Dissipation by two-dimensional flows at vanishing viscosity

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Setting of the problem

Many flows have the fascinating property of relaxing towards equilibrium in a finite time even when microscopic coupling parameters become formally negligible. This phenomenon, sometimes called „anomalous dissipation“, could perhaps be more appropriately termed „robust dissipation“, considering that it allows us to perform tasks as simple as pouring water without spilling on the other side.
What are the generic processes responsible for the relaxation of flows in this regime ?
In this study, we have focused on two-dimensional incompressible flows, and isolated two processes of robust dissipation. The first one is molecular energy dissipation by wall-produced quasi-singular structures, and the second one is the purely macroscopic production of incoherent enstrophy (as defined below) by nonlinear mixing.

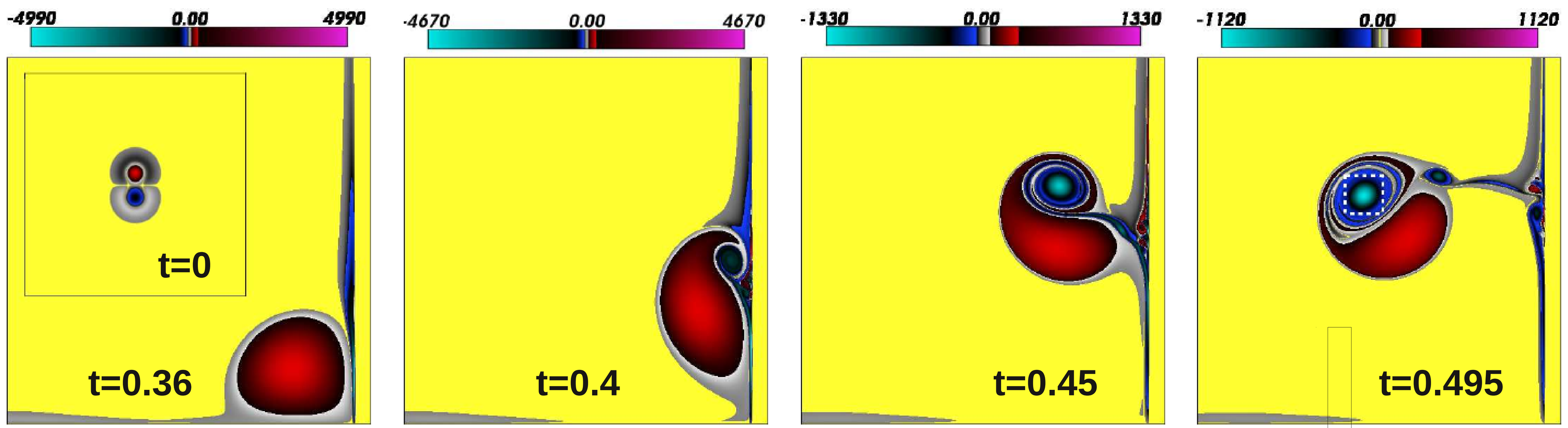
Molecular dissipation [1]

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -2\nu Z$$

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = -2\nu P + \boxed{\frac{\nu}{2}\oint_{\partial\Omega} \nabla(\omega^2)\cdot \mathbf{n}}$$

