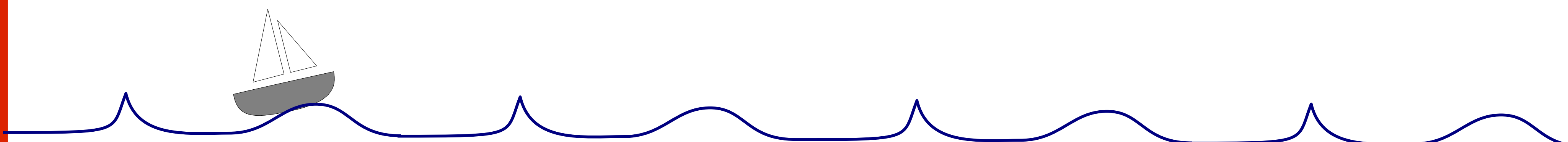
Coexistence of two dissipative mechanisms in two-dimensional turbulent flows



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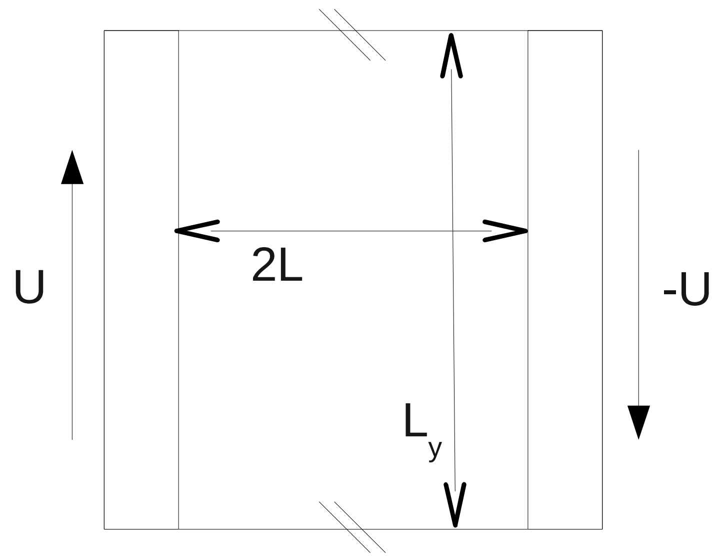
What are we looking for?

- To study some generic relaxation processes in turbulent flows.
- Toy model : **the 2D planar Couette flow**
  - no instability, no non-normal growth (conserved vorticity),
  - cannot study transition (not even bypass),
  - vorticity production **only nonlinearly** at the boundaries,
  - few results in literature (focus has been 3D).

$$\mathrm{Re} = \frac{UL}{\nu}$$



Questions :

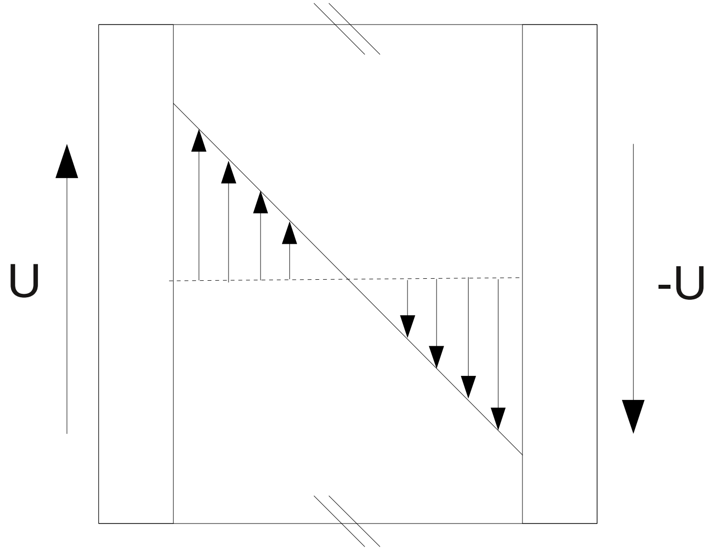
- Statistical equilibria ?
- Momentum transport ?
- Turbulent energy dissipation ?

