Extraction of coherent structures out of turbulent flows : comparison between real-valued and complex-valued wavelets

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CIRM, Marseille, 5th September 2007

Turbulence

Turbulence is a property of **flows** which involves
a **large number of degrees of freedom** interacting together.
It is governed by a **deterministic** dynamical system,
which is **irreversible** and **out of statistical equilibrium**.

Etymological roots of the word 'turbulence':
vortices *(turbo, turbinis)* and **crowd** *(turba,ae).*

Turbulent flows are solutions of the **Navier-Stokes equations** :

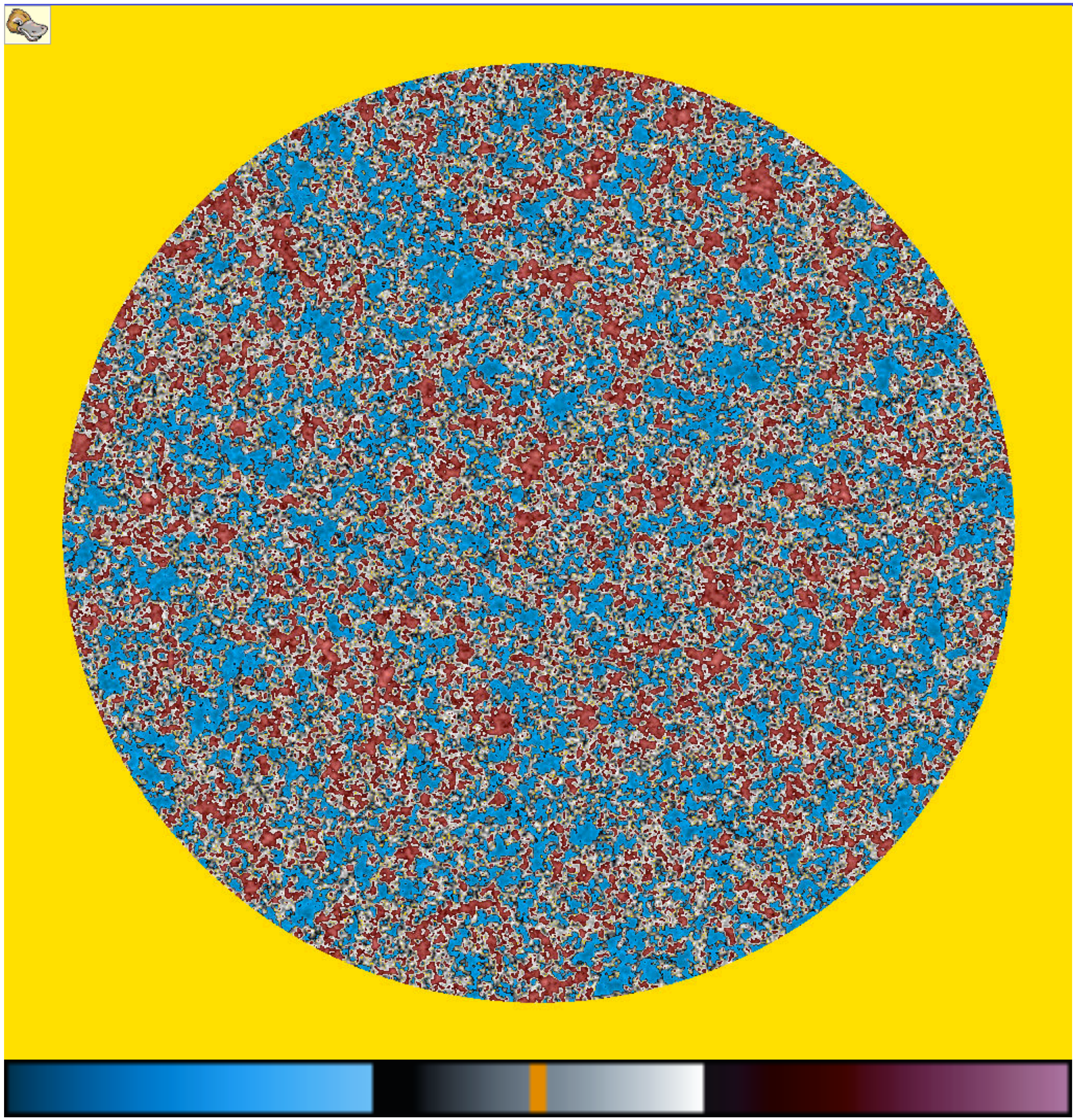
$$\partial_t \vec{\omega} + (\vec{v} \cdot \nabla)\vec{\omega} - \vec{\omega} \cdot \nabla \vec{v} = \nu \nabla^2 \omega + \nabla \times \vec{F}$$

$$\vec{\omega} = \nabla \times \vec{v} \qquad \text{and} \qquad \nabla \cdot \vec{v} = 0$$

ω vorticity, v velocity, F external force, ν viscosity and $\rho=1$ density,
plus initial conditions and boundary conditions

The **nonlinear term strongly dominates**
the viscous linear term and this is
quantified by the **Reynolds number**.

2D turbulent flow in a cylindrical container



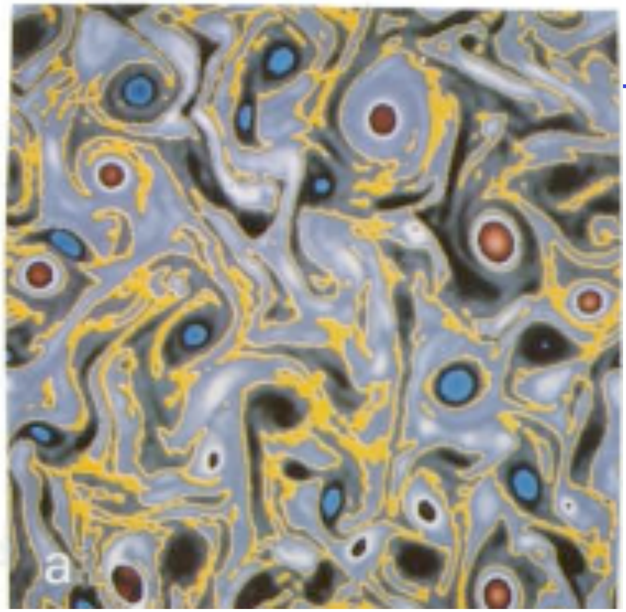
DNS
$N=1024^2$

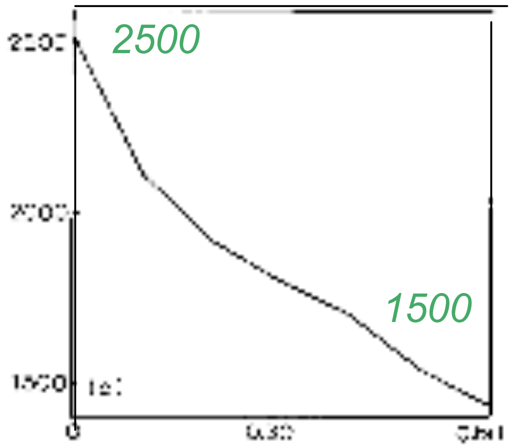
Random initial conditions

No-slip boundary conditions using volume penalization

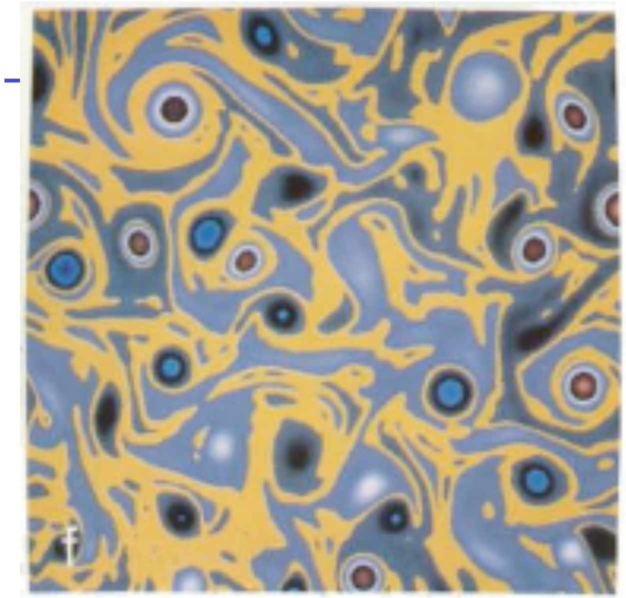
Schneider & Farge
Phys. Rev. Lett.,
December 2005

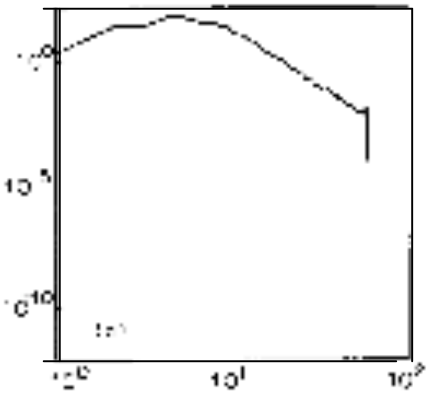
Reference simulation





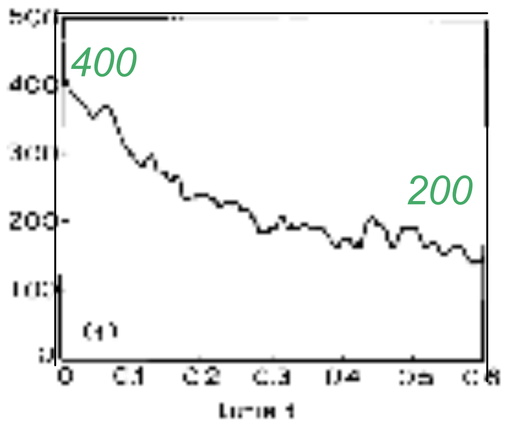
Enstrophy





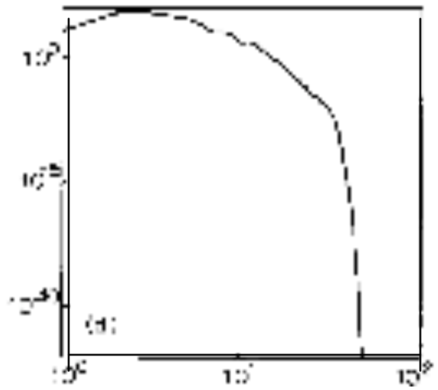
Energy spectrum

Initial time: t=0



Theoretical dimension

Time evolution

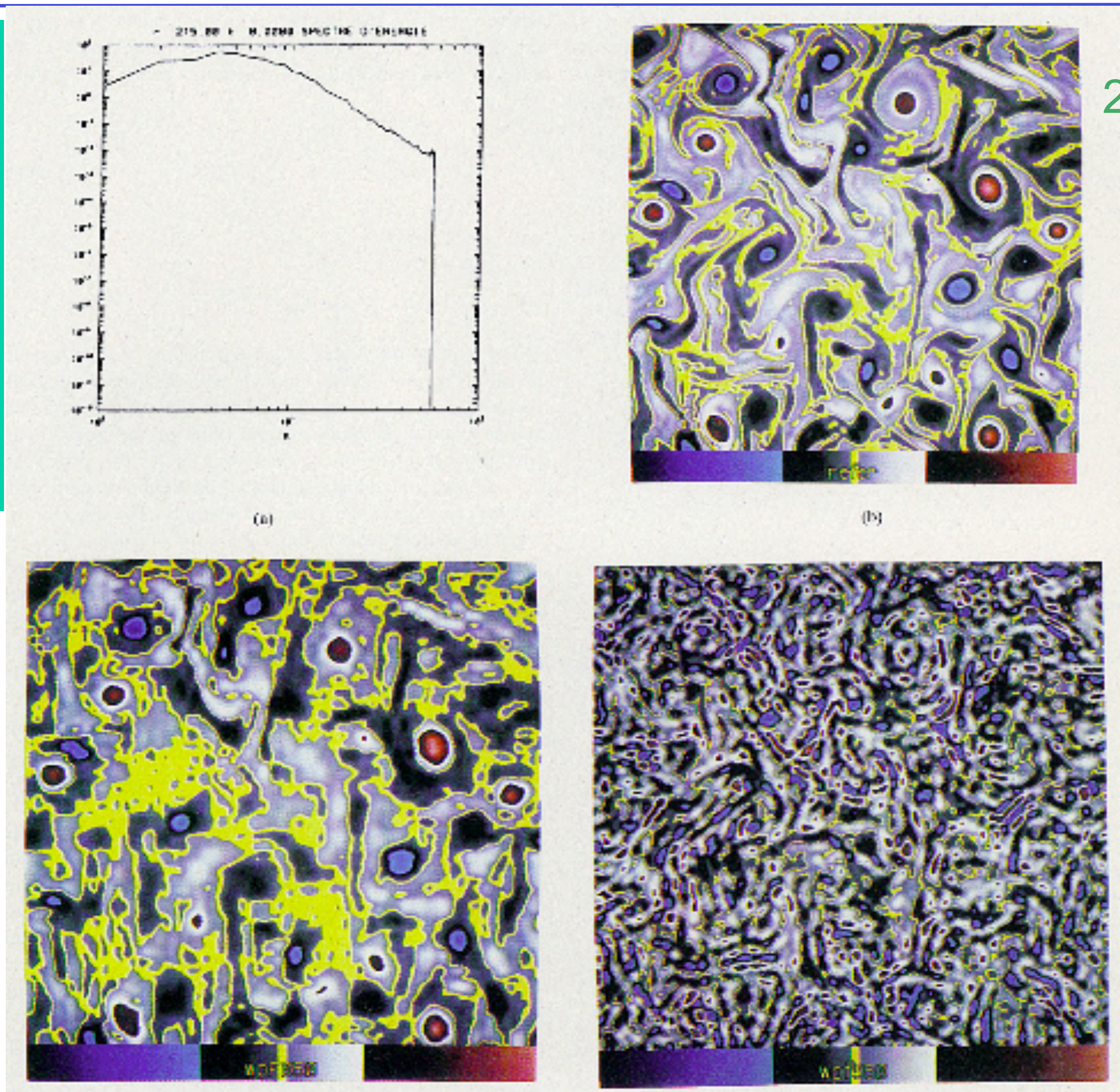


Energy spectrum

Final time: t=600

Wavelet Packet (WP) decomposition

Farge, Goirand, Meyer, Pascal and Wickerhauser
Improved predictability in 2D turbulent flows using wavelet packet compression
Fluid Dyn. Res., 10, 1992