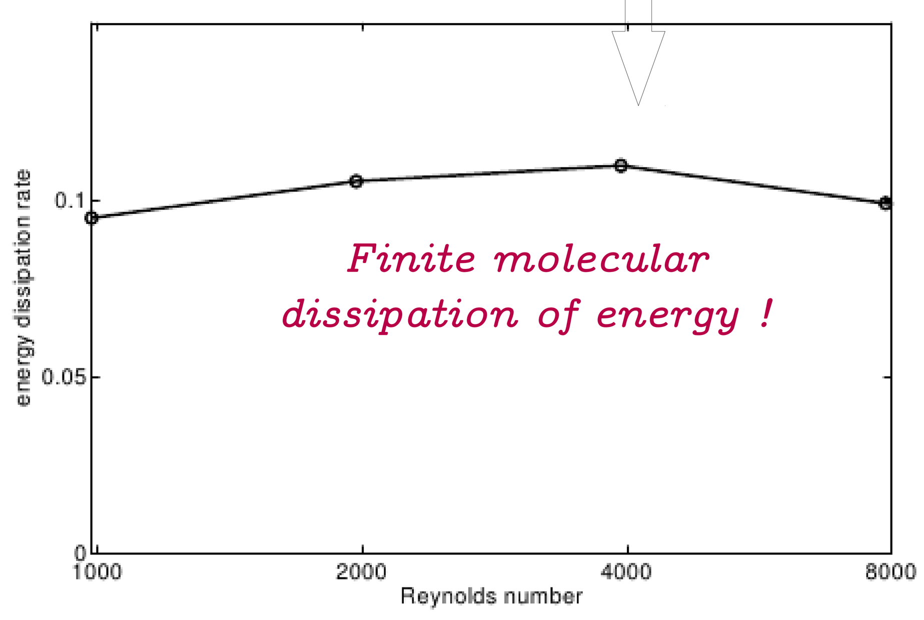
Dissipation of energy is possible only if sufficient enstrophy is produced at the boundary.

We have studied a dipole-wall collision for varying viscosity, and shown that intense structures are produced at the boundary, following the roll-up of a vortex sheet.



By integrating the energy dissipation rate in a region of fixed size centered on the main structure (dashed white box above), we show that we have isolated an **energy dissipating structure**.

$$\frac{\mathrm{d}E}{\mathrm{d}t} \xrightarrow[\nu\to 0]{} D > 0$$



Dissipation by the structure as a function of Re

These results where obtained using the volume penalization method discretized with a Fourier pseudo-spectral scheme.
We have shown that we obtain in this manner a good approximation of Navier boundary conditions

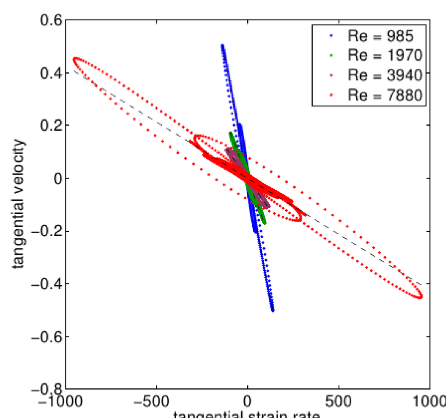
$$u_2 + \alpha(\mathrm{Re}, \eta, N)\partial_1 u_2 \simeq 0$$

with $\alpha \propto \mathrm{Re}^{-1}$

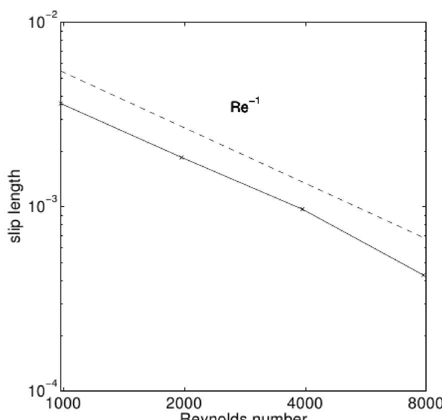
We conjecture that our results also apply to the case $\alpha = 0$

(no-slip boundary conditions) .

This could be checked by using a high resolution compact finite differences scheme in the wall-normal direction.



