CEMRACS 2010

“**WAVELET**” project

Particle-in-Wavelet scheme for the 1D Vlasov-Poisson equations

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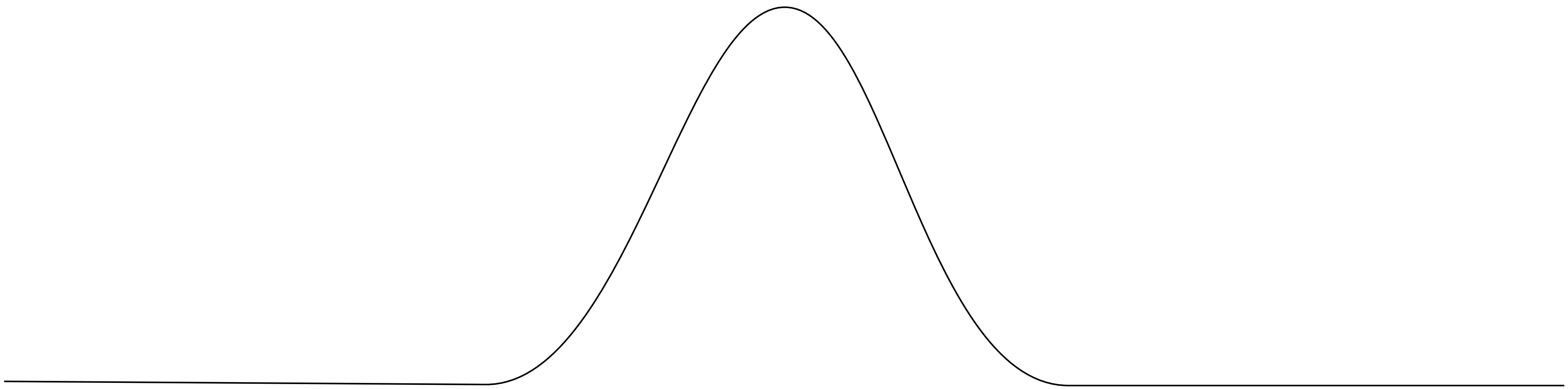
Outline

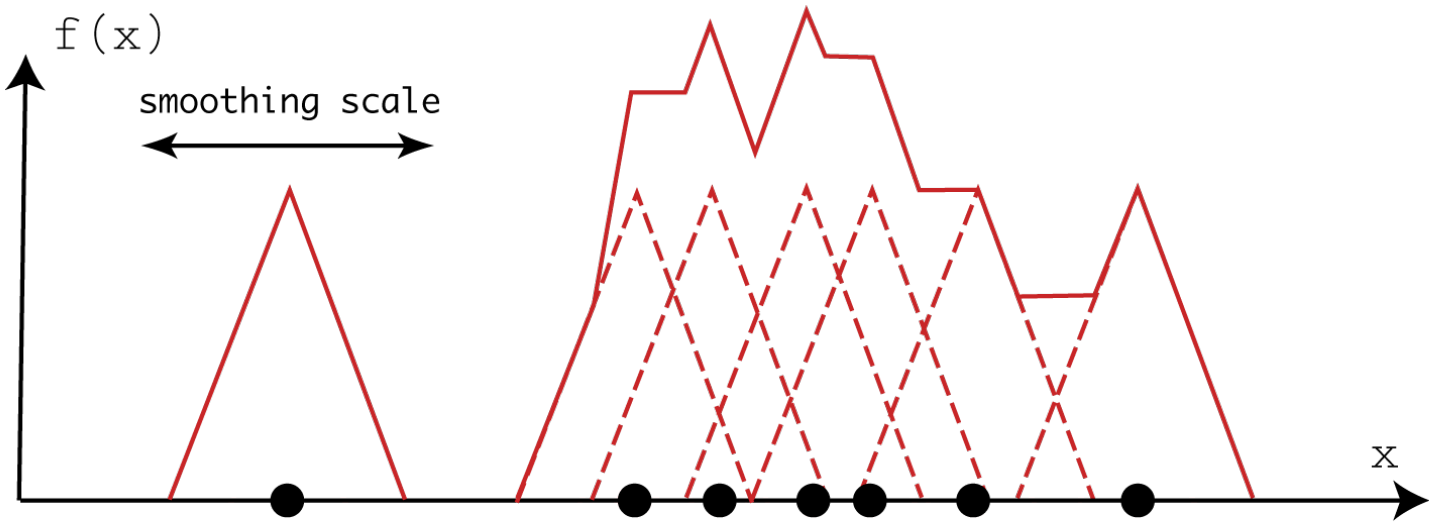
- Description of the scheme:
  - density estimation,
  - Poisson solver,
  - particle push.
- Test cases:
  - Landau damping,
  - two-streams instability.

Step 1
density estimation



Density estimation

- Goal: estimate a smooth probability density function from N independent realizations



Kernel method ⟺ Particle-in-Cell code

Empirical wavelet coefficients

Empirical density function = sum of Dirac distributions

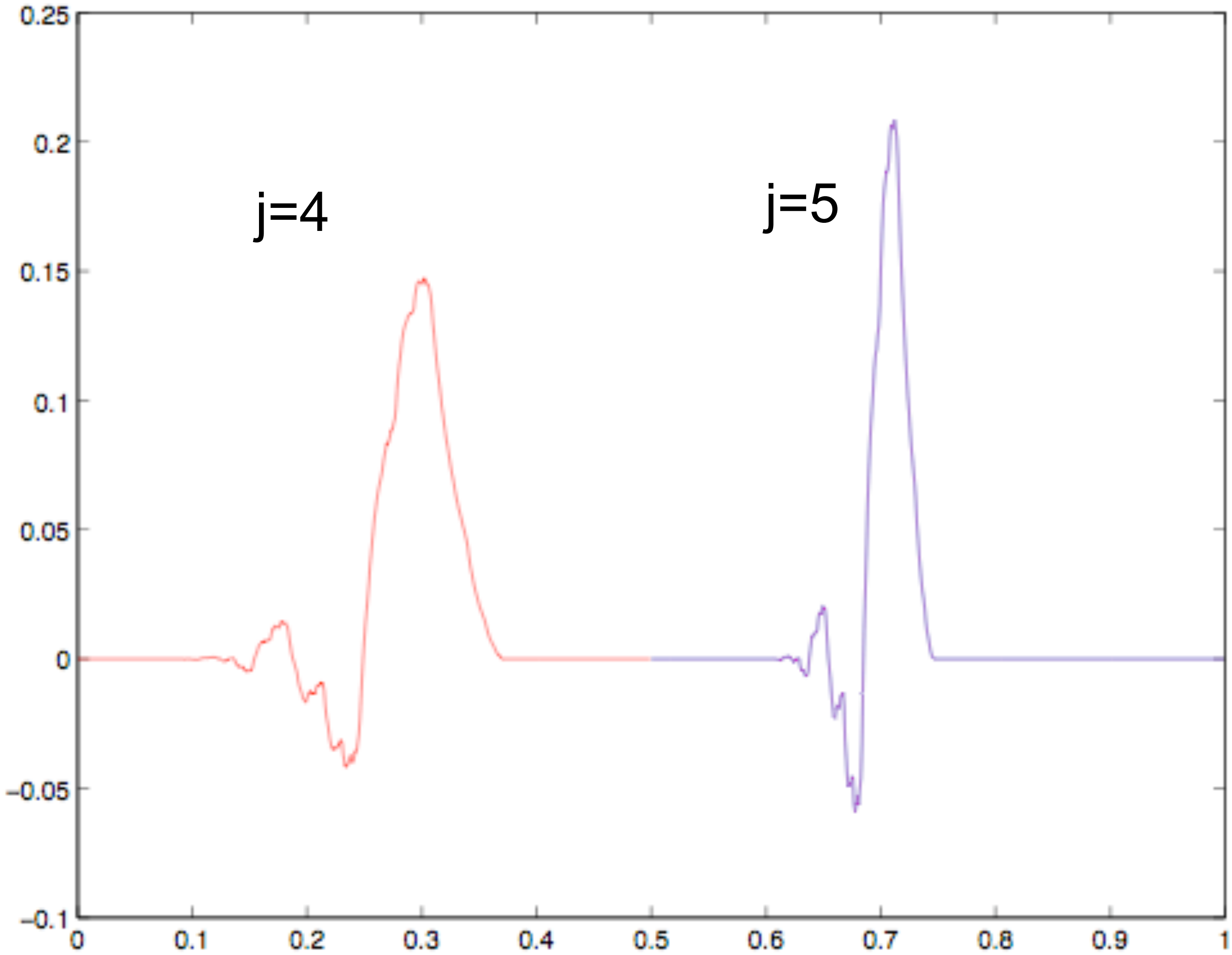
$$f^\delta(\mathbf{x}) = \frac{1}{N_p}\sum_{n=1}^{N_p}\delta(\mathbf{x}-\mathbf{X}_n)$$