2D turbulent flow computed by DNS and filtered using WP

Coherent flow:

strongest wavelet coefficients

Background flow:

weakest wavelet coefficients

t=0

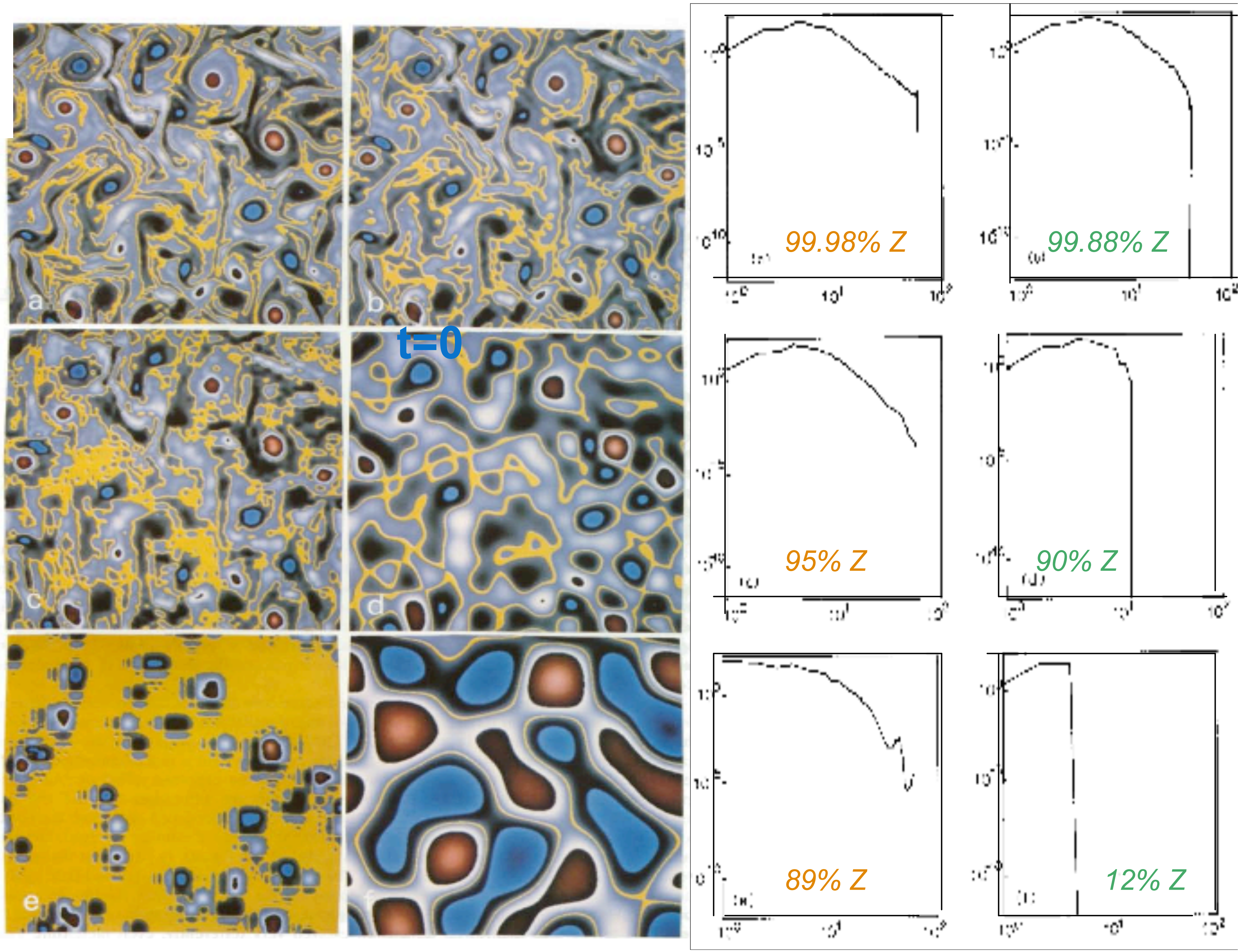
Wavelet Fourier Wavelet Fourier

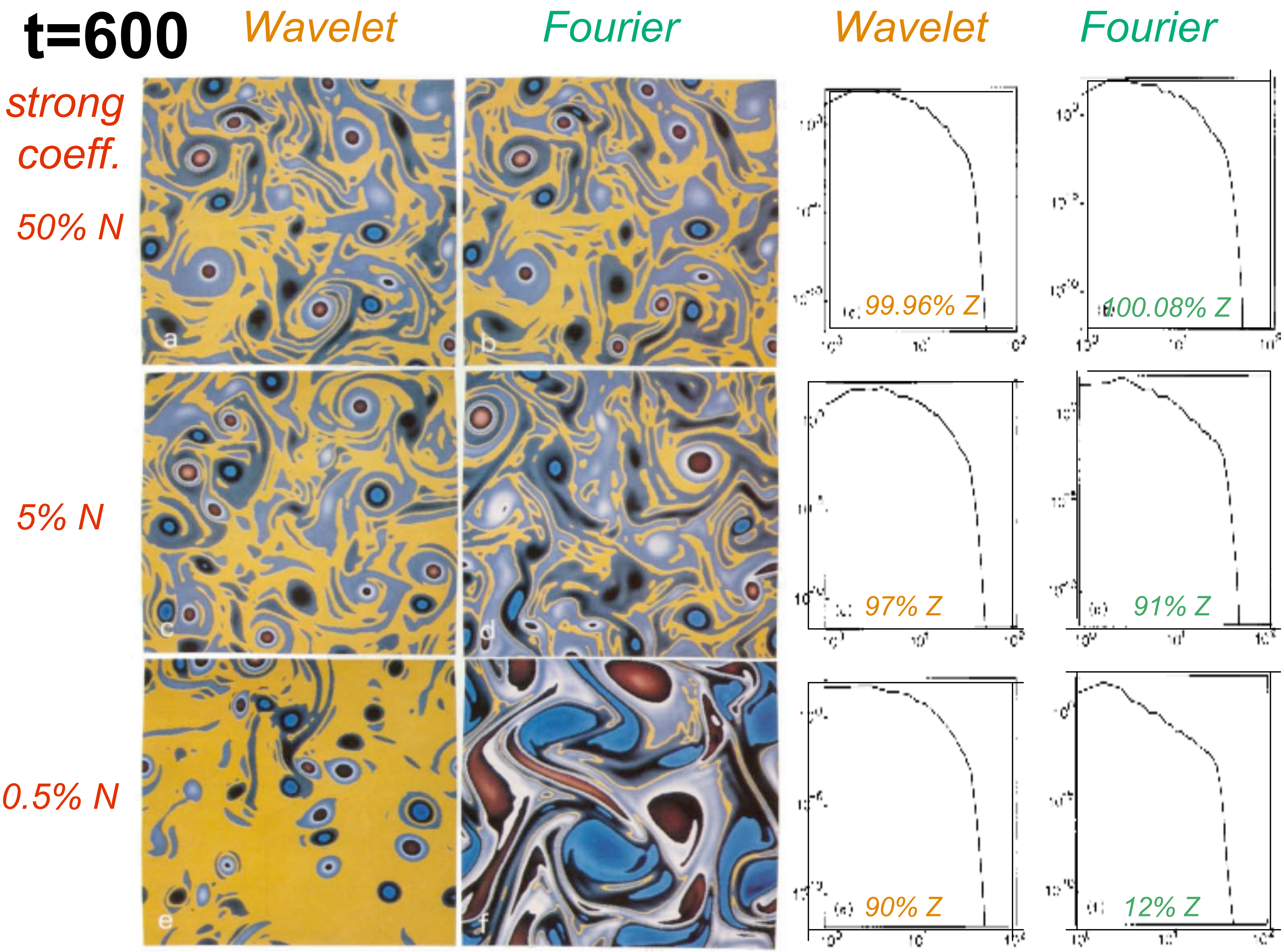
strong coeff:

50% N

5% N

0.5% N




t=600
Wavelet
Fourier
Wavelet
Fourier
strong coeff.
50% N
5% N
0.5% N
a
b
c
d
e
f
99.96% Z
100.08% Z
97% Z
91% Z
90% Z
12% Z

How to define coherent structures?

Since there is not yet a universal definition of coherent structures observed in turbulent flows (from laboratory and numerical experiments), we adopt an **apophetic method** :
instead of defining what they are, we define what they are not.

We propose the minimal statement:
'Coherent structures are different from noise'

$$\Longrightarrow$$

Extracting coherent structures becomes a **denoising problem**, not requiring any hypotheses on the coherent structures but only on the noise to be eliminated.

Choosing the simplest hypothesis as a first guess, we suppose we want to eliminate an additive Gaussian white noise, and use nonlinear wavelet filtering.

Farge, Schneider et al.
*Phys. Fluids, **15** (10), 2003*

Azzalini, Schneider and Farge
*ACHA, **18** (2), 2005*

Coherent Vortex Extraction

Farge, Schneider, Kevlahan, Phys. Fluids 11(8), 1999
Farge, Pellegrino, Schneider, Phys. Rev. Lett. 87(5), 2001

- Vorticity $\vec{\omega} = \nabla \times \vec{v}$ at resolution $N = 2^{3J}$
- Wavelet transform $\tilde{\vec{\omega}} = \langle \vec{\omega}, \psi_\lambda \rangle$
- Thresholding: $T = (4/3 Z \ln N)^{1/2}$

$$\tilde{\vec{\omega}}_C = \begin{cases} \tilde{\vec{\omega}} & \text{for} \quad |\tilde{\vec{\omega}}| \geq T, \\ 0 & \text{for} \quad |\tilde{\vec{\omega}}| < T \end{cases} \qquad \tilde{\vec{\omega}}_I = \begin{cases} \tilde{\vec{\omega}} & \text{for} \quad |\tilde{\vec{\omega}}| < T, \\ 0 & \text{for} \quad |\tilde{\vec{\omega}}| \geq T \end{cases}$$

- Inverse wavelet transform to reconstruct $\vec{\omega}_C + \vec{\omega}_I = \vec{\omega}$
- Apply Biot-Savart operator to reconstruct $\vec{v}_C + \vec{v}_I = \vec{v}$ with $\vec{v} = \nabla \times \nabla^{-2} \vec{\omega}$
- Remark: $Z = Z_C + Z_I$ (orth. dec.) and $E \approx E_C + E_I$
- Linear complexity, $\mathcal{O}(\mathcal{N})$