Navier correlation



Slip length

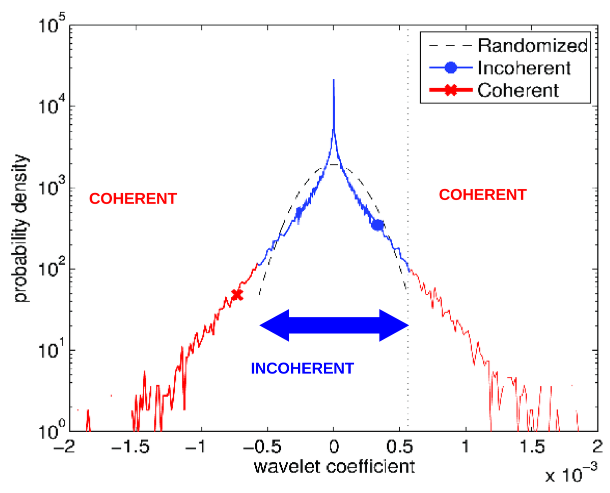
Another interesting aspect would be to study the collision (before detachment) using a boundary layer Ansatz (see poster by the same authors)

Flow dissipation [2]

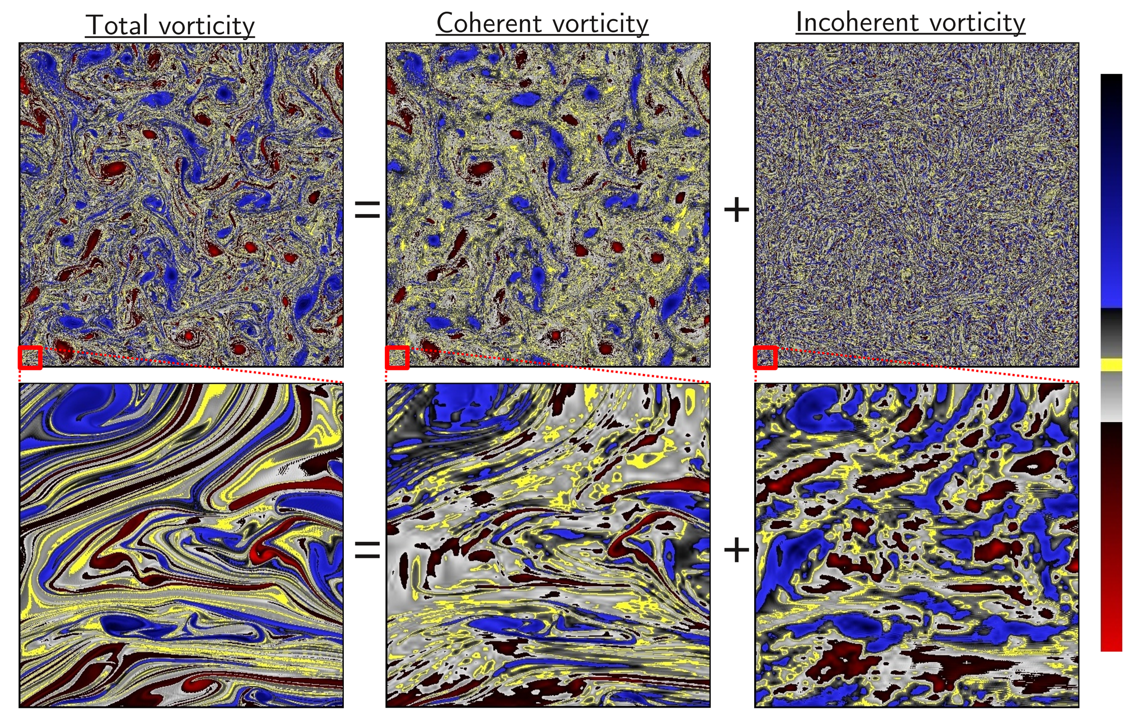
- **Scale-wise coherent vorticity extraction** is applied to split the flow between explicit and dissipated components:
  1. the vorticity field is expanded onto an orthogonal wavelet basis (Coiflet 2),
  2. within each scale and direction, a self-consistent threshold $\Theta_{j,\mu}$ is determined such that : $\Theta_{j,\mu} = q\sigma_{j,\mu}(\Theta_{j,\mu})$ where $\sigma_{j,\mu}$ is the standard deviation restricted to wavelet coefficients below the threshold. Below we take q=2.8
  3. coefficients below their threshold are **incoherent**, and those above are **coherent**,
  4. coherent and incoherent vorticity fields are reconstructed from the coefficients :