particle positions

Empirical wavelet coefficients := coefficients of f^δ

$$\overline{f}^\delta_\lambda \quad = \quad \langle f^\delta \mid \varphi_\lambda\rangle = \frac{1}{N_p}\sum_{n=1}^{N_p}\varphi_\lambda(X_n)$$

$$\tilde{f}^\delta_\lambda \quad = \quad \langle f^\delta \mid \psi_\lambda\rangle = \frac{1}{N_p}\sum_{n=1}^{N_p}\psi_\lambda(X_n)$$



R-Coiflet 1 scaling functions

Empirical wavelet coefficients

Problem: we need to compute $\varphi_\lambda(X_n)$

There is no analytical expression.

Proposed solution: use a lookup table !
initialized before the start of the computation.

+ interpolation by 2nd order Lagrange polynomials.

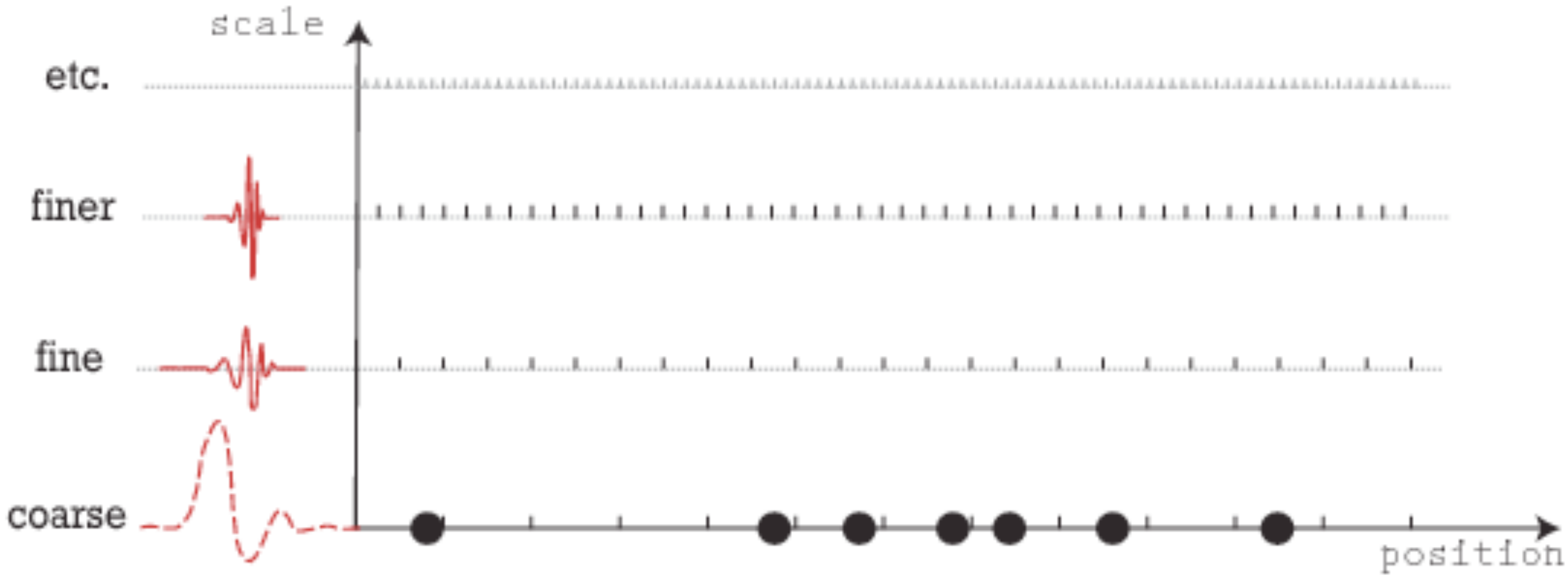
Cost proportional to N x S (S=6)
This is probably not optimal…

Wavelet based density estimation

$$f = \sum_{\lambda \in \Lambda_{\phi,L}} \overline{f}_\lambda \phi_\lambda + \sum_{\lambda \in \Lambda_{\psi,L}} \tilde{f}_\lambda \psi_\lambda$$

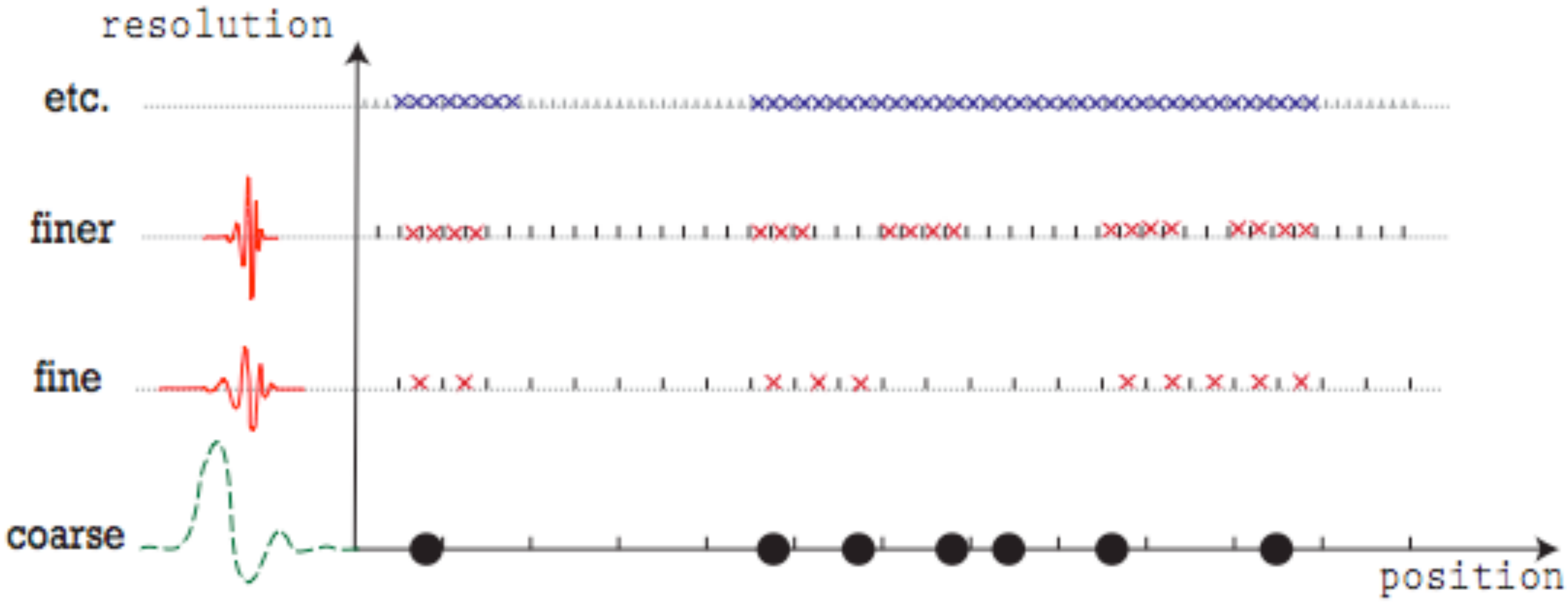
coarse scale approximation

details



Wavelet based density estimation

Denoising = setting to zero small wavelet coefficients (**thresholding**)
+ setting to zero fine scale wavelet coefficients (linear filtering, uniform smoothing)