M. Couette, *Ann. Chim. Phys. ser 6* **23**, p. 433 (1890)

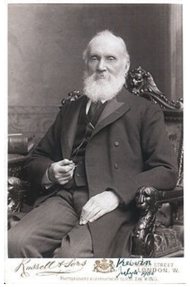
What are the expected states?

- Laminar steady state:

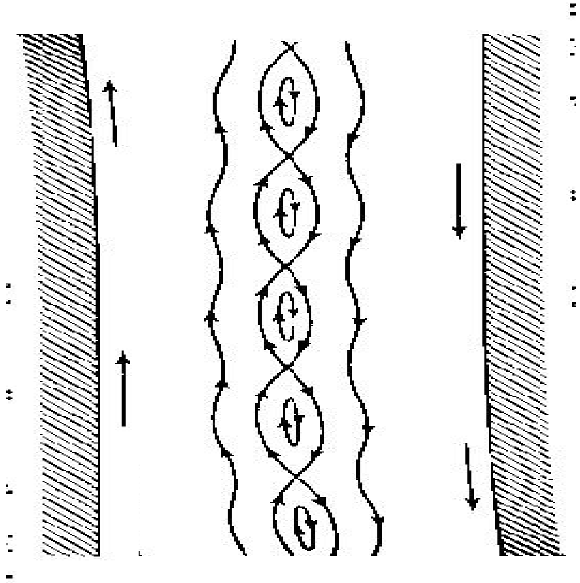
$$\mathbf{u} = -\frac{2Ux}{L}\mathbf{e}_y \qquad \omega = -\frac{2U}{L}$$



- Approach to Kelvin's «Cat's eyes» ?



(inviscid steady states)



- Turbulent steady state(s) ??
  - No linear growth mechanism !!
  - Sommerfeld paradox.
  - Long lived transients ?

U. Ehrenstein, M. Nagata, & F. Rincon, *Phys. Fluids* **20**, 064103 (2008)
F. Rincon, *Phys. Fluids* **19**, 074105 (2007)
Lord Kelvin, *Nature* **23**, p. 45 (1880)
D. Holm, J. Marsden & T. Ratiu (1986)

Numerical model

$$\mathbf{u} : \Omega \times]0,T[\to \mathbb{R}^2$$

$$\text{NSE}(\nu) \begin{cases} \partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} + \boldsymbol{\nabla} p = \nu \Delta \mathbf{u} \\ \boldsymbol{\nabla}\cdot\mathbf{u} = 0 \\ \mathbf{u}_{|\partial\Omega} = \pm U \mathbf{e}_y, \quad \mathbf{u}(\cdot,0) = \mathbf{u}_0 \end{cases}$$

$\to$ penalization + vorticity formulation + Fourier discretization

$$\mathbf{u} : \mathbb{T}^2 \times]0,T[\to \mathbb{R}^2$$

$$\text{PNSE}(\nu,\eta) \begin{cases} \partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} + \boldsymbol{\nabla} p = \nu \Delta \mathbf{u} - \frac{1}{\eta}\chi_{\pm}(\mathbf{u} \pm U\mathbf{e}_y) \\ \boldsymbol{\nabla}\cdot\mathbf{u} = 0 \\ \mathbf{u}(\cdot,0) = \mathbf{u}_0 \end{cases}$$

→ approximates **Navier** BC

$$u_y \pm \alpha \frac{\partial u_y}{\partial x} = 0$$

with slip length $\alpha = \sqrt{\eta\nu}$ at a precision O(η).