We apply this procedure to homogeneous isotropic decaying 2D flows in the enstrophy cascade regime.

$$\omega = \sum_{\lambda\in\Lambda} \widetilde{\omega}_\lambda \psi_\lambda = \underbrace{\sum_{\lambda\in\Lambda_C} \widetilde{\omega}_\lambda \psi_\lambda}_{\text{COHERENT}} + \underbrace{\sum_{\lambda\in\Lambda_I} \widetilde{\omega}_\lambda \psi_\lambda}_{\text{INCOHERENT}}$$

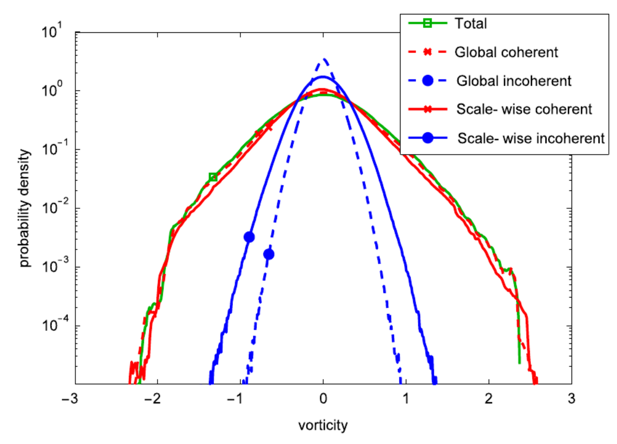


The **scale-wise wavelet coefficients PDFs** has exponential tails. The thresholding (pictured above by blue and red colouring) separes the tails from the core. Unfortunately, the core is still far from a Gaussian (dashed line).

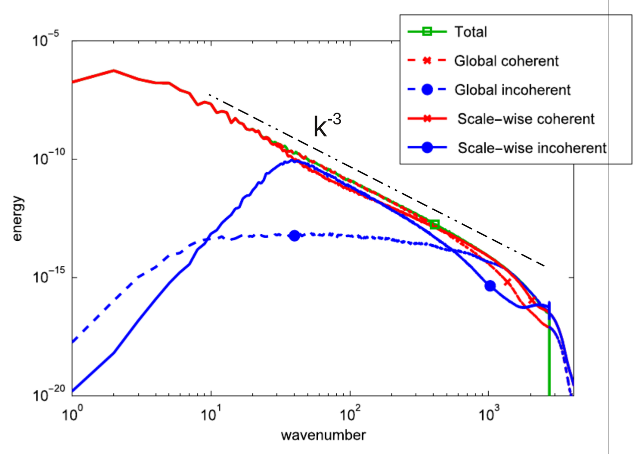


However, the **vorticity PDFs** in physical space is closer to a Gaussian for the incoherent part than for the total.

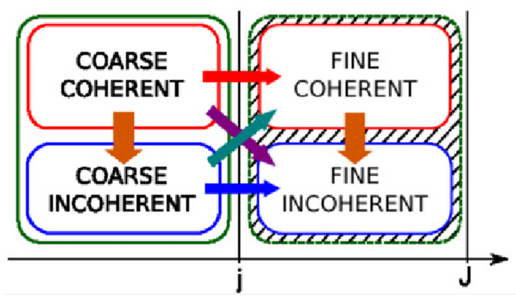
The **energy spectra** of the coherent and incoherent parts are almost parallel in the inertial range, while the coherent part dominates in the dissipative range. This suggests that the incoherent part is produced by randomization due to mixing by the nonlinear term.