Choice of parameters

COARSE SCALES	INTERMEDIATE SCALES	FINE SCALES
keep all	nonlinear threshold	discard all

L | J

$$2^{dL} := N_p^{\frac{1}{3}}$$

keep only coefficients such that:

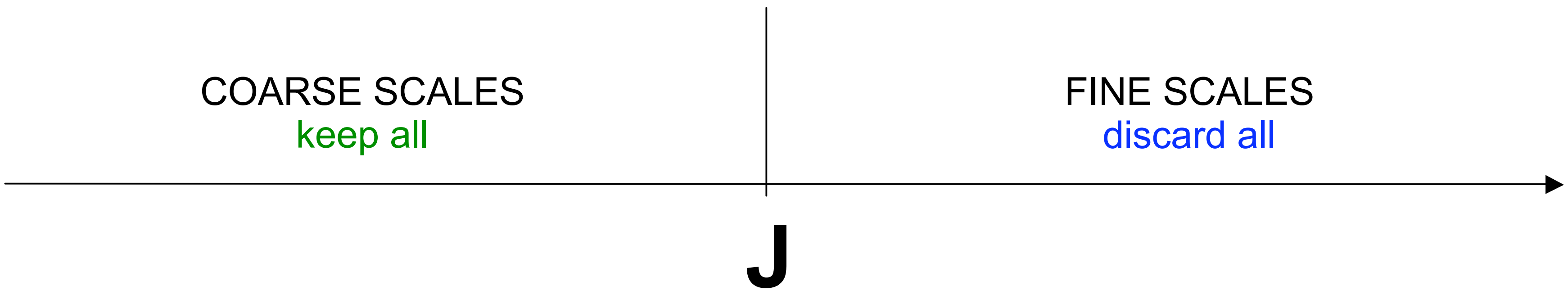
$$\left|\tilde{f}_{\lambda}^{H}\right| \geq \sqrt{\frac{j}{4N_p}}$$

$$2^{dJ} := \frac{N_p}{\log_2 N_p}$$

In this presentation we shall only consider **linear thresholding**, so we impose

L = J

Choice of parameters



In this presentation we shall only consider **linear thresholding**, so we impose

$$\mathbf{L = J}$$

Step 2
Poisson solver

Galerkin discretization

- We adopt a Galerkin discretization of the Laplace operator in the scaling function basis at the finest resolved scale,

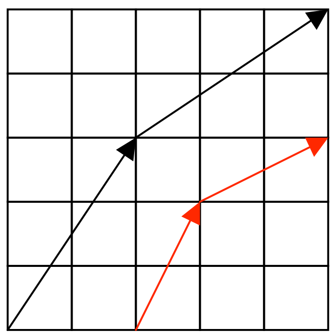
$$-L\overline{\phi} = 1 - \overline{\rho}$$

- L is a circulating matrix, equivalent to applying a finite difference operators to the scaling function coefficients.

Inversion

- For inversion we use the conjugate gradient method,
- We use a diagonal preconditioner in wavelet space : the number of iterations is almost independent on resolution.

Step 3
Particle push



Electric field derivation

- Galerkin projection of the gradient operator in the scaling function basis,

$$\overline{E} = -G\overline{\phi}$$

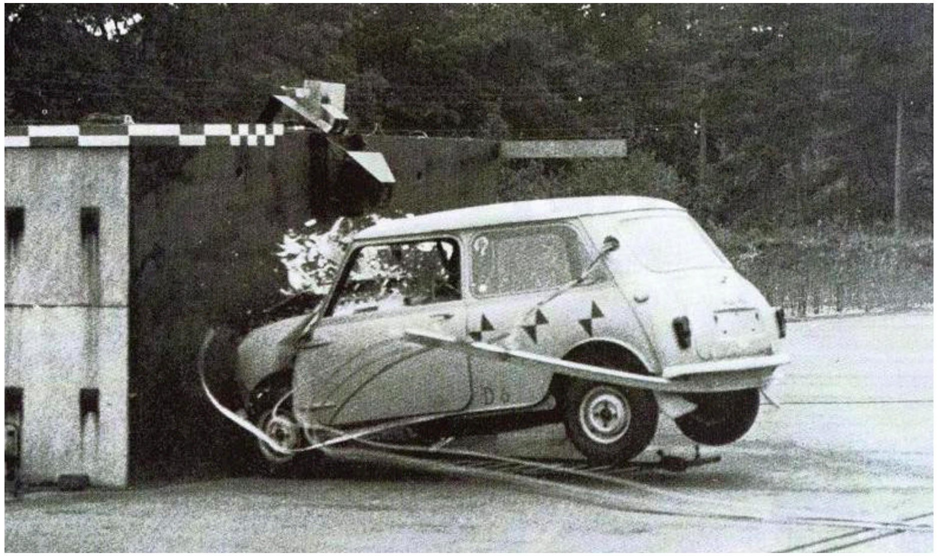
- As before, equivalent to finite difference operator.

- (L symmetric + G antisymmetric + LG=GL) => no self forces

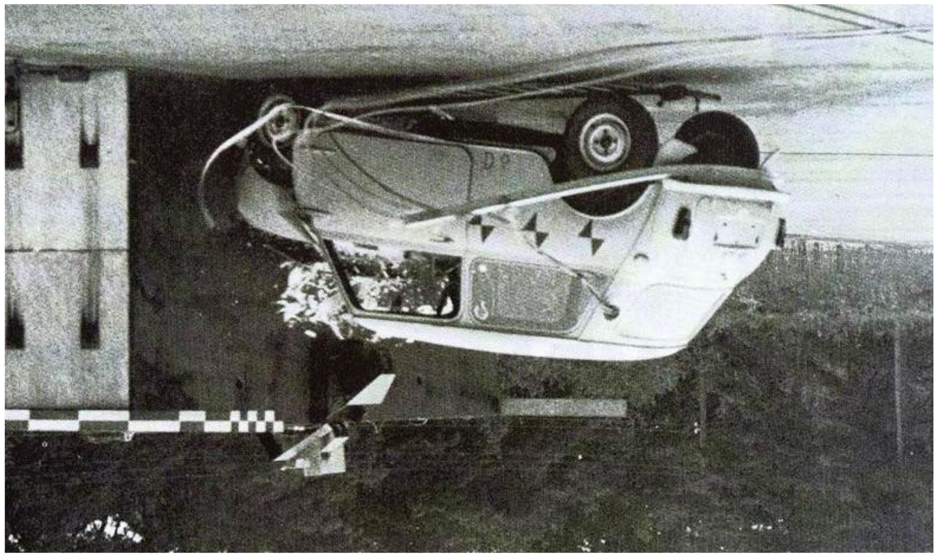
Electric field interpolation

- Interpolation using the same lookup-table method as for the density estimation step.