2D vortex extraction using wavelets

in laboratory experiment

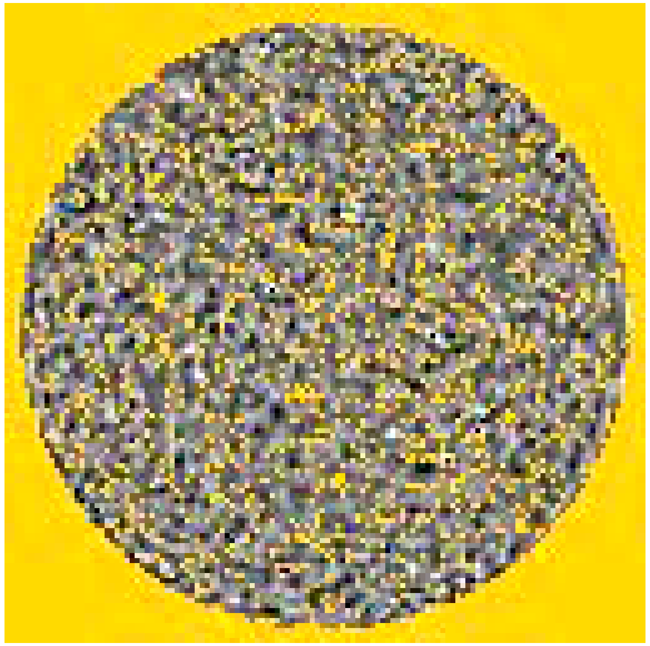
PIV
$N=128^2$

Total vorticity
100% E
100% Z

2% N

Coherent vorticity
99% E
80% Z

98% N



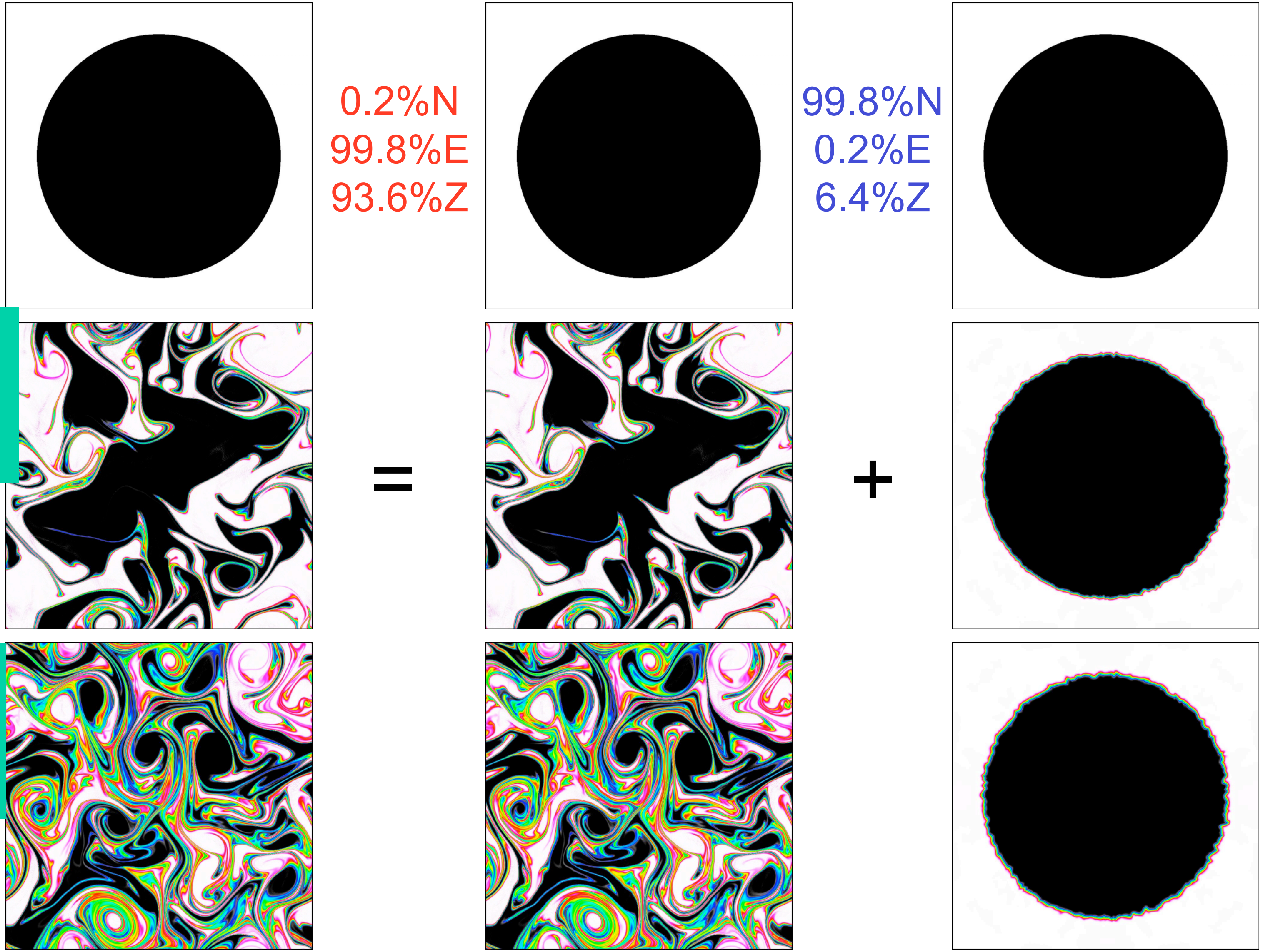
Incoherent vorticity
1% E
3% Z

Passive scalar advection
in numerical experiment

DNS
$N=512^2$

Beta, Schneider, Farge 2003, Chemical Eng. Sci., 58

Beta, Schneider, Farge 2003, Nonlinear Sci. Num. Simul., 8



Total flow

Coherent flow

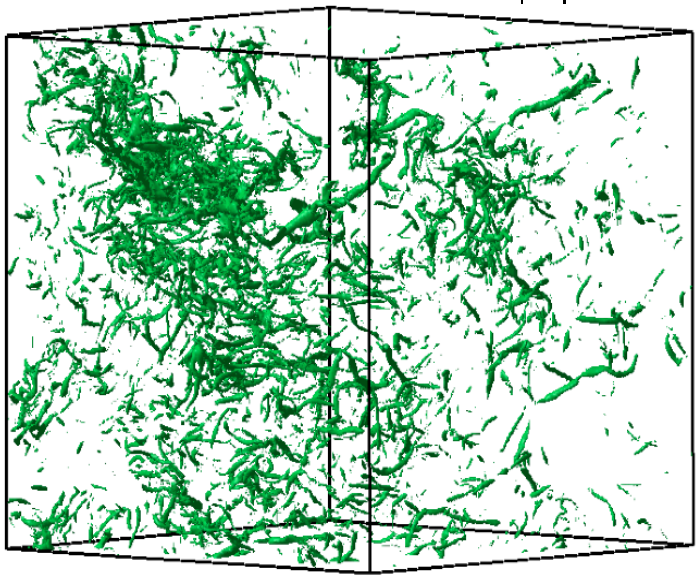
Incoherent flow

Modulus of the 3D vorticity field

computed by Yukio Kaneda et al.