Vorticity PDFs



Spectra

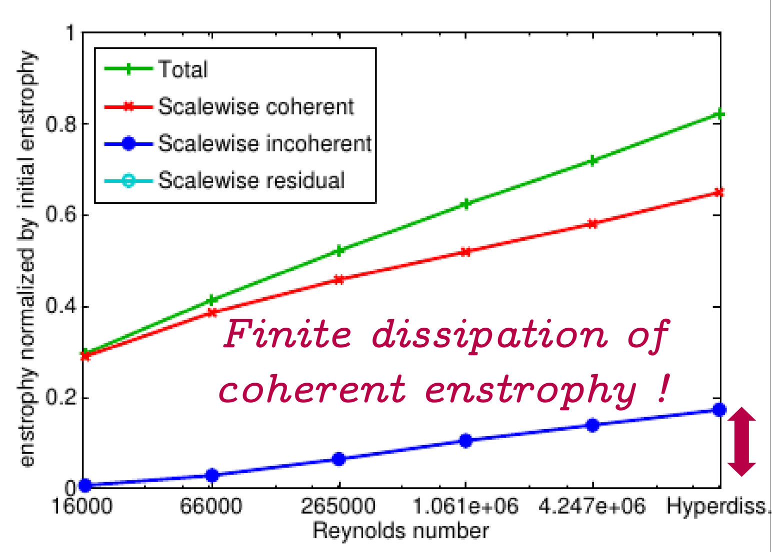


	Scale-wise (%)
$C_\le \to C_>$	48.4
$C_\le \to I_>$	15.9
$C_> \to I_\le$	−20.2
$I_\le \to I_>$	24.8
$<\to>$	100
$C_\le \to I_\le$	51.3

Nonlinear enstrophy transfers

To check this we have computed the **entrophy transfers** between the various components due to the nonlinear term. We show that the rate of production of incoherent enstrophy by the nonlinear term is 50% of the rate of the total enstrophy cascade flux.

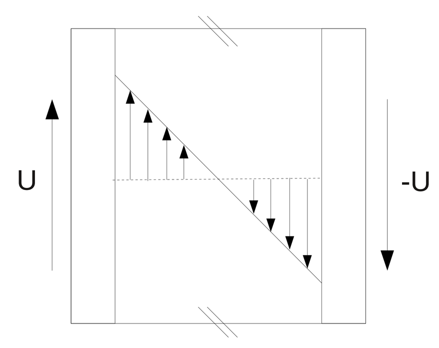
By measuring the amount of incoherent enstrophy for increasing Reynolds number (and fixed time), we observe **finite dissipation of coherent enstrophy in the vanishing viscosity** limit, whereas the dissipation of total enstrophy has been shown to go to zero.



Enstrophy as a function of Reynolds number

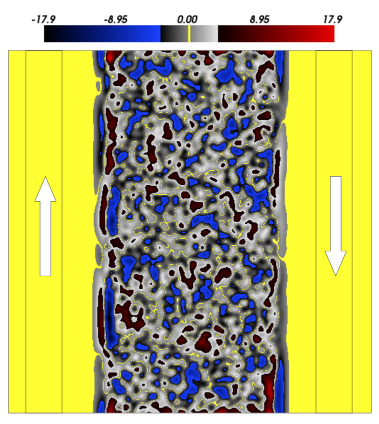
Combining the two mechanisms ?

We would like to take the 2D couette flow as a toy model of a situation where the two mechanisms isolated above are present simultaneously and interact with each other.