C. Navier, *Mém. Acad. Sci. Inst. France* **6** p. 389 (1823)
P. Angot, C.-H. Bruneau, P. Fabrie, *Numer. Math.* **81**, p.497 (1999)
KS & MF, *PRL* **95**, 244502 (2005)
RNVY, MF, KS, *PRL* **106,** 184502 (2011)
RNVY, D. Kolomenskiy, KS, *preprint* (2011)

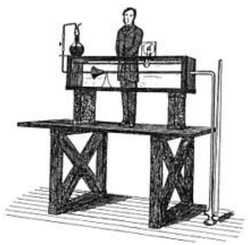
Method

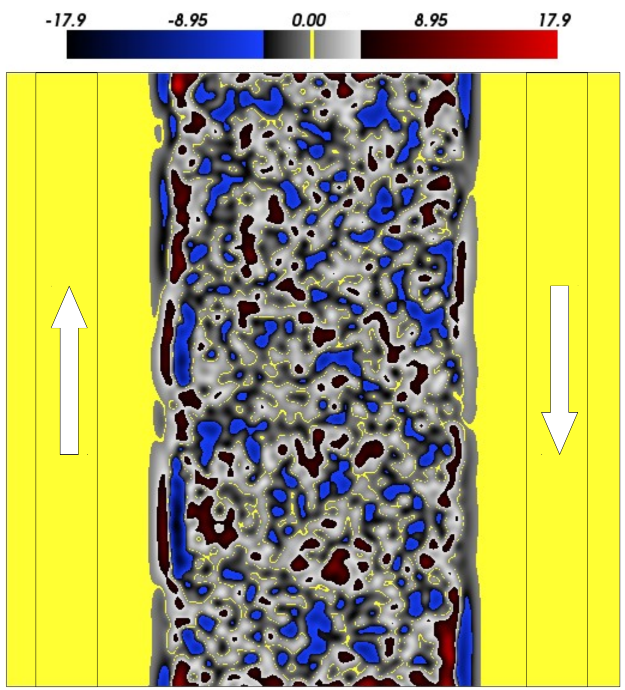
- Keep initial condition fixed.
- Two Reynolds numbers:
  - Re_U based on external shear velocity U
  - Re_0 based on initial RMS velocity

$$\sqrt{\langle \mathbf{u}_0^2 \rangle} = 0.232U$$

- In the future: vary viscosity.
- In this presentation: a few examples.
- Always substract laminar shear profile when presenting results.

(!! carefully correct for penalization effect to have stationnary solution of PNSE)