- Time discretization using 3rd order Runge-Kutta



Test cases



Reference and competitor

- reference solutions obtained using

semi-Lagrangian solver

  - 3rd order spline interpolation
  - grid size 2049^2

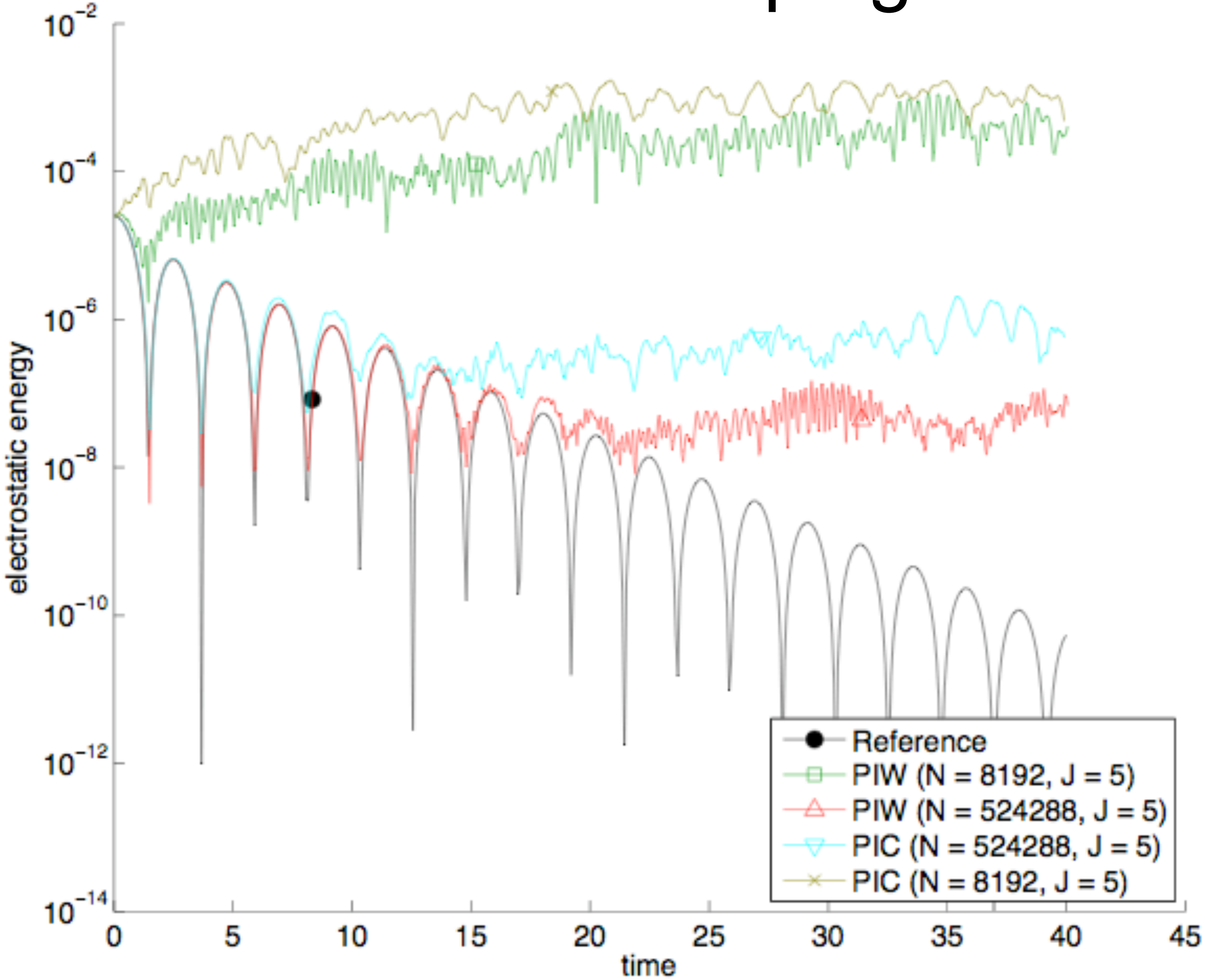
- competing solutions obtained with classical

PIC solver

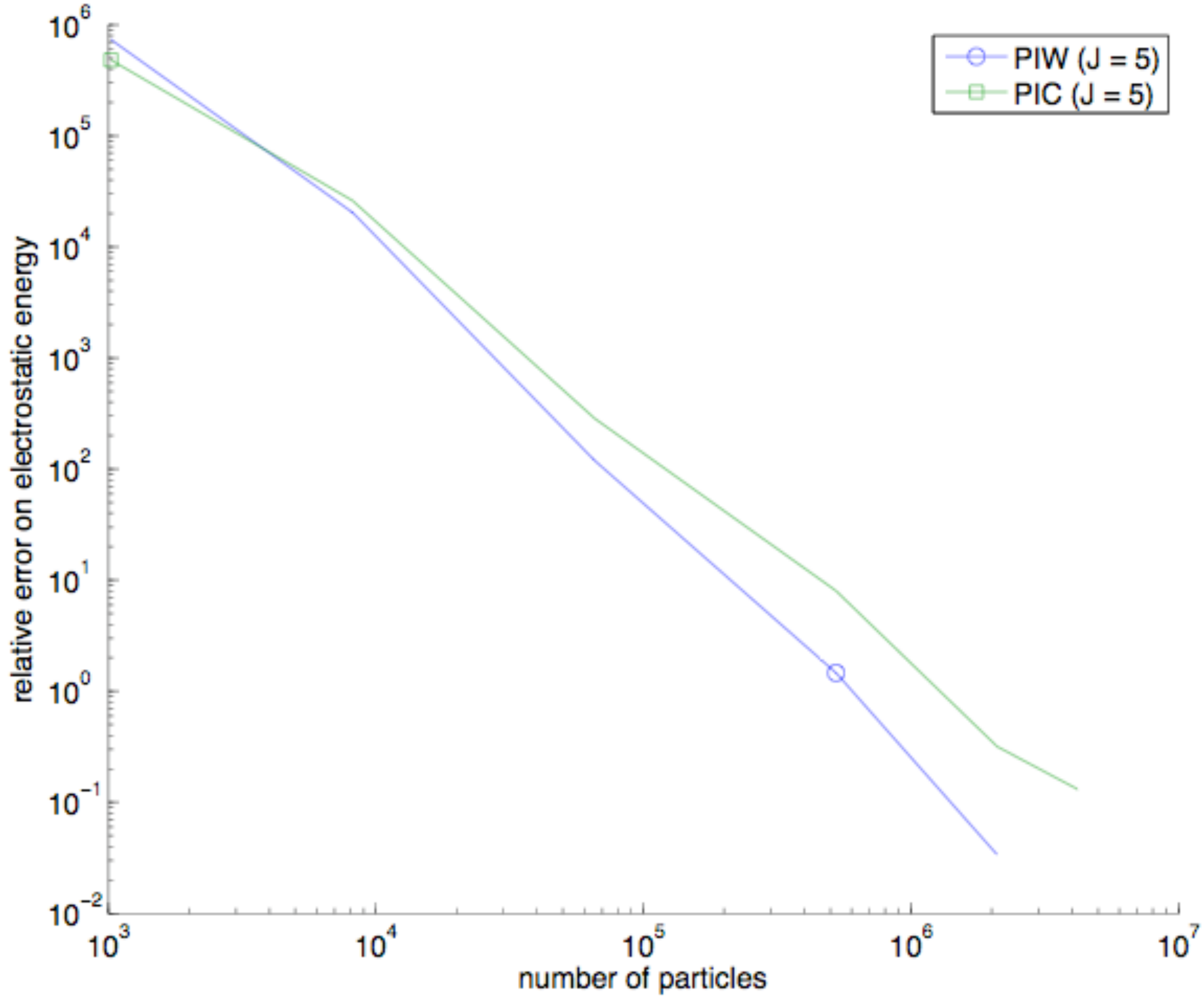
  - triangular charge assignment function

- **VLASY** platform much appreciated.