DNS
$N=2048^3$

$|\omega|=5\sigma$



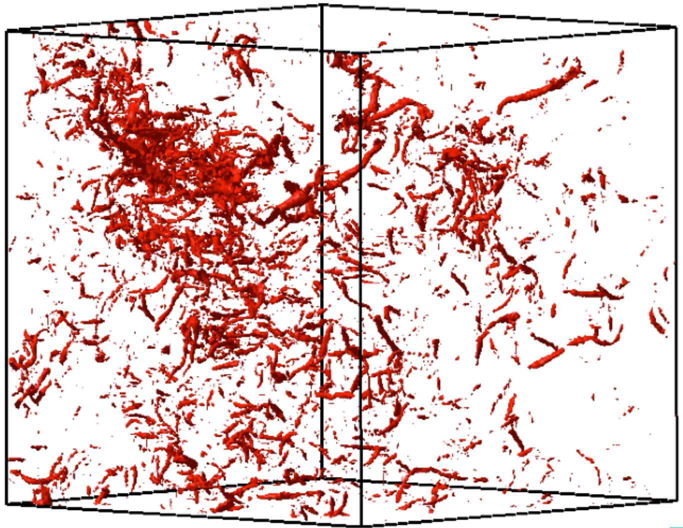
with $\sigma=(2Z)^{1/2}$

Coherent vorticity

2.6 % N coefficients
80% enstrophy
99% energy

Incoherent vorticity

97.4 % N coefficients
20 % enstrophy
1% energy



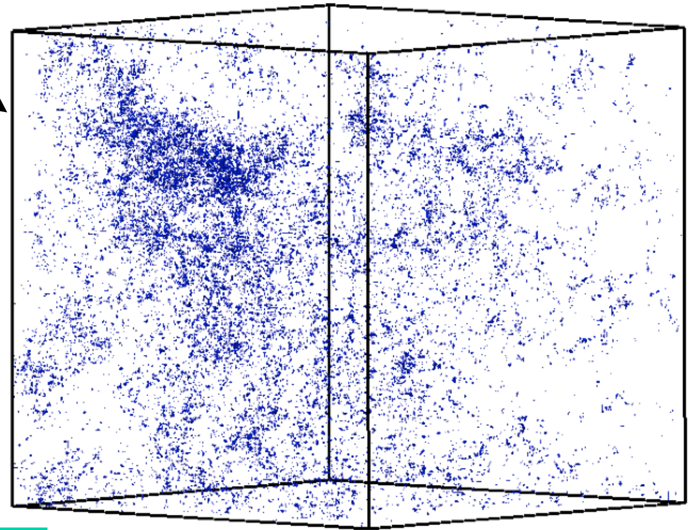
$|\omega|=5\sigma$

Total vorticity

$R_\lambda=732$
$N=2048^3$
Visualization at 256^3

+

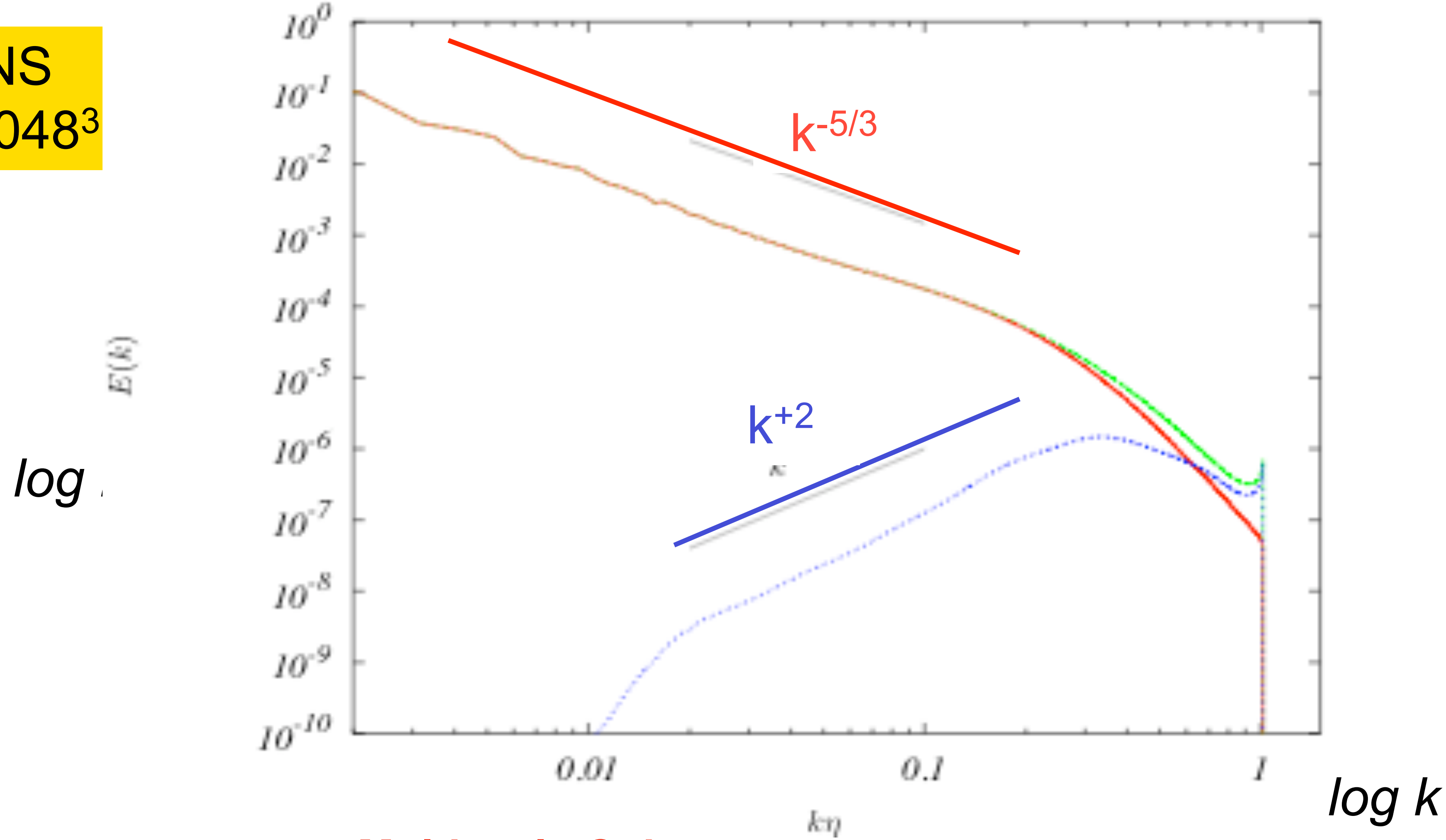
Submitted to Phys. Fluids



$|\omega|=5/3\sigma$

Energy spectrum

DNS
N=2048^3

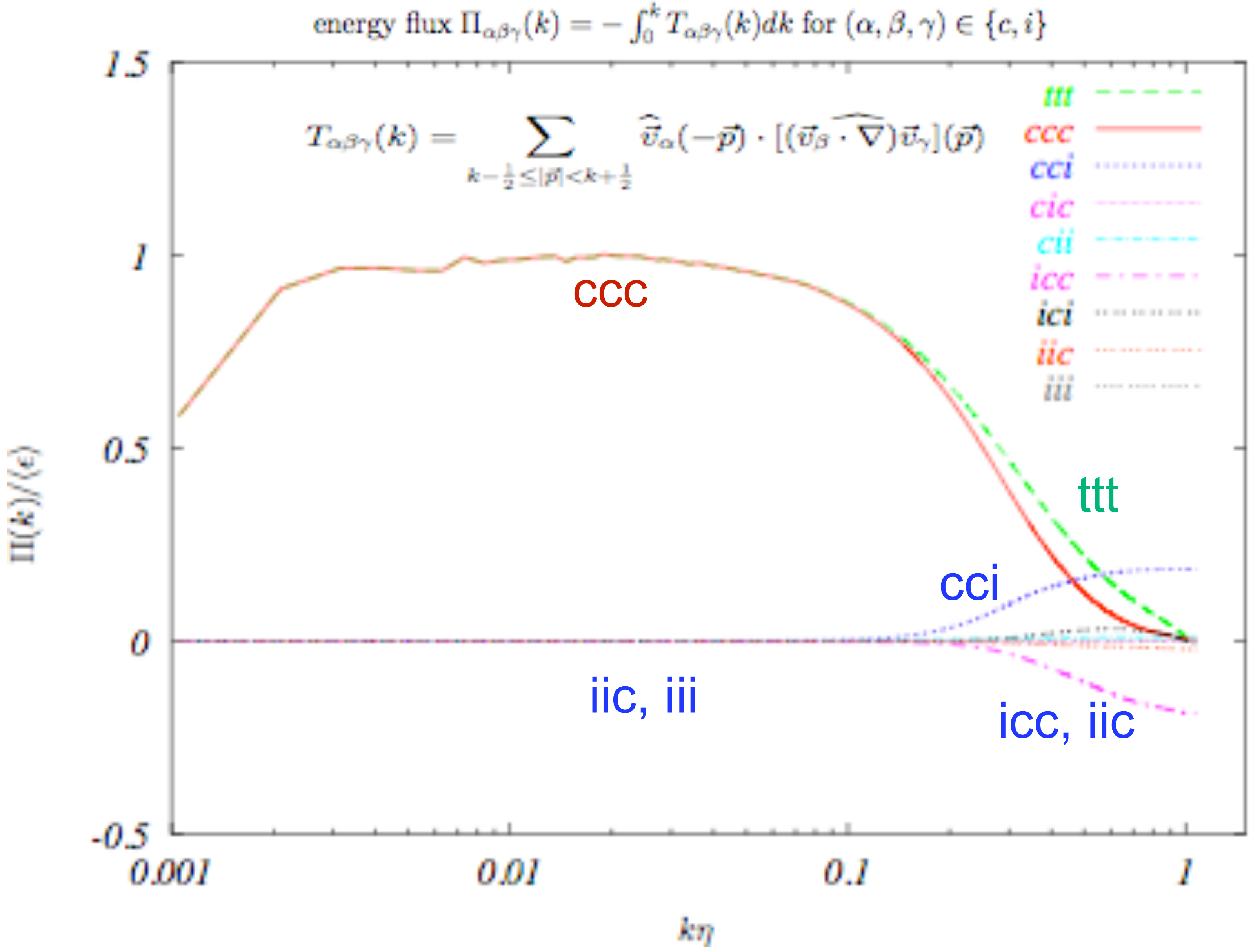


2.6 % N coefficients
80% enstrophy
99% energy

Multiscale Coherent
$k^{-5/3}$ scaling, i.e.
long-range correlation

Multiscale Incoherent
k^{+2} scaling, i.e.
energy equipartition

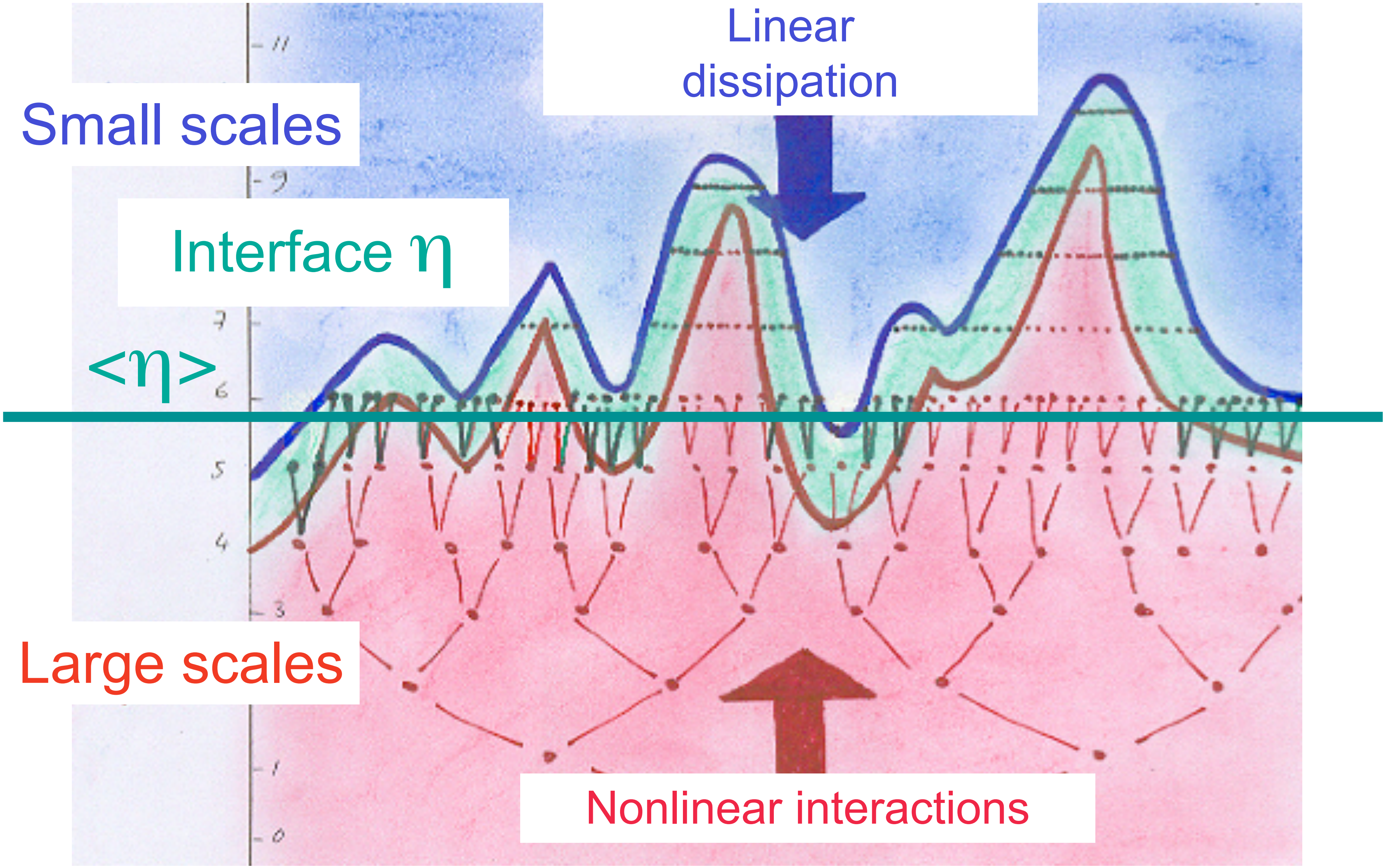
Energy flux



energy flux $\Pi_{\alpha\beta\gamma}(k) = -\int_0^k T_{\alpha\beta\gamma}(k)dk$ for $(\alpha, \beta, \gamma) \in \{c, i\}$
$T_{\alpha\beta\gamma}(k) = \sum_{k-\frac{1}{2} \leq |\vec{p}| < k+\frac{1}{2}} \widehat{\vec{v}}_\alpha(-\vec{p}) \cdot [(\widehat{\vec{v}_\beta \cdot \nabla)\vec{v}_\gamma}](\vec{p})$
ttt
ccc
cci
cic
cii
icc
ici
iic
iii
1.5
1
0.5
0
-0.5
0.001
0.01
0.1
1
$\Pi(k)/\langle\epsilon\rangle$
$k\eta$
ccc
ttt
cci
iic, iii
icc, iic

New interpretation of the energy cascade

Wavelet space viewpoint



Mathematical image processing meeting, CIRM Luminy, Sept. 5, 2007

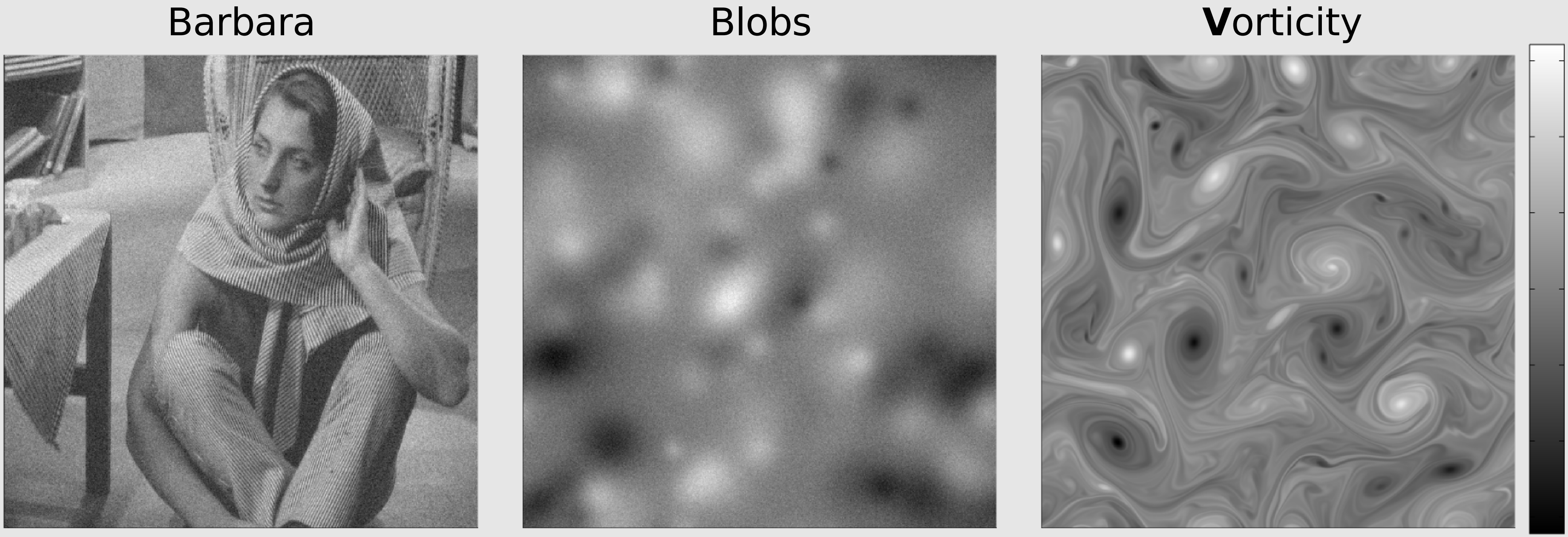
Extraction of coherent structures out of 2D turbulent flows:

comparison between real-valued orthogonal wavelet bases and complex valued wavelet frames

R. Nguyen van yen[1], M. Farge[1], K. Schneider[2]
thanks to the collaboration of N. Kingsbury[3]

[1] LMD, ENS Paris
[2] LMSNM-GP, Université d'Aix-Marseille
[3] Signal Processing Group, Cambridge University

Introduction (1/2)

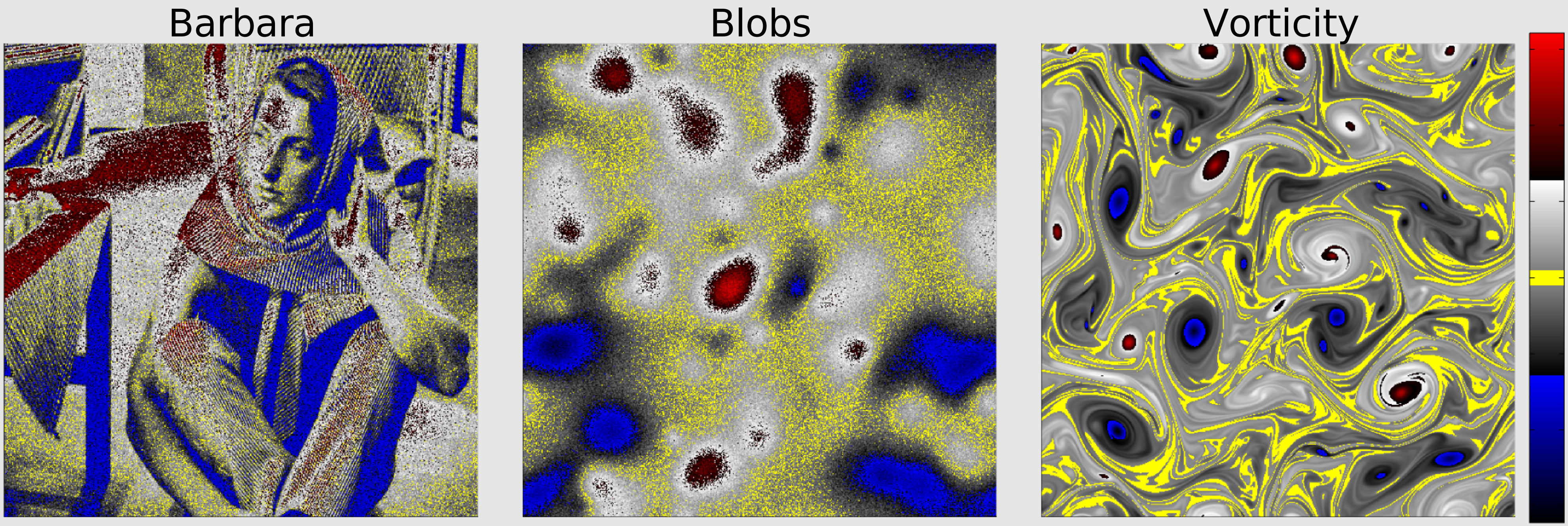


1. Barbara: classical visual image example (good for comparison with denoising algorithms)

2. Blobs: sum of randomly centered, periodized Gaussian functions

3. Vorticity: obtained by solving the Navier-Stokes equation (credit B. Kadoch)

Introduction (2/2)



(colour palette optimized to visualize vorticity)

- *Barbara* and *Blobs* are artificially supplemented with a noise (SNR 14dB), either white (shown above) or correlated (not shown).
- *Vorticity* is taken fresh from the numerical simulation, but **modelled** as containing a noise **of dynamical origin**. It is intrinsically a zero mean **fluctuating quantity**.

Outline

1. Model

   extracting coherent structures in the wavelet denoising framework

2. Iterative algorithm

   practical implementation of the extraction procedure

3. Results

   numerical study of the algorithms in academic and practical situations

Part I

Model

Coherent structures

There is no tractable and widely accepted definition

We propose a minimal hypothesis : coherent structures are not noise

Extracting coherent structures amounts to removing the noise

Hypotheses need to be made, not on the structures, but on the noise

Hypotheses on the noise (1/2)

As a starting point, we suppose the noise to be:

- **additive**,
- **stationary**,
- **Gaussian**,

which yields the decomposition :

$$\omega = \omega_C + \omega_I$$

coherent structures

incoherent "noise"

Do we need an hypothesis on the **correlation** ?

Hypotheses on the noise (2/2)

- Analytical results in 1D, together with numerical experiments in 2D, suggest that denoising is possible only below a certain “level of correlation”.
- To define such a critical correlation, we would need to choose a parametric model.
- We limit ourselves to 2 simple models:
  - white noise,
  - long range correlated and isotropic noise, with a power spectrum decaying like

$$E(k) \propto k^{-\frac{1}{2}}$$

and a random phase.