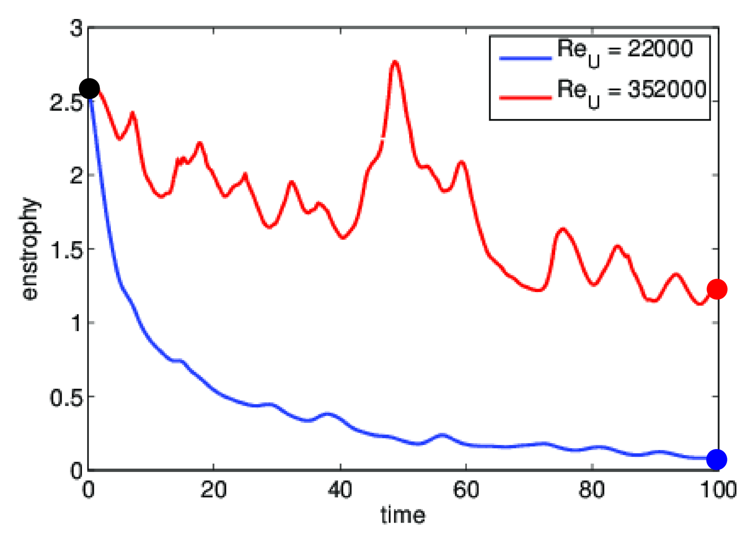


In 2D there is no linear mechanism for creation of vorticity. Vorticity can be produced only nonlinearly at the boundary.
Moreover, the background strain tends to kill small perturbations.

For these two reasons, we need to take an **initial condition** with vorticity larger than the background strain. We choose a random vorticity field adjusted so that the stream function vanishes close to the walls.