Landau damping



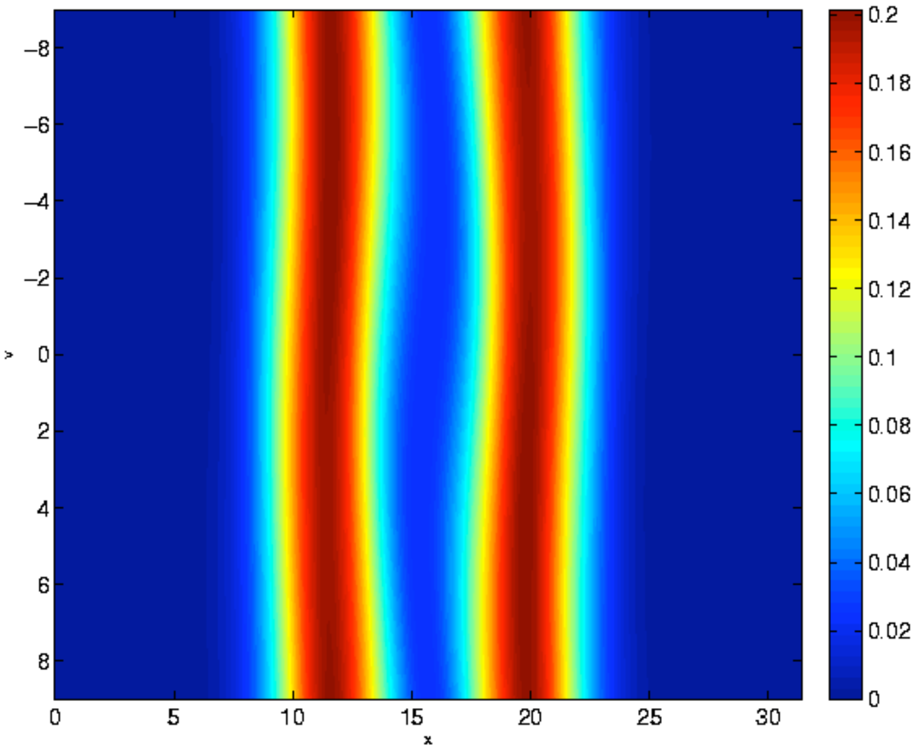
Landau damping: convergence



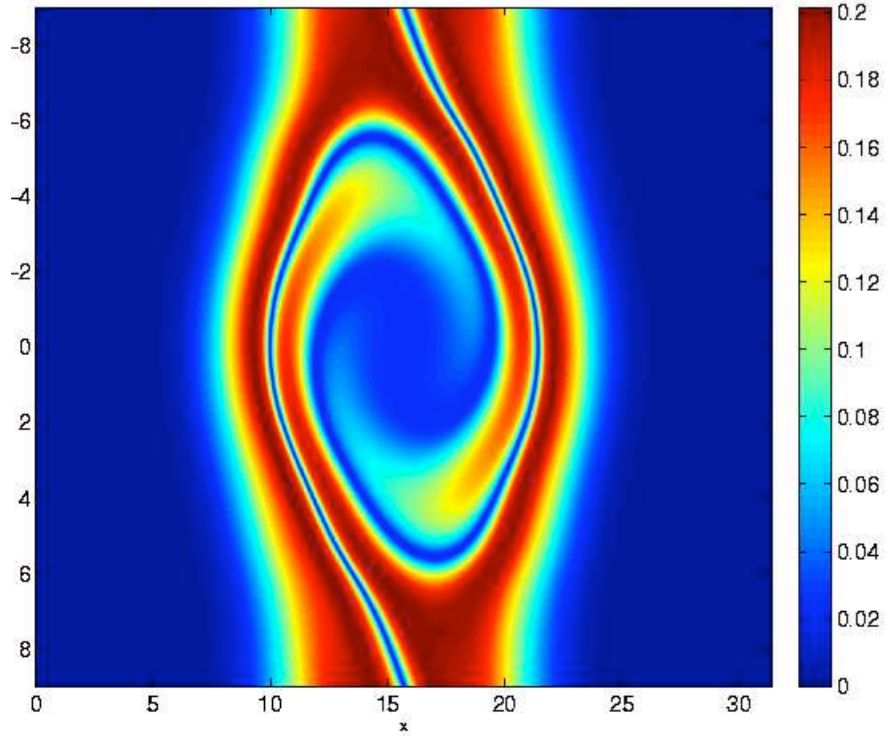
L^2 error on the electric field at $t = 20$