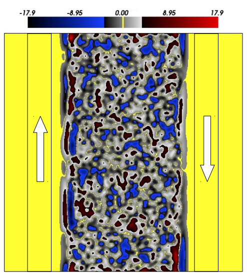


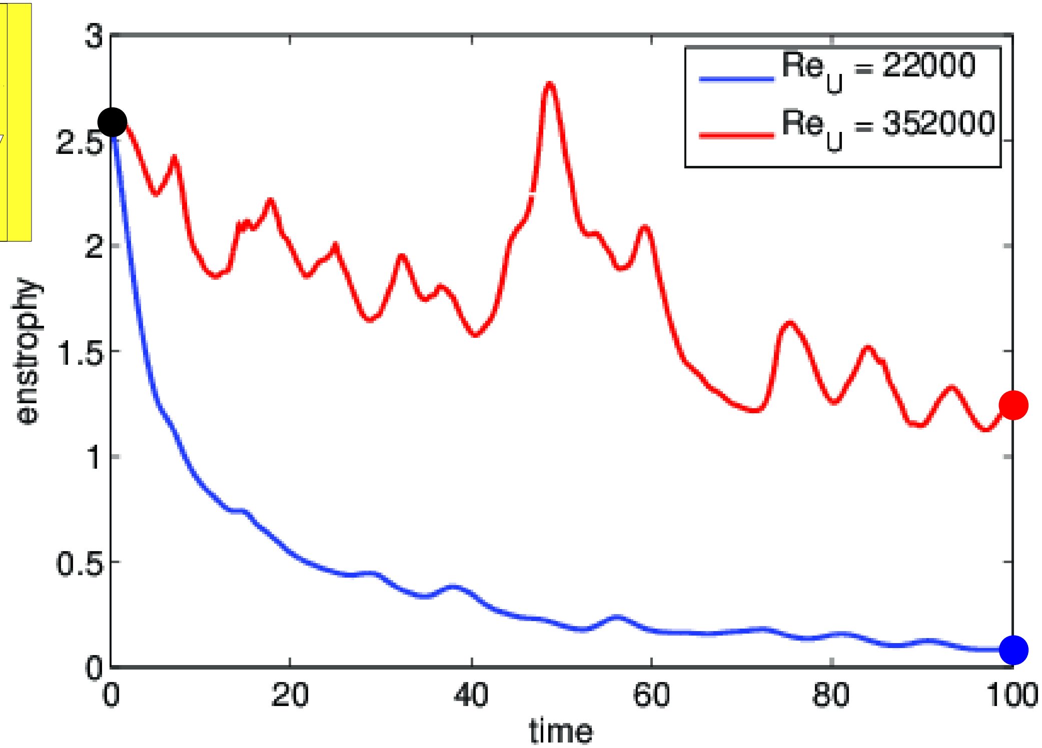
Initial vorticity fluctuations

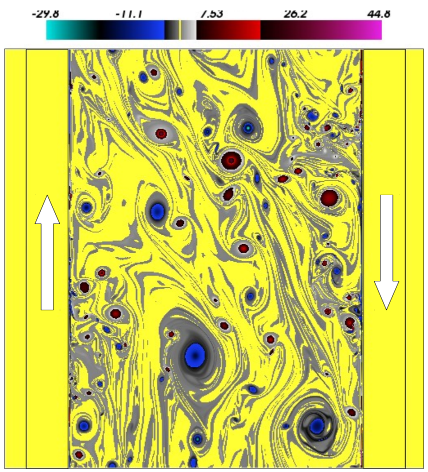
Results overview

$$\mathrm{Re}_U = 22000$$ → decay to «cat's eyes»

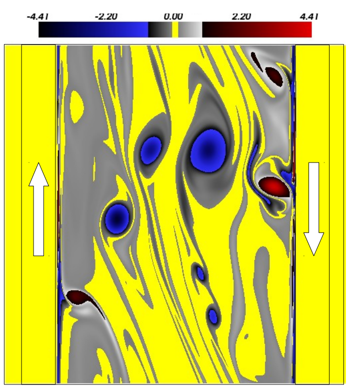
$$\mathrm{Re}_U = 352000$$ → solution remains turbulent, and then ?

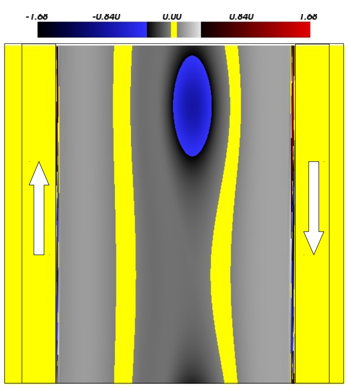




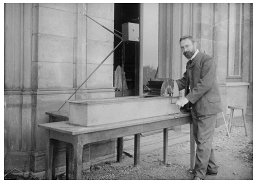


?