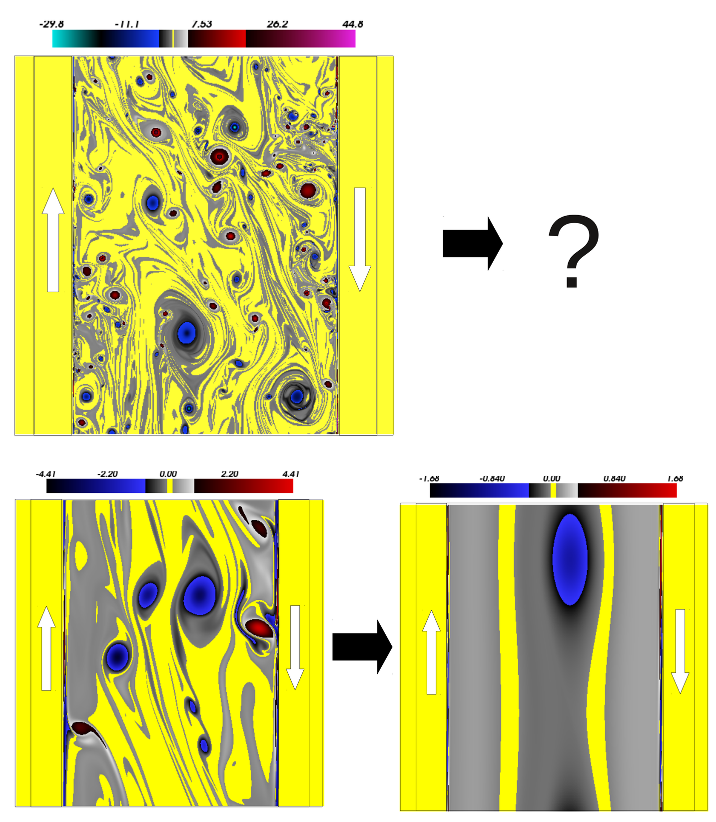
Initial vorticity fluctuations

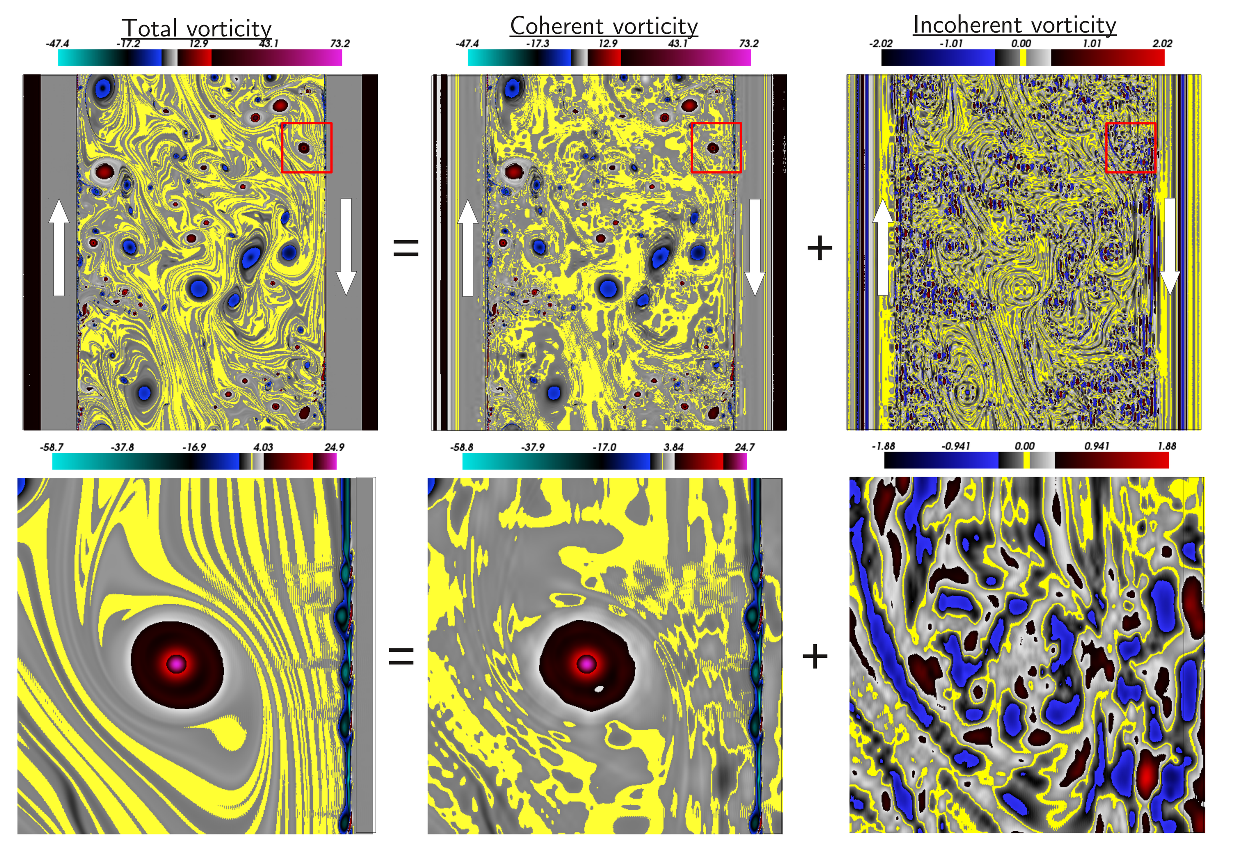


Partial results are shown there for 2 representative Reynolds numbers.

For the smaller Reynolds, we observe rapid decay towards an asymetric flow similar to the „Kelvin cat's eyes“, which then decays linearly on viscous timescales.

For the higher Reynolds, the decay is not observed during the course of the computation. Many structures similar to the ones we observed above for the dipole-wall collision are produced and subsequently interact. The eventual fate of the solution is unknown : will it eventually decay back to the stationary linear profile, or remain time-dependent ?





The split between coherent and incoherent vorticity can be done in this case like in the homogeneous case, but due to the penalization model, the incoherent and coherent vorticity fields leak outside of the domain. The boundary layer is captured in the coherent part.
The compression achieved is approximately of a factor 10.

Outlook and (many!) open questions

- What does this point of view tell us on the physics of the two-dimensional Couette flow ? What are the roles of the dissipation mechanisms with respect to momentum transport across the cell ?
- What is the behavior at large times for large Reynolds numbers ? How is this related to dissipation ?
- Does this lead to a useful modeling framework ? In the homogeneous case we proposed a maximum entropy model for the incoherent part, does this extend to the wall bounded case ?
- Is the regime that we compute relevant in some way for oceanic turbulence ?

References :
[1] RNVY, MF & KS, *PRL* **108** (2011)
[2] RNVY, MF & KS, *Physica D* 10.1016/j.physd.2011.05.022 (2011)

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