Differences with visual image denoising

Three differences that will be discussed in this talk...:

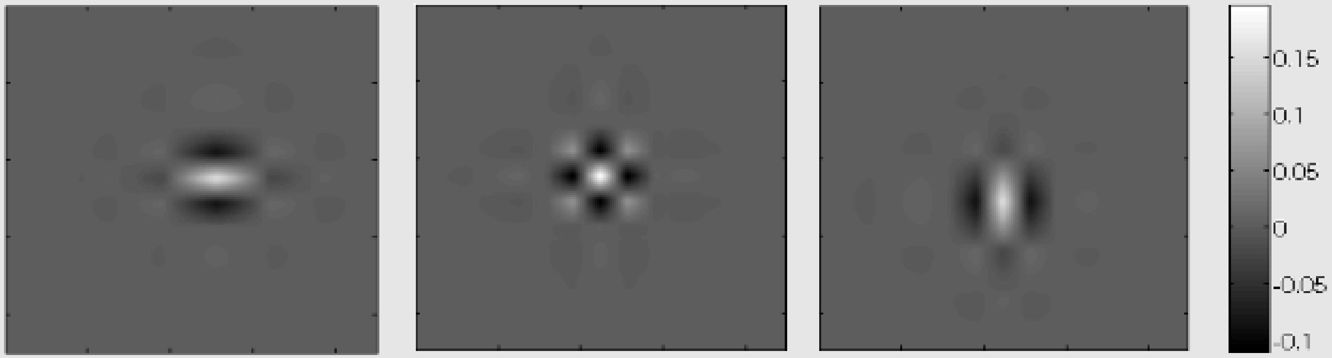
- our goal is actually to **compress** the flow and denoising is only a tool,
- since there is no reference noiseless vorticity field (such as Lena), **quantifying performance** is difficult,
- the **incoherent part** is used to estimate performance.

...and some more not to be addressed here:

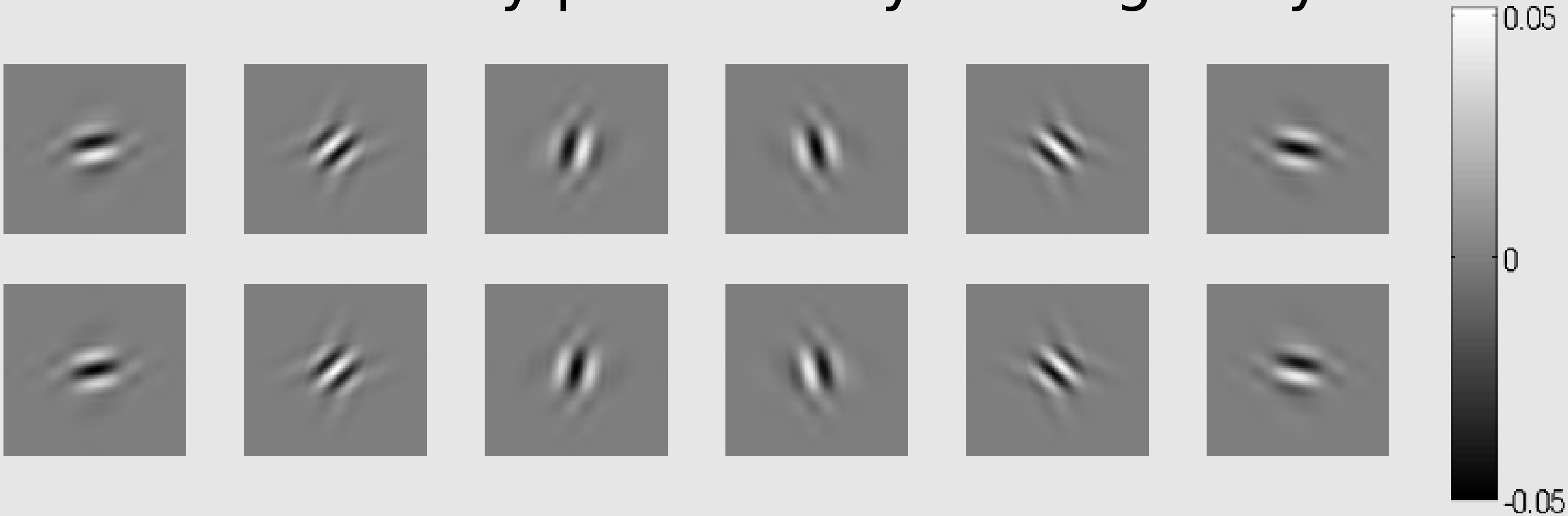
- our goal is to preserve the **time evolution**,
- **computational efficiency** is then a critical issue,
- the real challenge is actually **3D** Navier-Stokes,
- vorticity is then a **vector field**.

Two wavelet families to compare (1/2)

Real wavelets : we use separable Coiflet 12 filters



Complex wavelets : we use DTCWT filters,
kindly provided by N. Kingsbury



Two wavelet families to compare (2/2)

Real wavelets: The real orthogonal wavelet transform preserves whiteness. It has been shown to possess good decorrelating properties when applied to particular kinds of Gaussian, correlated noises.

Complex wavelets: the DTCWT uses a quadtree of real separable wavelet filters followed by orthogonal linear combinations. The decorrelating properties thus remain those of real wavelets.

There are, however, correlations between the wavelets themselves.

Consequently, the energy conservation is lost as soon as we manipulate (i.e. threshold) the coefficients.

Part II

Thresholding procedures

Principle of wavelet thresholding

Goal: eliminate from a given set of wavelet coefficients those that are likely to be realisations of Gaussian random variables

- Thresholding methods developed since Donoho & Johnstone have proven useful for denoising images
- We have to stick to **hard thresholding** because we want to have good compression and idempotence
- [Azzalini et al., *ACHA,* '05] have proposed an **iterative method** to determine the threshold value
- Generalization to complex wavelet coefficients is straightforward

Iterative algorithm

Given a set of wavelet coefficients (1)

$$\Lambda^{(l)}$$

Compute the variance (2)

$$\sigma_{(l)}^{2} = \sum_{\Lambda^{(l)}} |X_\lambda|^2$$

Eliminate outliers (3)

$$\Lambda^{(l+1)} = \{ \lambda / |X_\lambda| < \chi \sigma_{(l)} \}$$

Return to (1) unless (4)

$$\Lambda^{(l+1)} = \Lambda^{(l)}$$

Choosing a set of wavelet coefficients

Either **global** thresholding,

or **scale by scale** thresholding:

- Previously applied for denoising ([Johnstone & Silvermann] and others).
- In 2D, we propose to treat each subband separately.
- For statistical reasons, we restrict ourselves to subbands containing at least 32x32 coefficients.

Choosing the quantile parameter χ (1/2)

- Needs to be adjusted depending on the application
- Minimizing the global denoising error for the Lena image using real wavelets leads to:

$$\chi_{real}=3.1$$

- But this value is too small for vorticity fields since we want to achieve high compression. We then arbitrarily choose:

$$\chi_{real}=6.0$$

- Statistical interpretation: when we feed a pure Gaussian noise to the algorithm, $1/10^3$ coefficients are retained in the first case, and only $1/10^9$ in the second case

Choosing the quantile parameter χ (2/2)

- We want these probabilities to remain the same when using complex wavelets
- The squared moduli of the complex wavelet coefficients of a standard Gaussian white noise are approximately independent **khi-square** random variables with **2 degrees of freedom**
- This leads to the following relationship:

$$\chi^2_{complex} \simeq \frac{\chi^2_{real}}{2}$$

Part III

Results

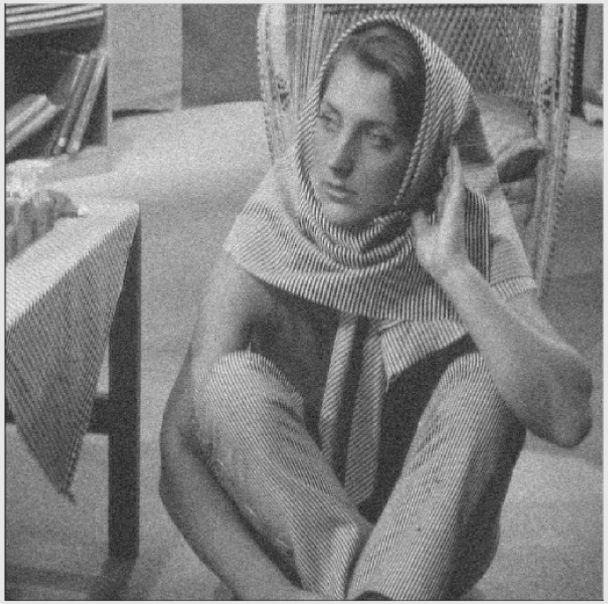
Part III : Outline

For the 3 fields shown in the introduction, we will show:

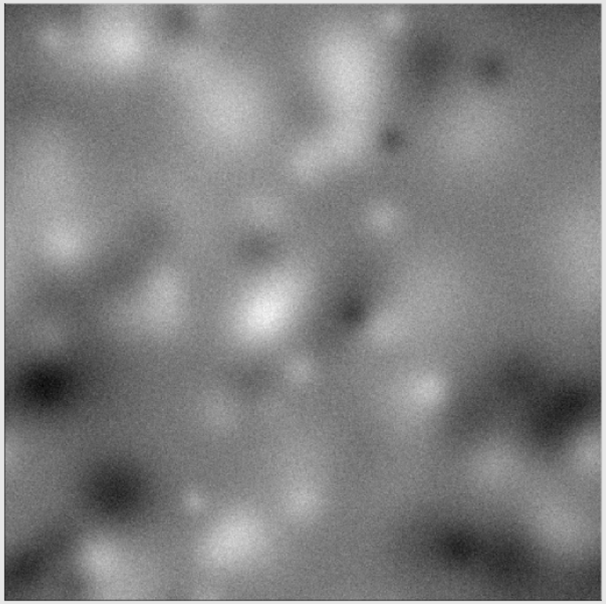
- their wavelet coefficients and the effect of thresholding
- the denoising efficiency of both algorithms
  - in the presence of white noise,
  - in the presence of correlated noise
- the compression properties
- the reconstructed noise, and some estimates of its Gaussianity

The 3 fields (1/2)

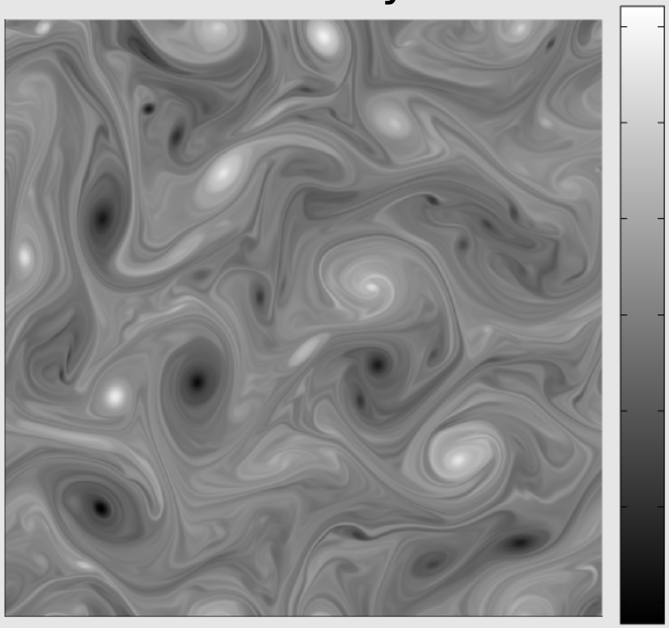
Barbara



Blobs



Vorticity



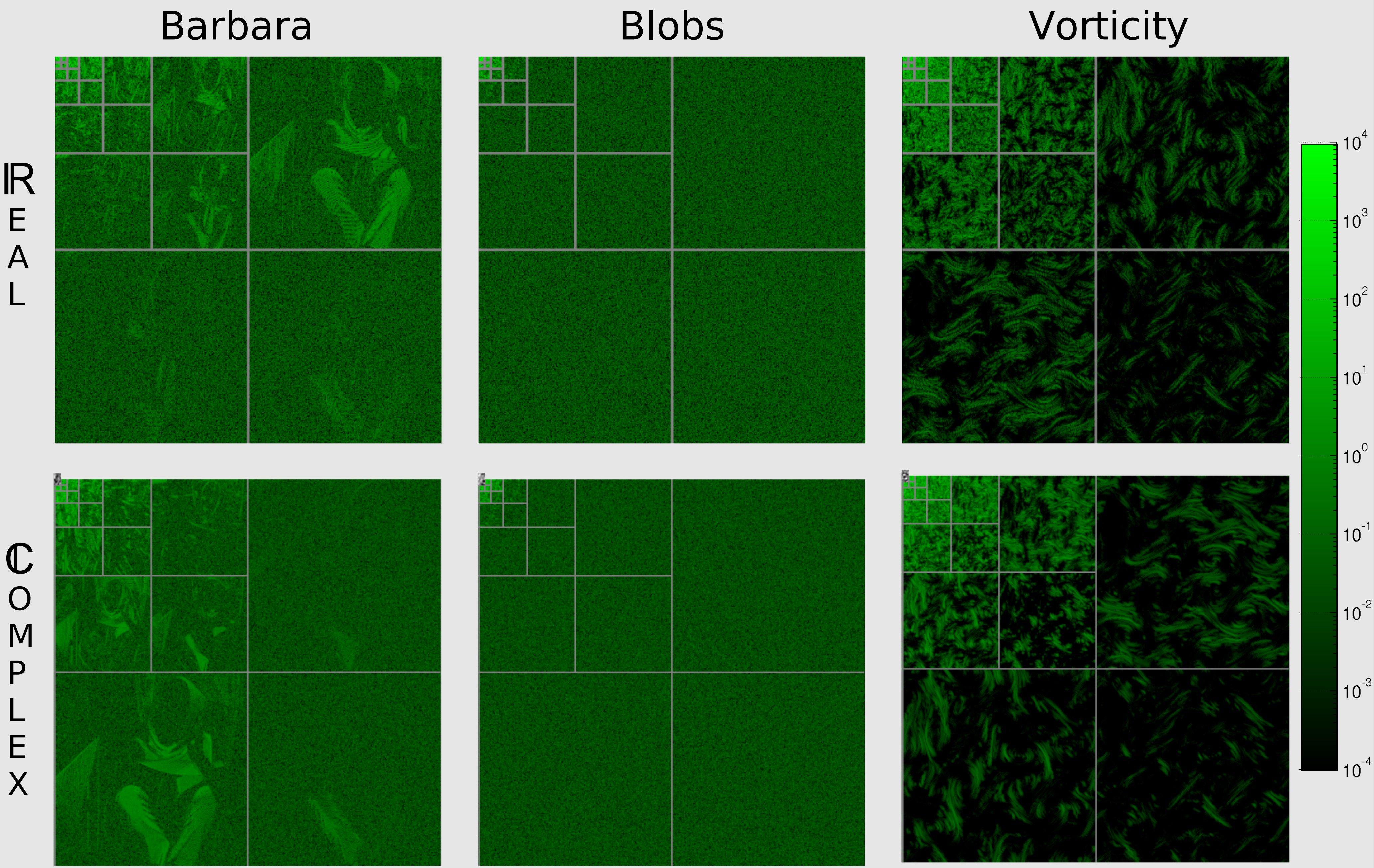
1. <u>Barbara</u>: classical visual image example (good for comparison with denoising algorithms)