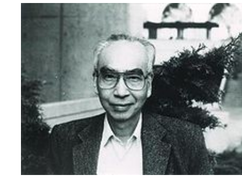




t = 600

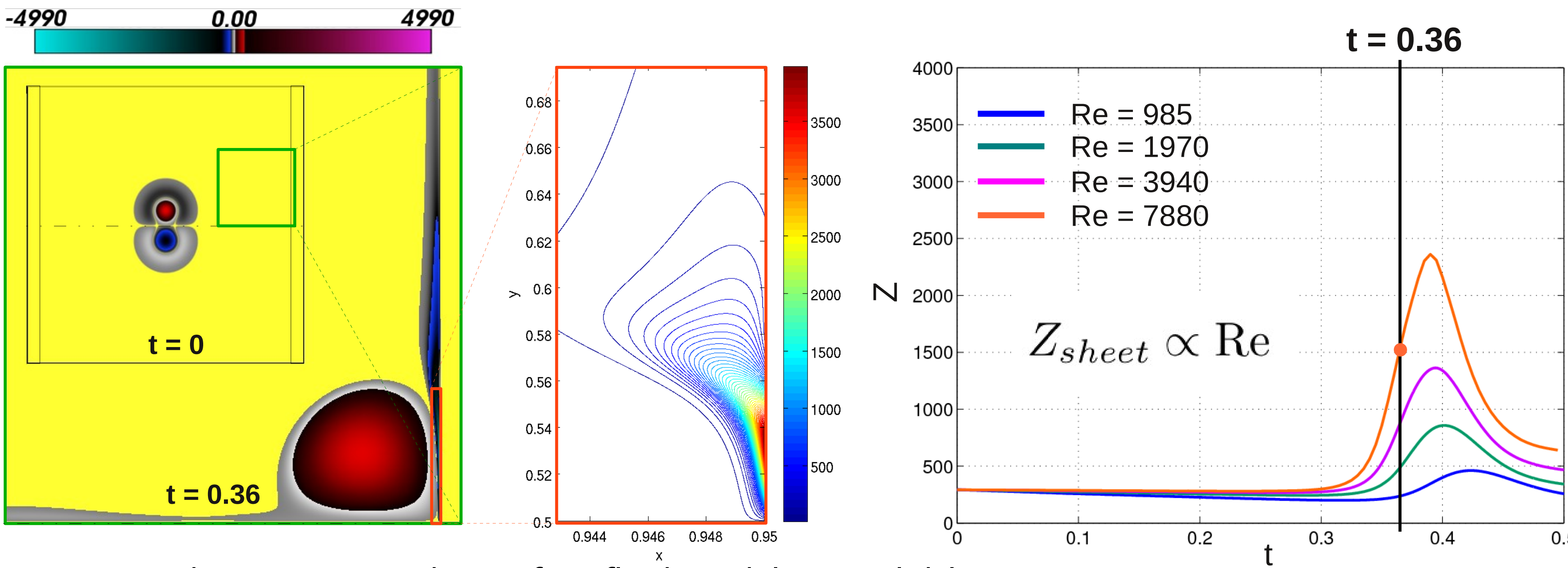


Vortex-wall interaction



- At vanishing viscosity, vortex-wall interactions result in the production of **intense vortex sheets** and subsequently **energy dissipating structures**.

- This is better seen when considering a simplified dipole-wall collision:



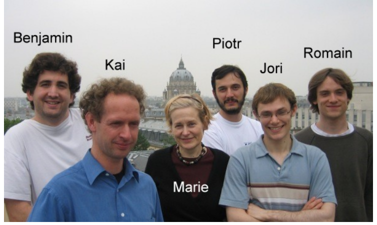
$\rightarrow$ at vanishing viscosity the perfect fluid model is invalid !

A first mechanism of dissipation : molecular dissipation at singularities.

L. Prandtl, *ZAMM* **1**, p.431 (1921)
J.M. Burgers, *Proc. KNAW* **26**, p.582 (1923)
T. Kato *in Sem. Nonlin. Par. Diff. Eq.* (1984)
RNVY, MF, KS, *PRL* **106,** 184502 (2011)

Production of randomness

- The wall produces localized vortices as we have seen, but nevertheless the flow maintains a random aspect.
- This is due to nonlinear mixing in the bulk.
- Same effect can be seen in 2D homogeneous isotropic turbulence and leads to k^{-3} spectrum.
- How to quantify it?



How to quantify it?