Two-streams instability

snapshots of reference solution

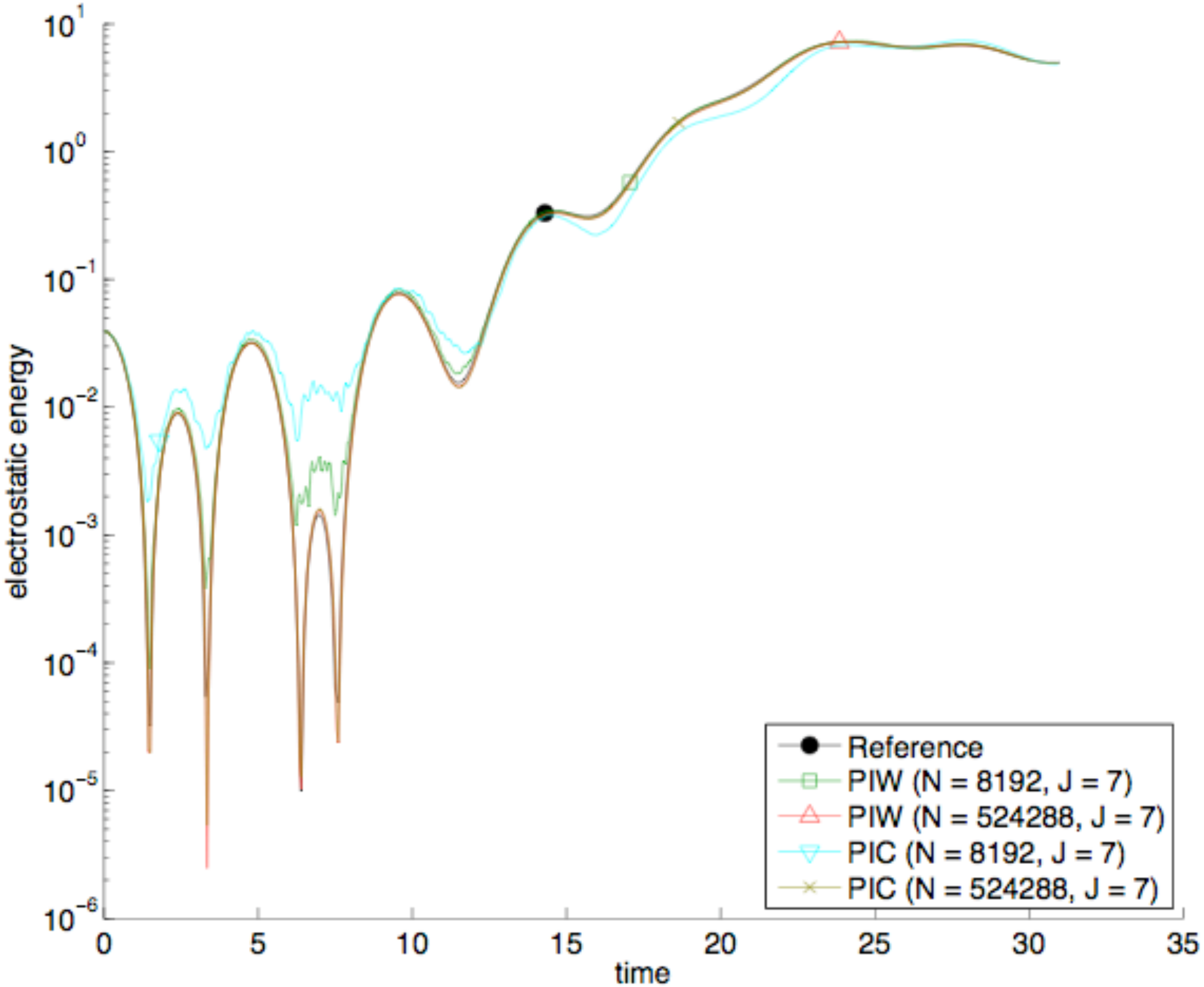


t = 10



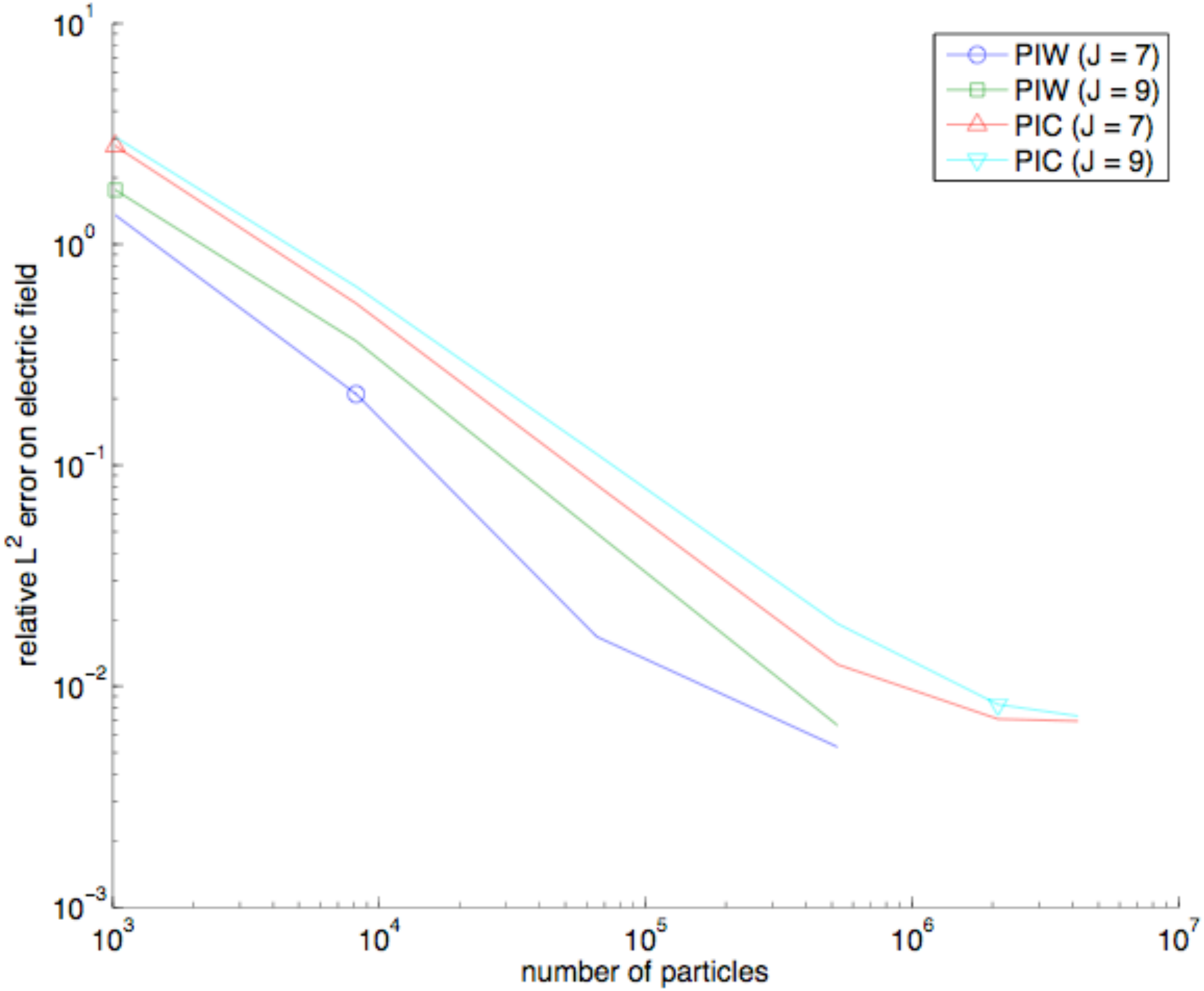
t = 30

Two-streams instability



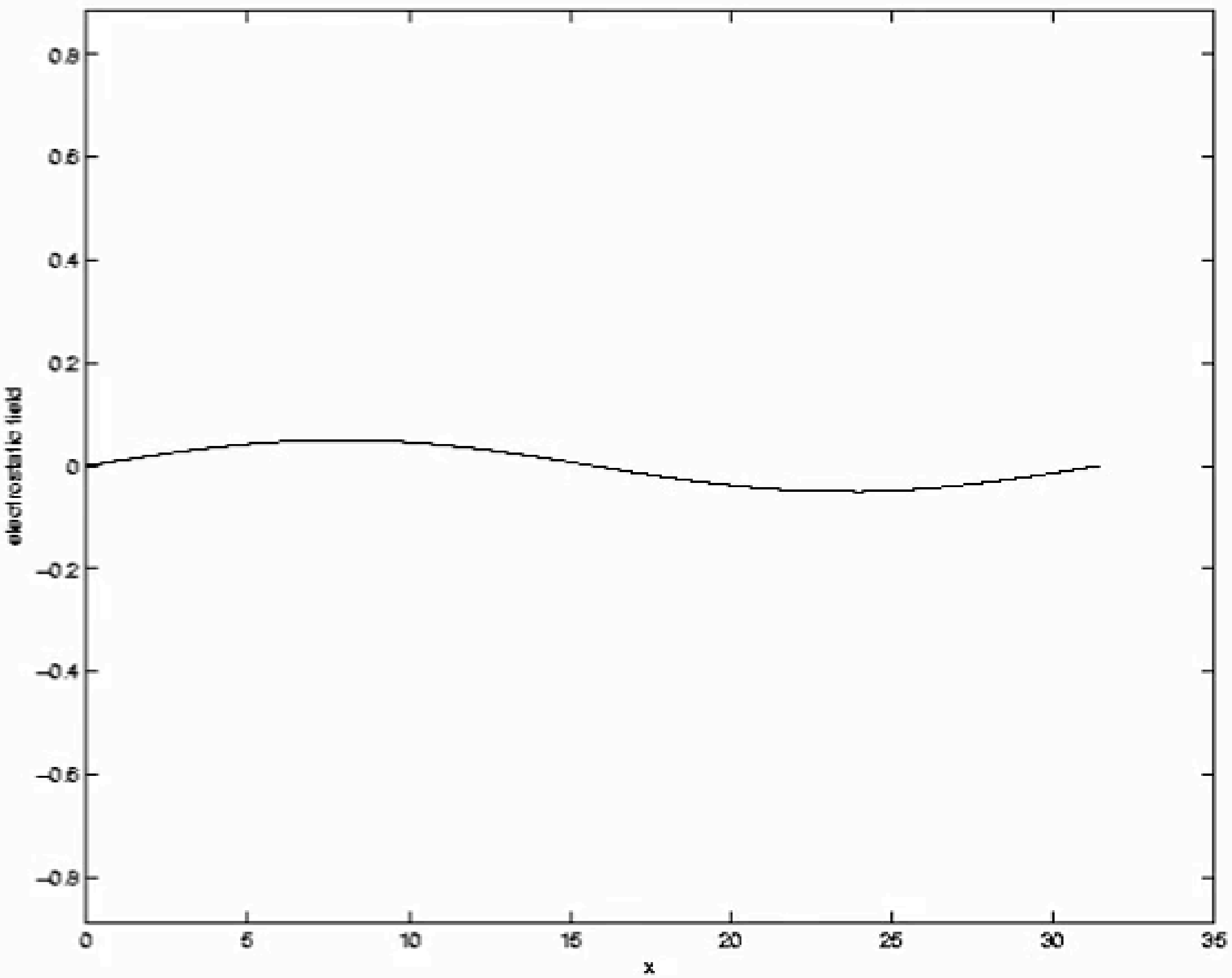
<u>Time evolution of electrostatic energy</u>