2. <u>Blobs</u>: sum of randomly centered, periodized gaussian functions

3. <u>Vorticity</u>: obtained by solving the Navier-Stokes equation (credit B. Kadoch)

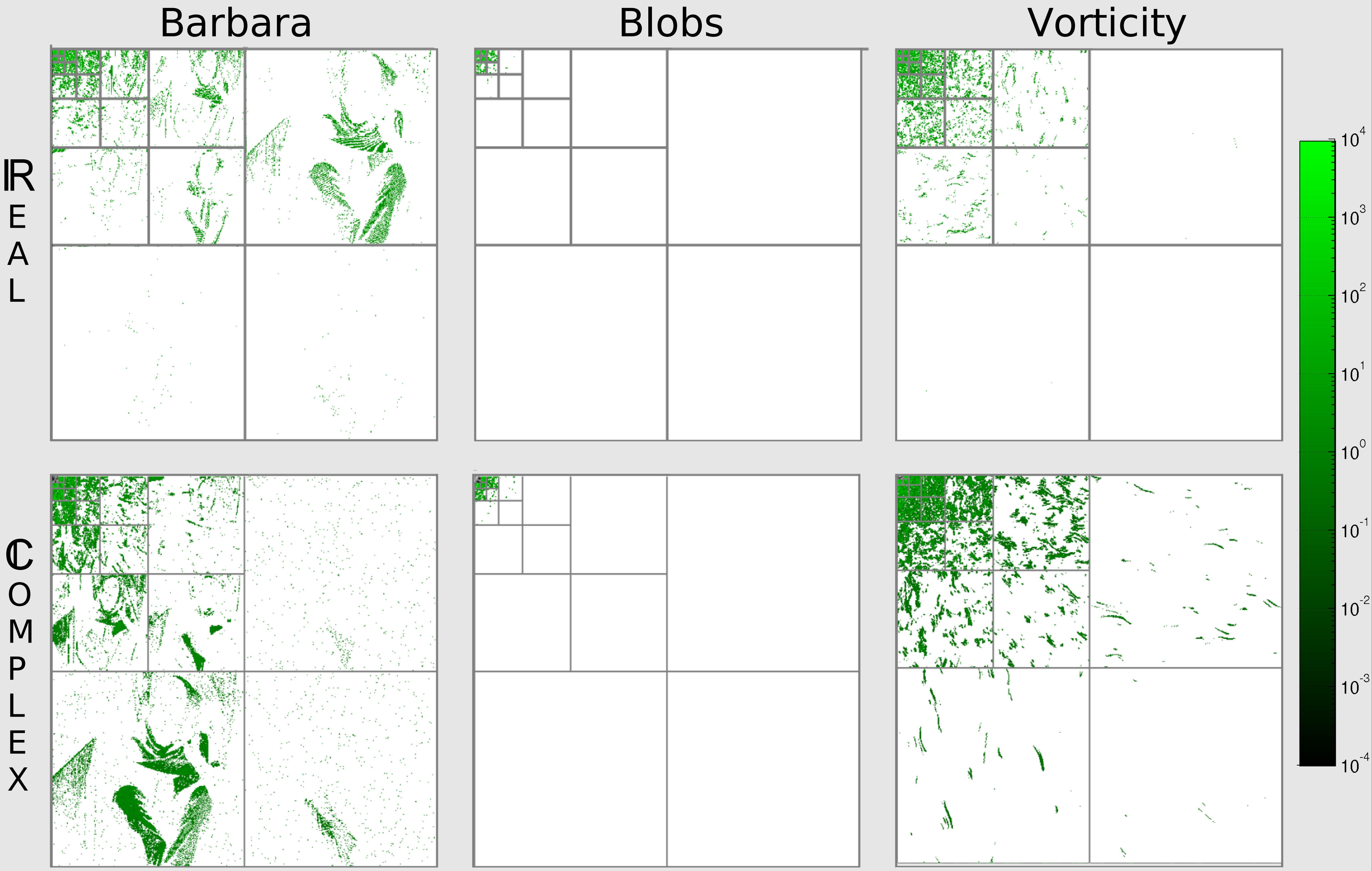
...all having 512x512 pixels

Wavelet coefficients (before thresholding)



Only 3 of the 6 complex wavelet directions are shown here

Wavelet coefficients (after thresholding)



Coefficients below the threshold are now coloured white

Denoising gains for white noise

	Barbara	Blobs	Vorticity
Real	2,0	15	?
Complex	6,1	20	?
Real Scale by scale	0,37	15	?
Complex scale by scale	5,4	20	?

(decibels)

- Bad scale by scale performance for Barbara because the threshold is overestimated in some crowded subbands (striped pattern)
- Complex wavelets are better for denoising (already clear from previous studies)

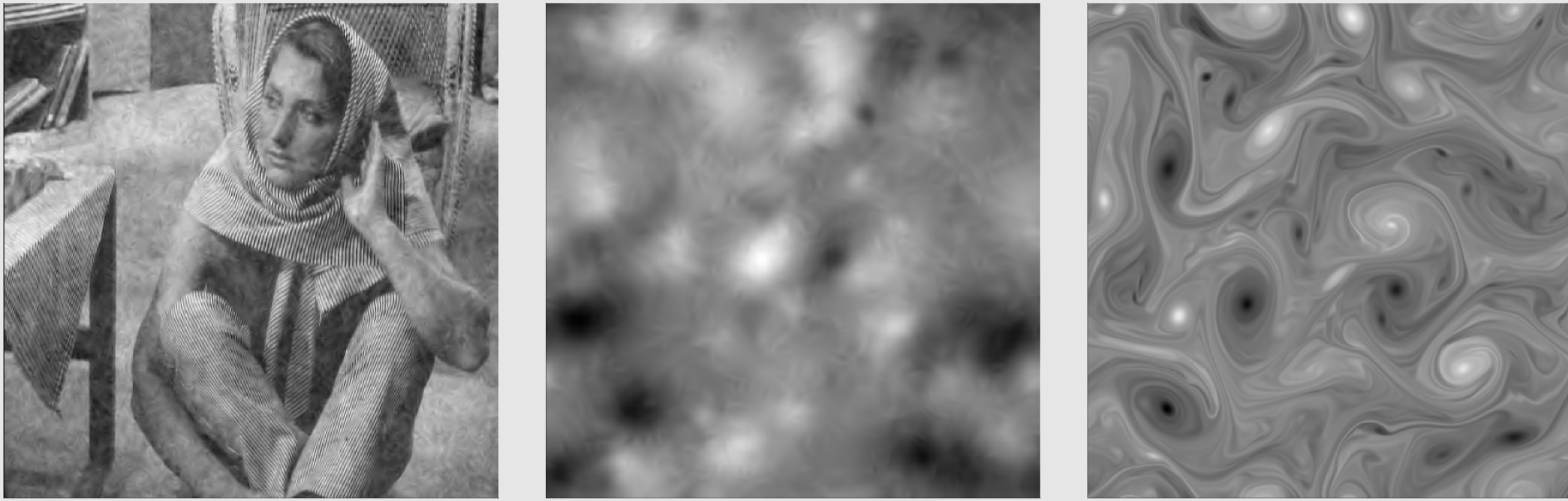
Denoising gains for correlated noise (1/2)

	Barbara	Blobs	Vorticity
Real	0,71	5,7	?
Complex	**2,2**	5,7	?
Real scale by scale	-0,53	6,3	?
Complex scale by scale	**2,8**	7,1	?

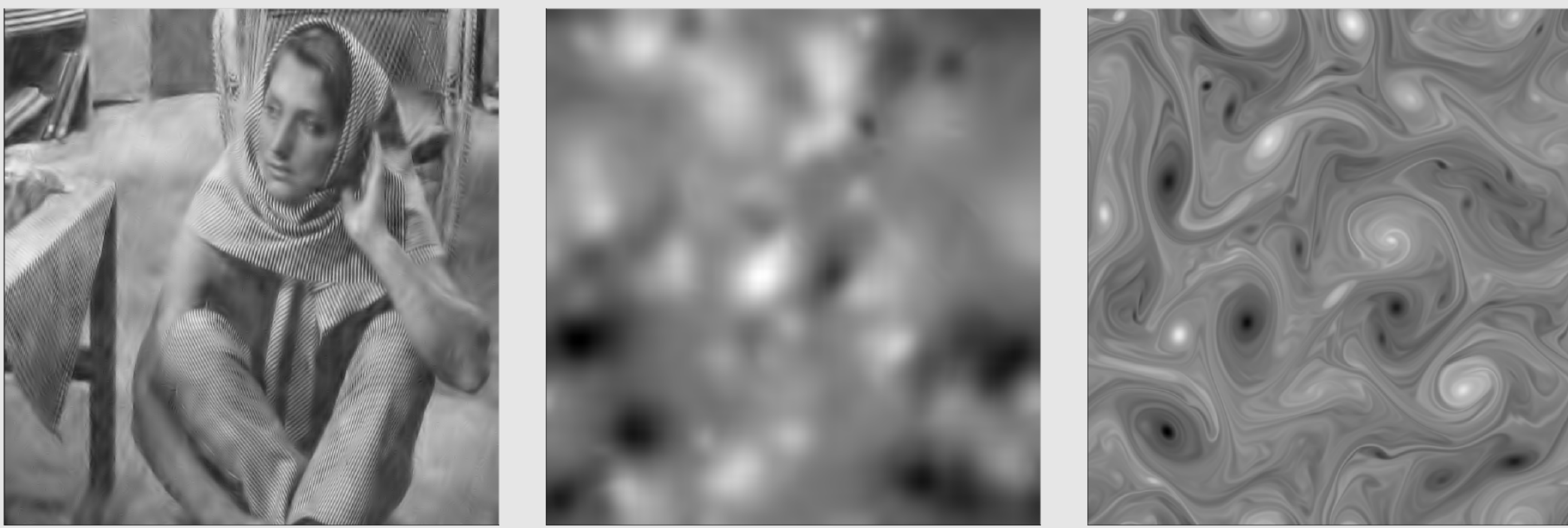
(decibels)

- For a given SNR, correlated noise is nastier.
- Scale by scale thresholding helps.
- Visual illustration on the next slide.

Denoising gains for correlated noise (2/2)



Global thresholding of complex wavelet coefs.



Scale by scale thresholding of complex wavelet coefs.

Compression

Percentage of coherent coefficients (% of 512x512)

	Barbara	Blobs	Vorticity
Real	4,30	0,2	**2,6**
Complex	34,9	1,1	**29,6**
Real scale by scale	2,94	0,2	**6,6**
Complex scale by scale	30,7	1,1	**44,5**

Energies of the coherent and incoherent parts (% of total)

	Barbara		Blobs		Vorticity	
Real	88,3	12	91,6	8,4	97,1	2,9
Complex	89,7	7,5	91,5	8,2	**98,7**	**0,4**
Real scale by scale	85,7	14	91,6	8,4	98,7	1,3
Complex scale by scale	88,6	7,9	91,5	8,2	**99,1**	**0,3**

- Scale by scale algorithm behaviour on vorticity fields is not satisfactory up to now.
- Energies do not add up to 100% in the DTCWT case.
- Complex wavelets pick up more directional features.