- Wavelet tool to quantify flow randomization:

Scale-wise coherent vorticity extraction (SCVE)

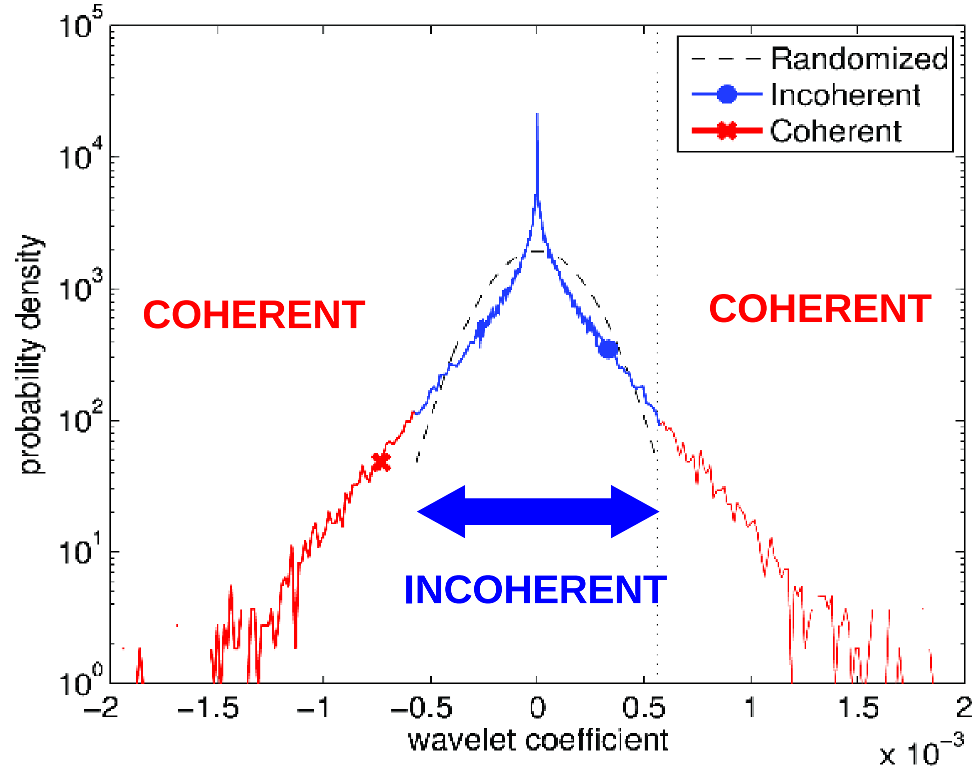
$$\omega = \sum_{\lambda \in \Lambda} \widetilde{\omega}_\lambda \psi_\lambda$$

- Split the wavelet coefficients of vorticity into two sets.
- The sets are defined by a threshold depending on scale and direction.