Two-streams instability: convergence



<u>L^2 error on the electric field at t = 10</u>

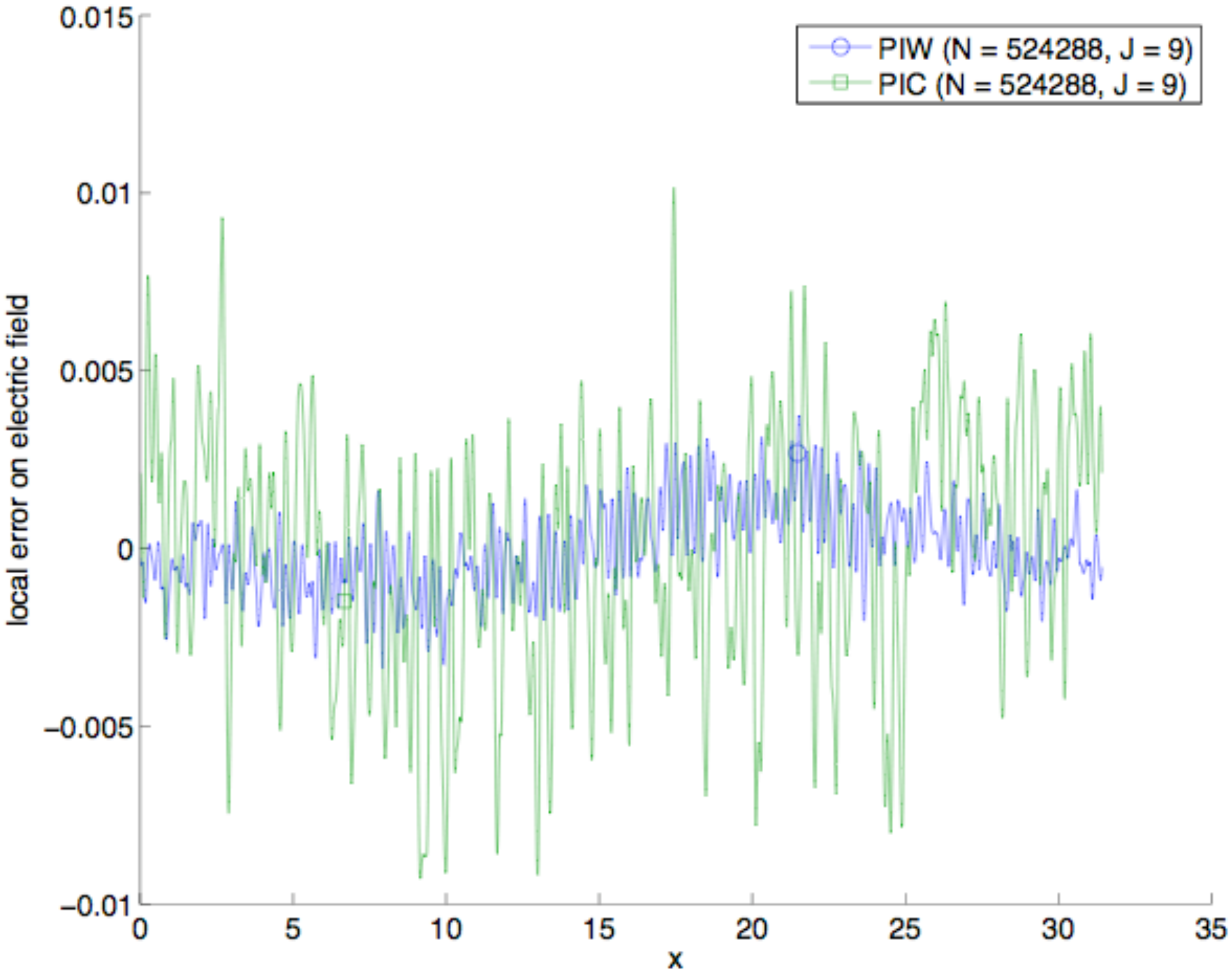
Two-streams instability



<u>Movie of electric field from t = 0 to t = 30</u>

Two-streams instability: denoising

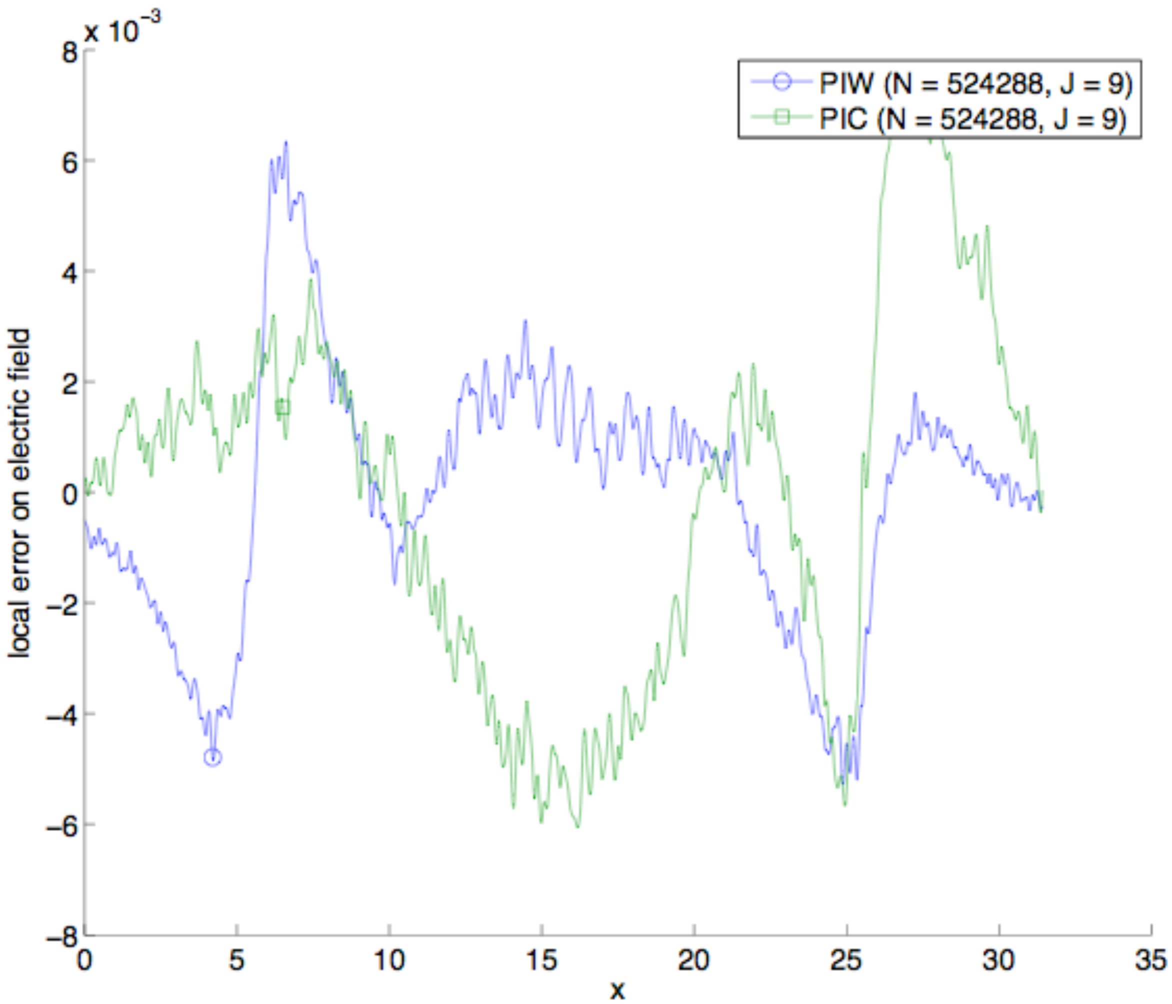
t = 30



Error on the electric field as a function of x

Two-streams instability: denoising

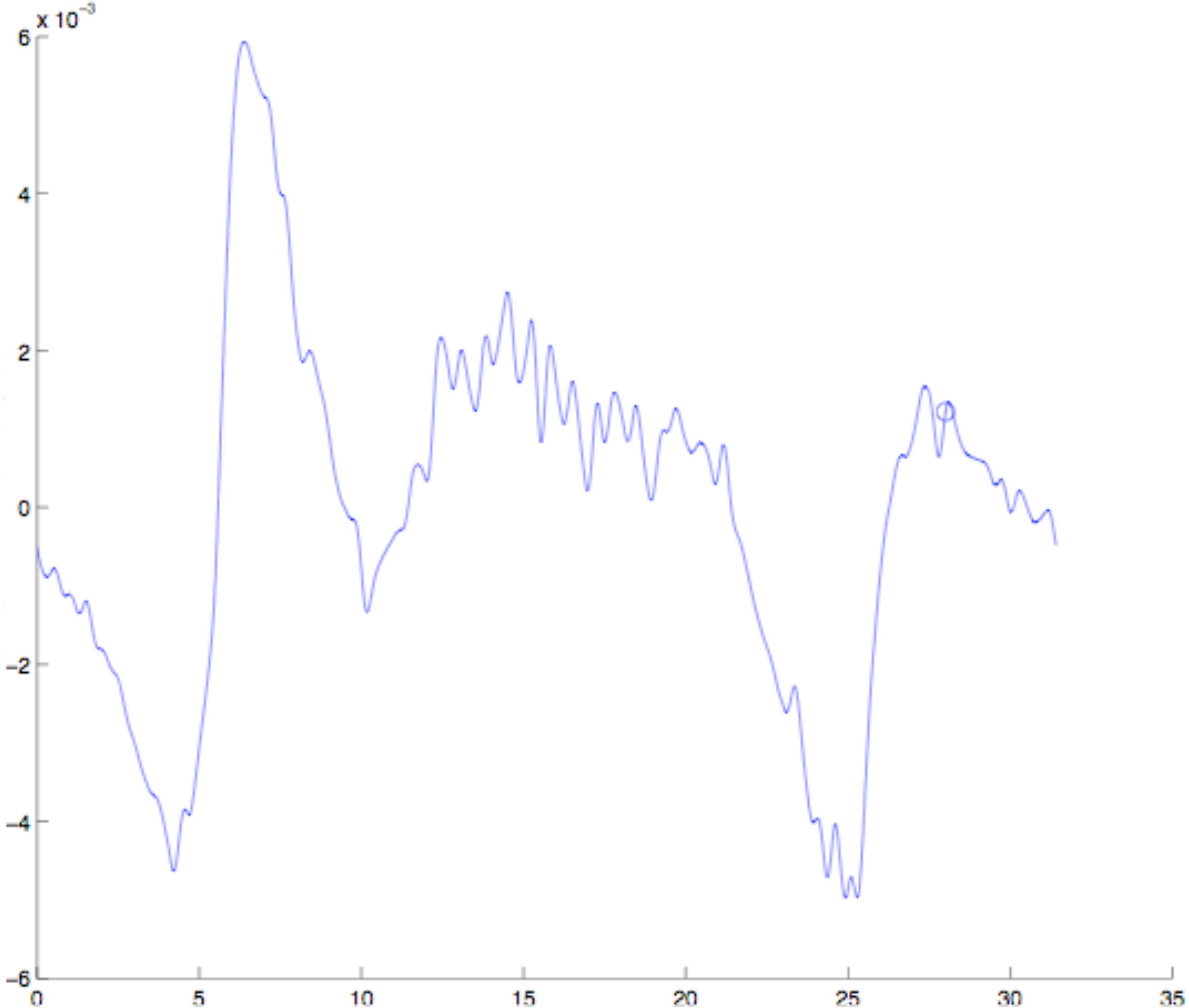
t = 30



Yes, this can be denoised !

Two-streams instability: denoising

t = 30



Yes, this can be denoised !

Conclusion

✓ Particle-in-Wavelet scheme proposed and successfully implemented in 1D,

✓ Without denoising, behaves like a high order PIC scheme,