Properties of the reconstructed noise

- We have to characterize the incoherent part a posteriori since it is produced by the nonlinear dynamics
- For this, qualitative appreciation is not sufficient, and we will use statistical tools
- Here, we will check the Gaussianity of the noise.
- For sorted data $X_{(i)}$,

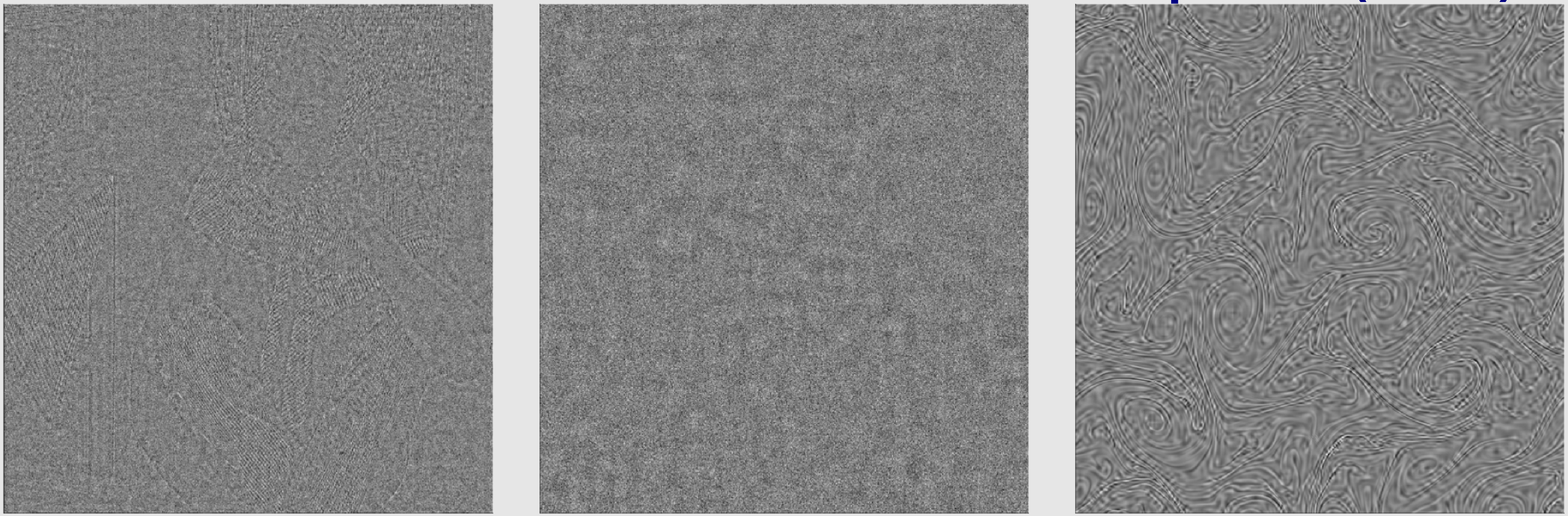
  the **normal probability plot** is the set

$$\{(X_{(i)}, Y_i)\}$$

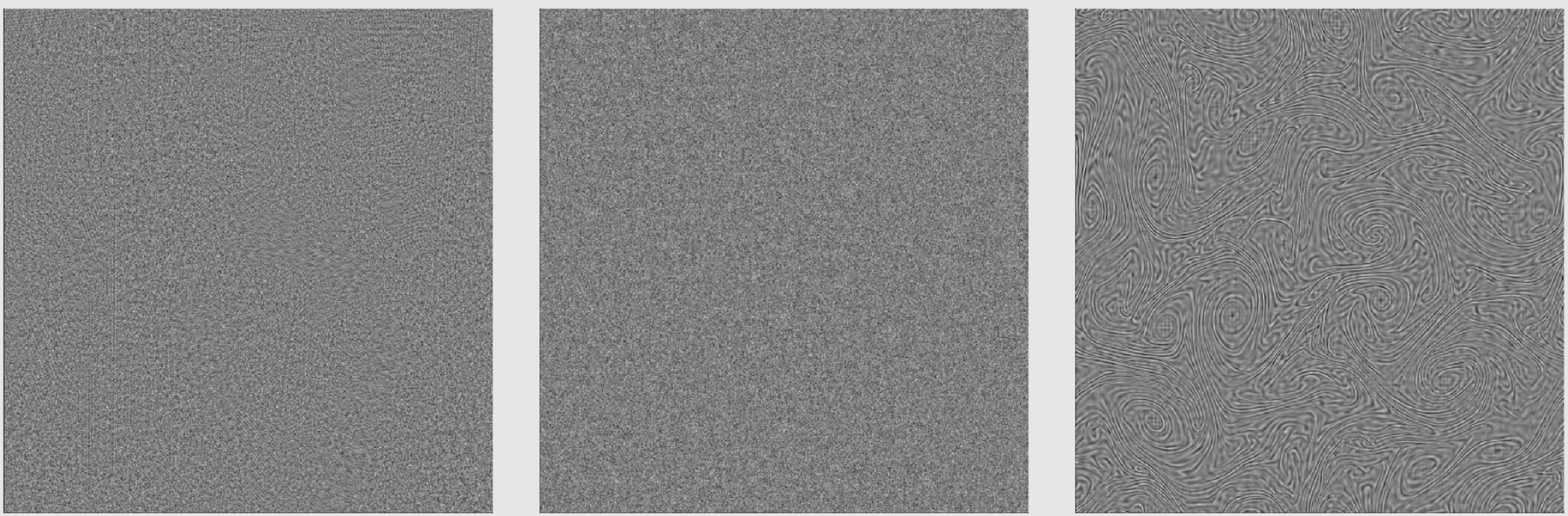
where $$Y_i = F^{-1}\left(\frac{i}{n}\right)$$

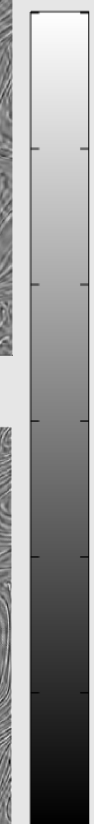
with $$F(x) = \int_{-\infty}^{x} \exp\left(\frac{-x^2}{2}\right) \frac{dx}{\sqrt{(2\pi)}}$$

Visualization of the incoherent part (1/2)



Global thresholding of **real** wavelet coefs.



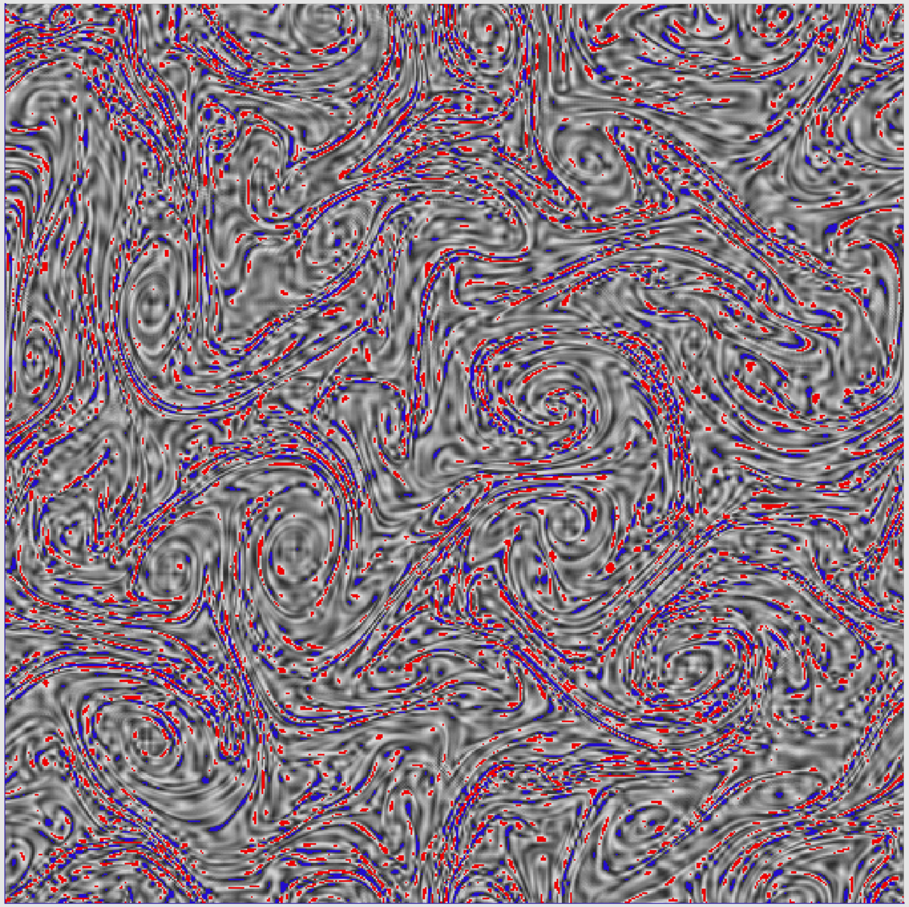


Global thresholding of **complex** wavelet coefs.

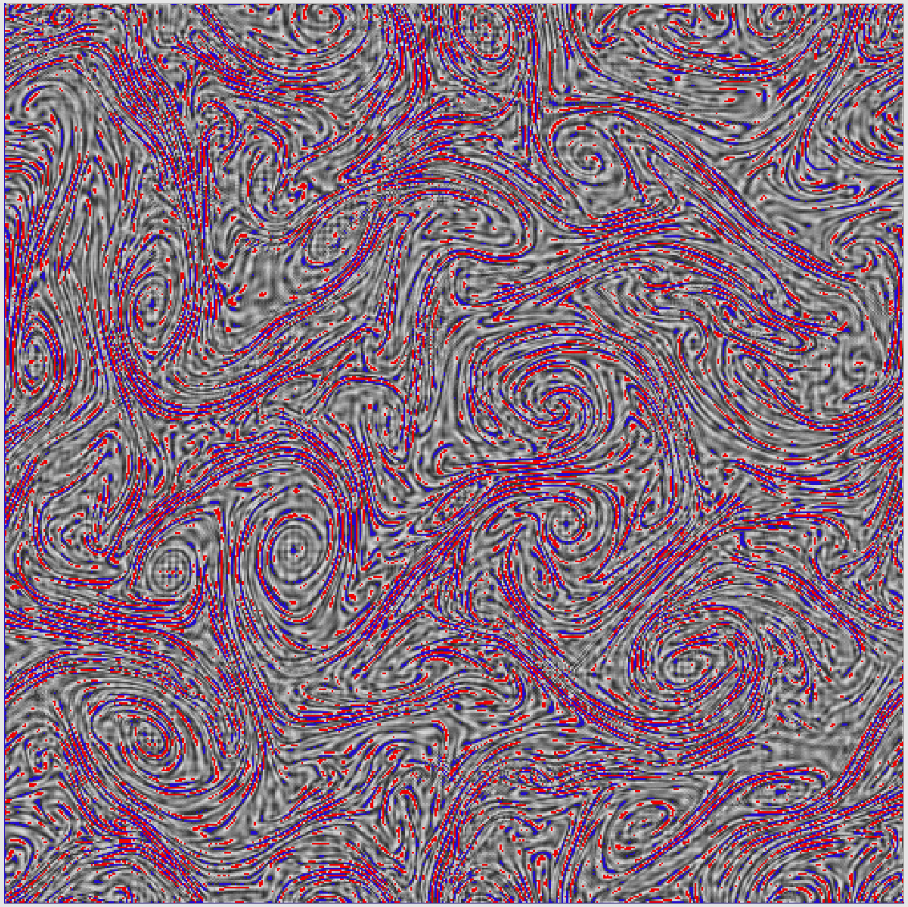
Visualization of the incoherent part (2/2)

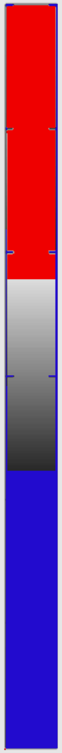
And if zoom on the vorticity field :