$$|\widetilde{\omega}_\lambda| \leq \Theta_\lambda \iff \lambda \in \Lambda_I$$

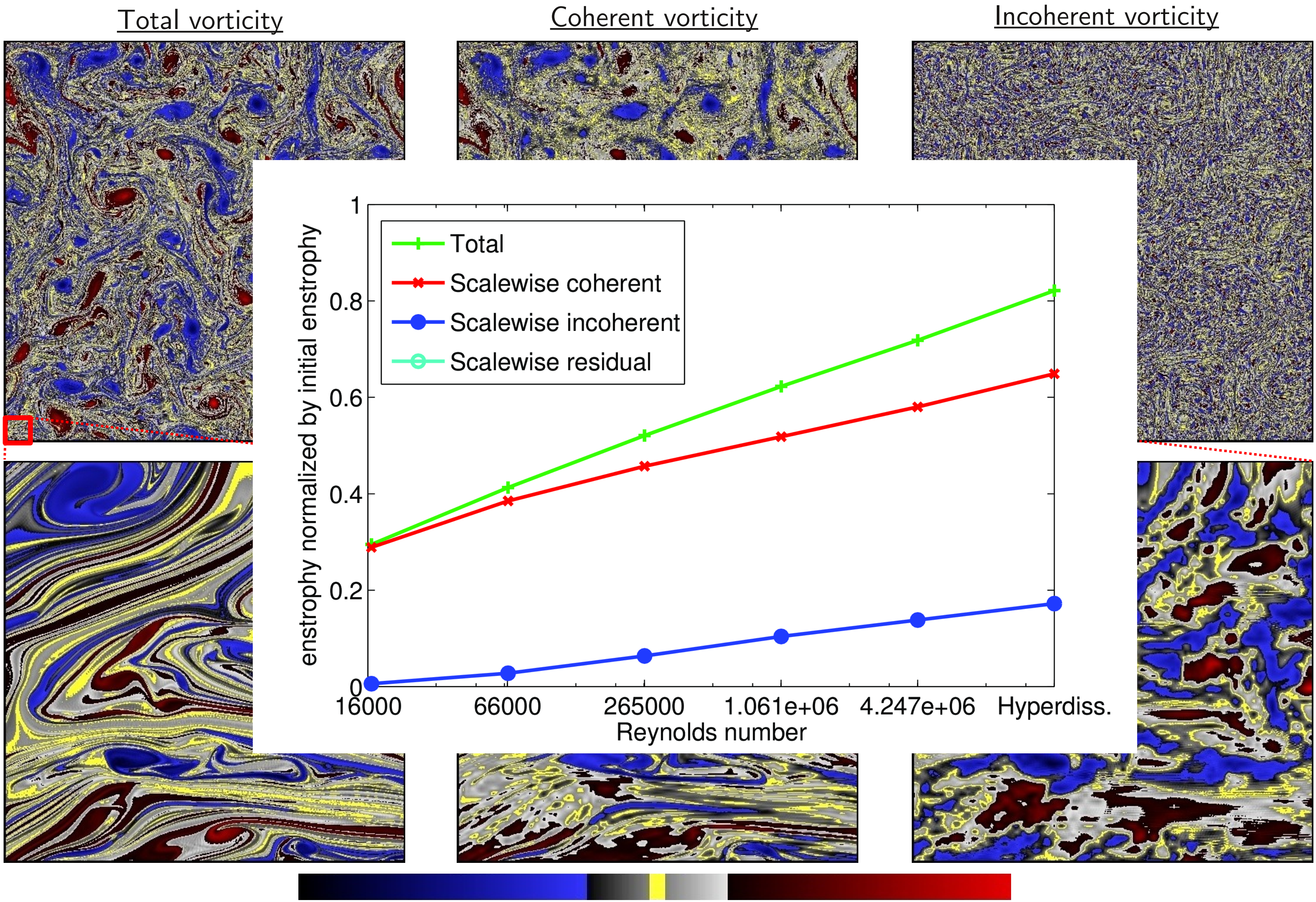
- The thresholds are determined self-consistently by:

$$\Theta_\lambda = q\sigma(\Theta_\lambda)$$



M. Farge, K. Schneider & N. Kevlahan, *Phys. Fluids* **11**, p. 2187 (1999)
RNVY, MF, KS, *Physica D* doi:10.1016/j.physd.2011.05.022, in press.

