewise compression rates (Fig. 11). In the wall-less case, the compression rate decreases in the inertial range and increases again only in the dissipative range, while for the wall-bounded flow it always increases when going to finer scales.
- snapshots of **squared velocity gradient** (Fig. 5, 10) reveal that at large Reynolds number molecular dissipation is more **spotty** in the wall-bounded case than in the wall-less case. This is due to the vortices that have been produced at the walls.



11. Scalewise compression rates in the wall bounded case, it was computed only from the subdomain shown in the inset



INTERPRETATION

- **For wall-less turbulence,** we have shown that the total wavelet compression rate does not improve with Reynolds number. We have related this with the fact that the scalewise compression rate doesn't improve at fine scales for a fixed Reynolds number.
- This could mean:
  a) either that wall-less 2D turbulence is **non-intermittent**,
  b) or that coherent vorticity extraction needs to be improved to obtain better compression rates.
- **For wall-bounded 2D turbulence the picture seems entirely different**[4],
- indeed, we have shown that the compression rate improves like $Re^{0.4}$,
- once the first detachment events have taken place, dissipative structures take over and govern the late time statistics of the wall-bounded flow in the whole domain,
- in future work, we would like to interpret the transfer of energy from coherent to incoherent parts of the flow as **turbulent dissipation**.
- Adaptive algorithms[3] are needed to follow coherent structures efficiently in wall-bounded flows.

The parallel C++ numerical code is available online under the GPL licence : http://justpmf.com/romain/kicksey_winsey
The software is in an alpha stage of development but support can be provided on demand.

References:
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