✓ Improvement expected with denoising, but not tested yet.

Perspectives

$\Rightarrow$ Convergence studies with nonlinear thresholding,

$\Rightarrow$ Adaptivity, 2D version,

$\Rightarrow$ Biggest issue: we cannot get rid of large scale noise!

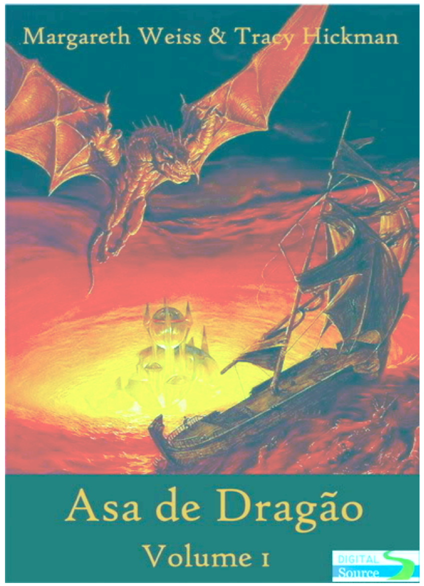
- Hybrid approach: Eulerian Wavelet-Galerkin for the coarse scales, PIW for fine scales ?

Thank you !

and thanks to Matthieu Haefele
and all CEMRACS organizers

An alpha version of the
Kicksey-Winsey C++ platform
is available online under the GNU GPL.

Visit http://justpmf.com/romain
and do not hesitate to ask me.



The reference for WBDE is: *JCP* **229**, p. 2821 (2010)