Results for 2D HIT



Results for 2D Couette

Perspectives

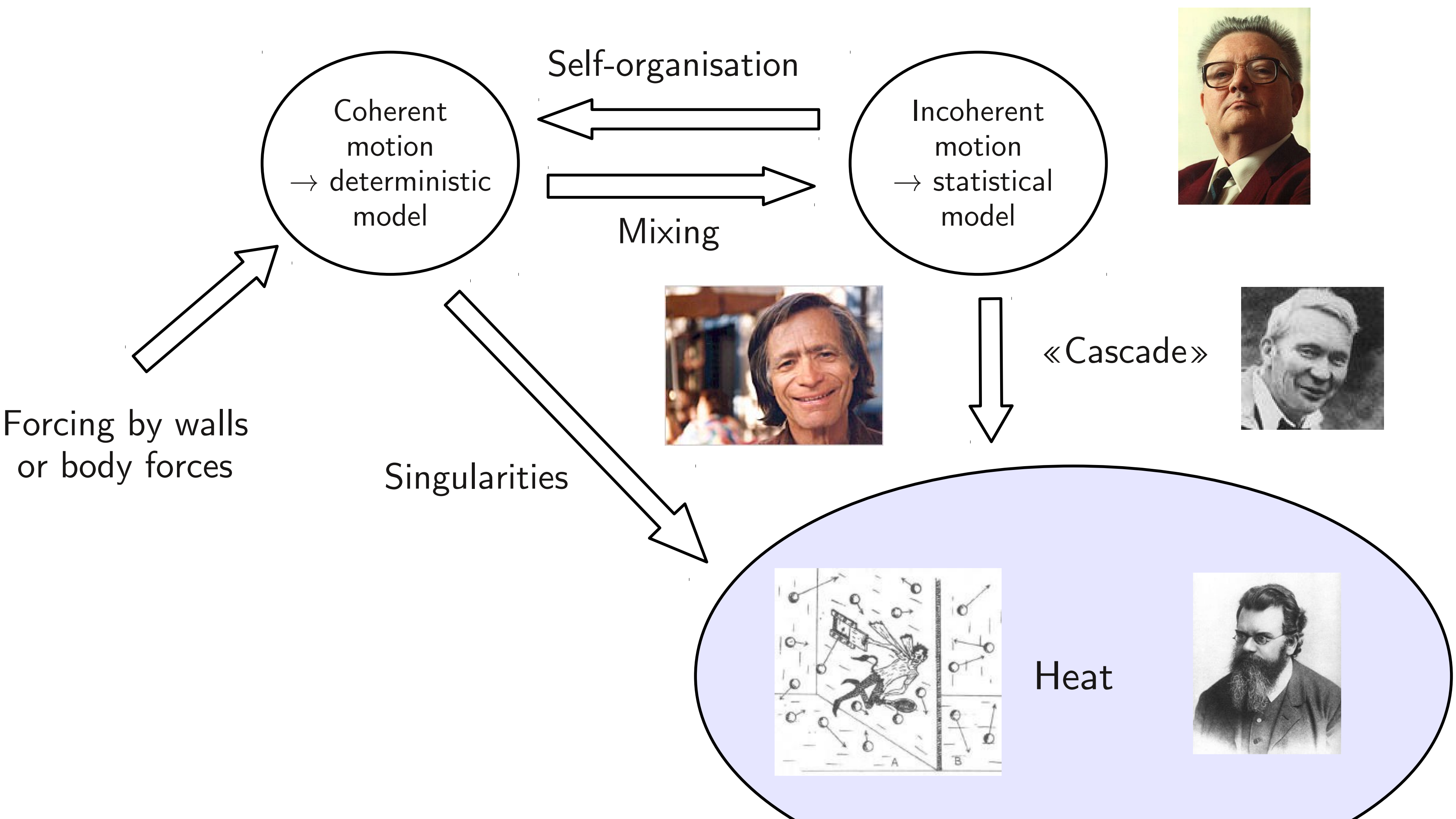
Momentum transport

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle u_y \rangle = -U\nu \int_{\pi_+} \frac{\partial u_y}{\partial x} + U\nu \int_{\pi_-} \frac{\partial u_y}{\partial x}$$

$$\phi_{\mathrm{laminar}} = \frac{2U\nu}{L}$$

Perspective: modelling

- Extensive study for varying Reynolds number.
- Design and validate statistical models for incoherent part.



Thank you !

And thanks to Rupert Klein, Dmitry Kolomenskiy, Claude Bardos, Jean-Marcel Rax and others for stimulating discussions!

C++ code download : http://justpmf.com/romain
Papers : http://wavelets.ens.fr