<u>Real wavelets</u>



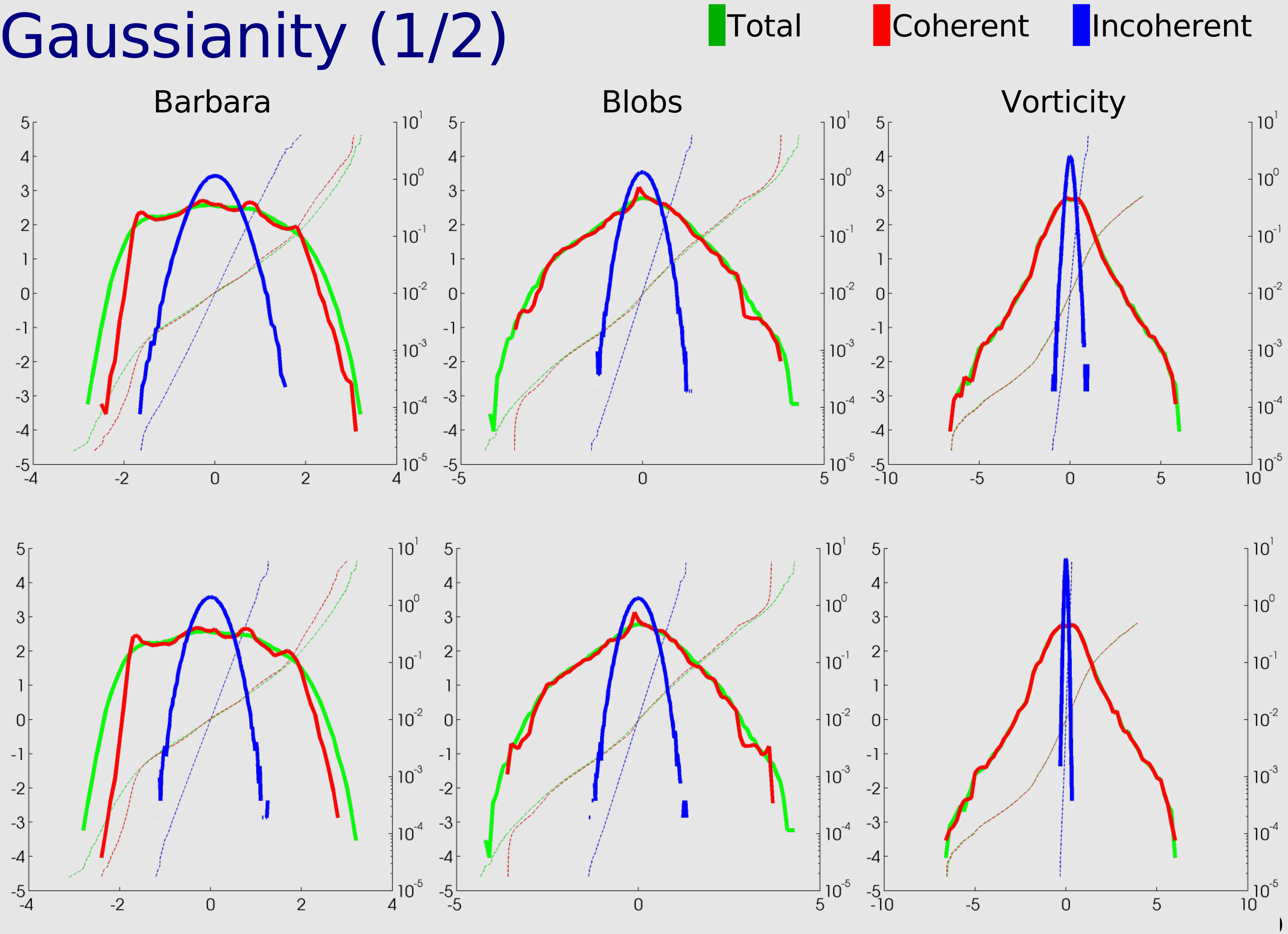
<u>Complex wavelets</u>





Qualitative features : local anisotropy, presence of long filaments

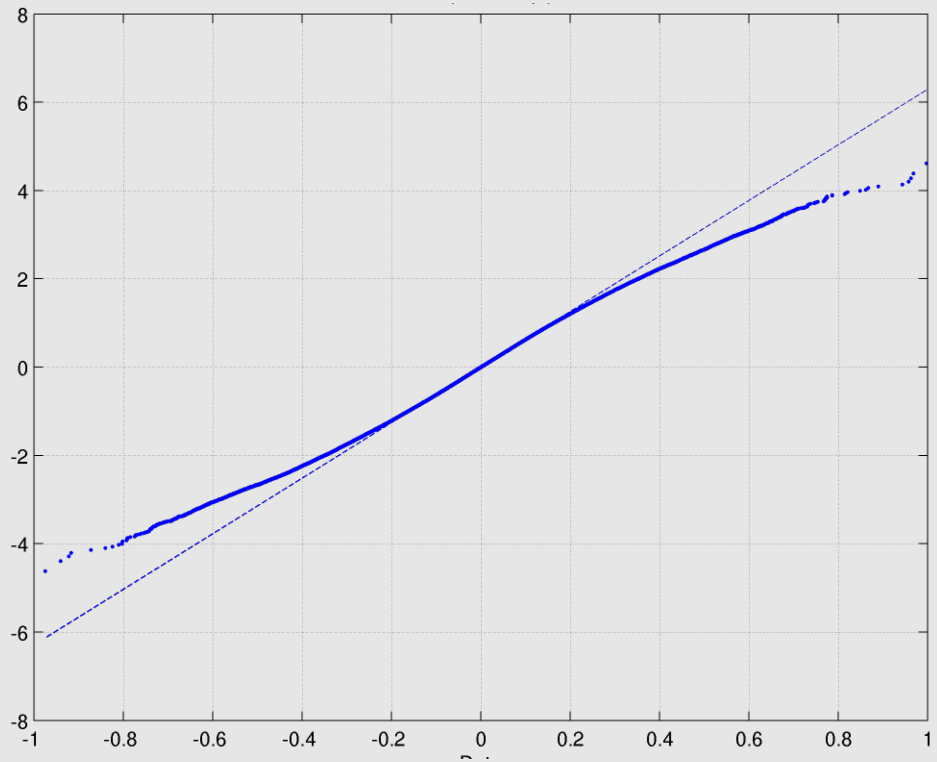
Gaussianity (1/2)



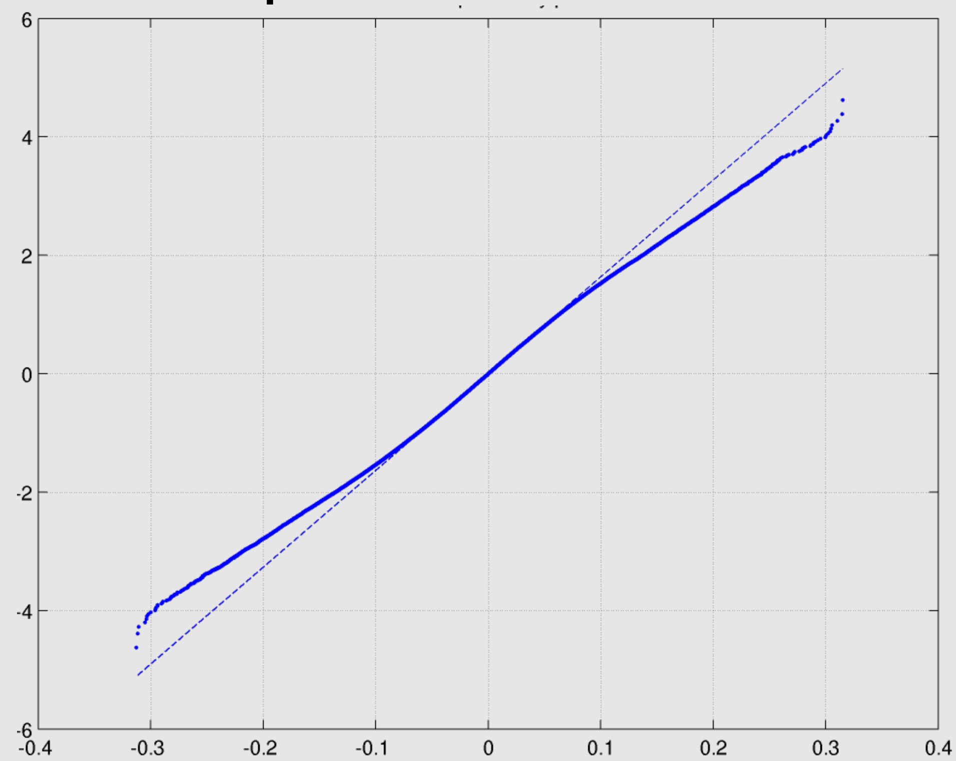
Gaussianity (2/2)

Total | Coherent | Incoherent

Real wavelets



Complex wavelets



	Blobs		Vorticity	
	Real	Complex	Real	Complex
Skewness (Total)	-0,18		0,03	
Skewness (Coherent)	-0,21	-0,21	0,03	0,03
Skewness (Incoherent)	0,00	0,00	0,00	-0,01
Flatness (Total)	3,48		5,74	
Flatness (Coherent)	3,56	3,56	5,92	5,83
Flatness (Incoherent)	3,00	3,00	3,62	3,50

Conclusion

- Complex wavelets have been applied to the extraction of coherent structures in turbulent flows.
- We have found an incoherent component which is closer to being Gaussian in the complex case, and displays new local anisotropy features.
- Translation invariance could have nice consequences that haven't checked here, for example preservation of local extrema.
- The DTCWT will be considered in future studies taking time evolution into account.
- In the end, lack of orthogonality is a serious issue

Papers on Wavelets and turbulence:
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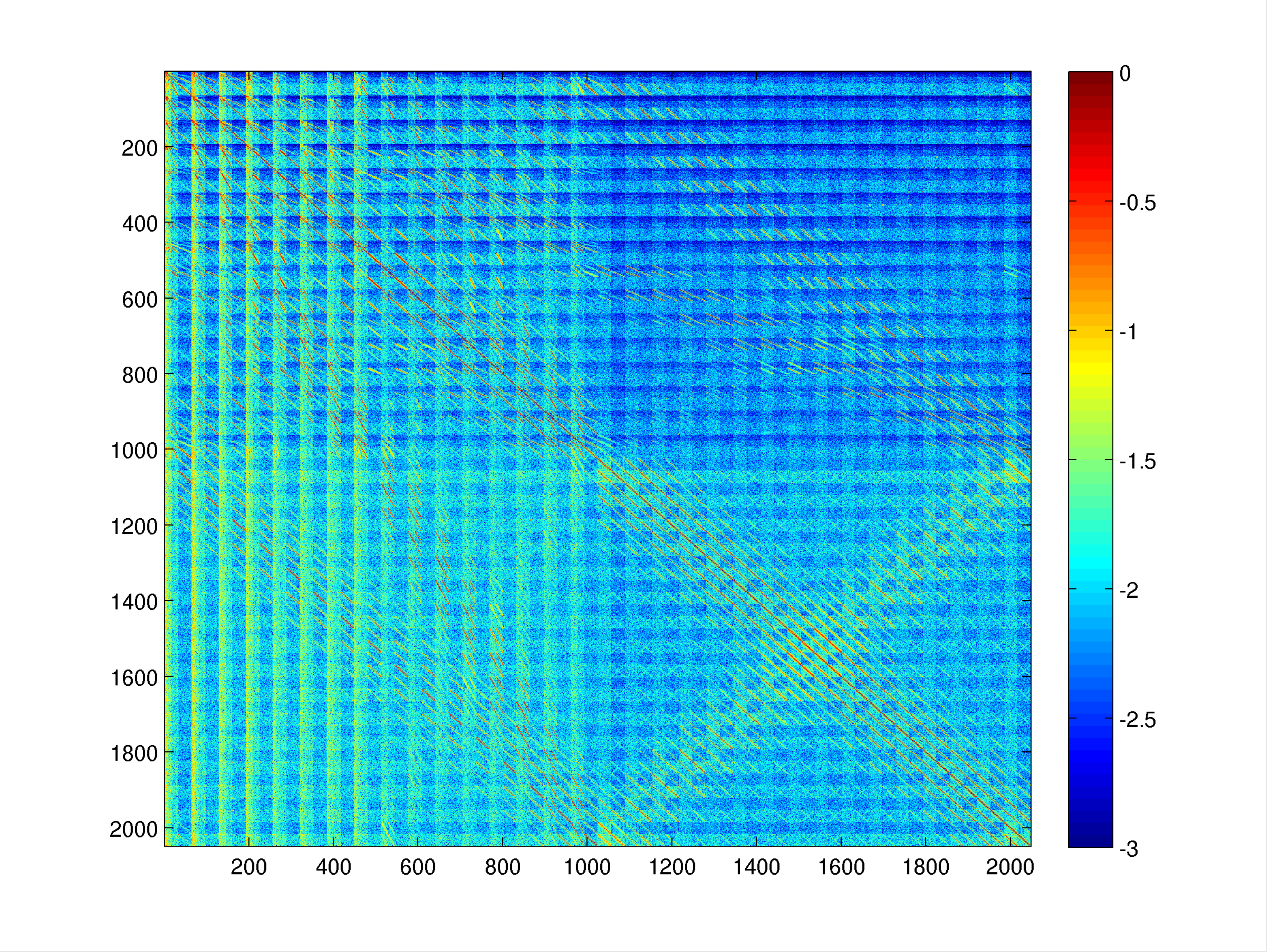
Additional slide (“decorrelation” with DTCWT)



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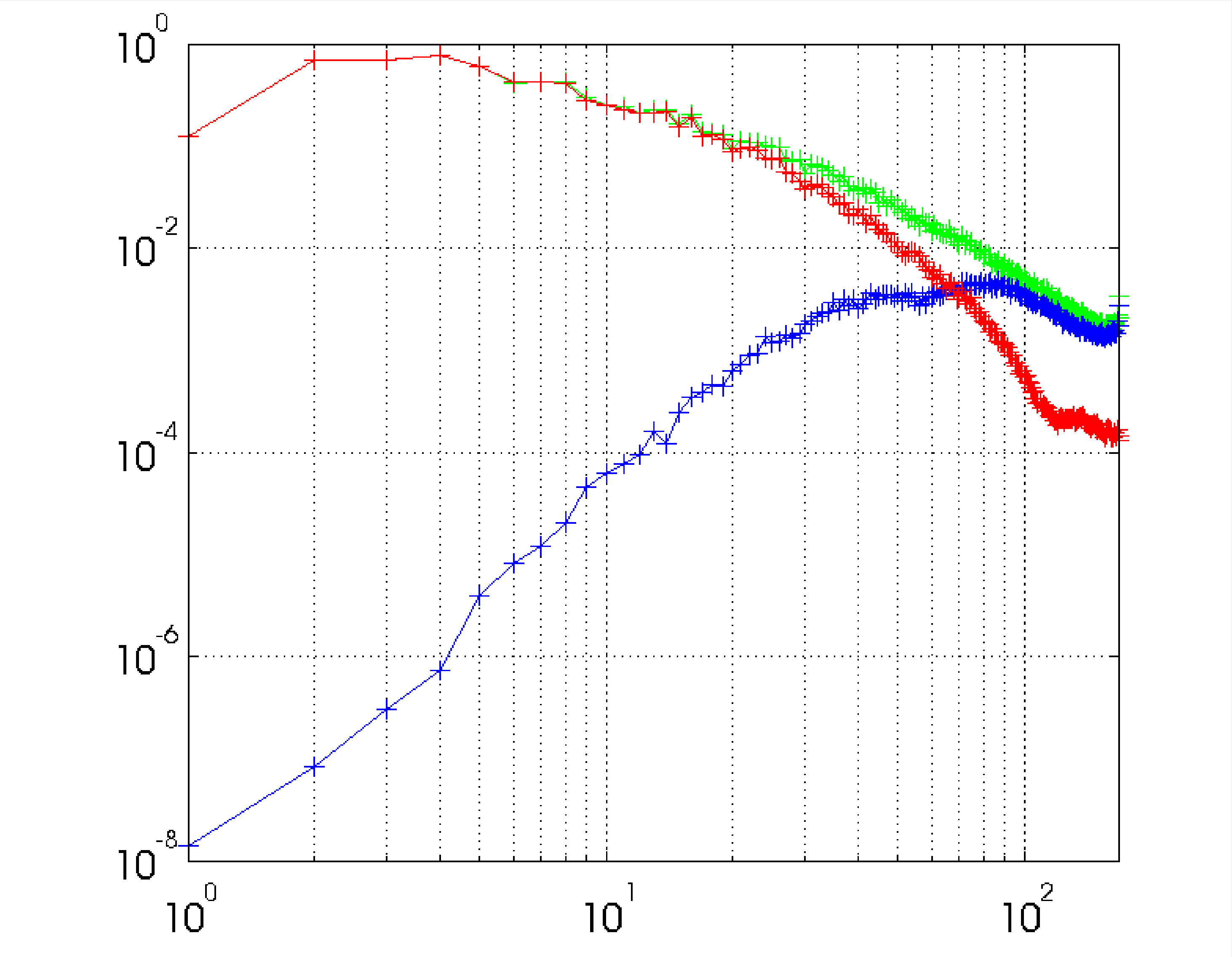
Additional slide (spectra with DTCWT)



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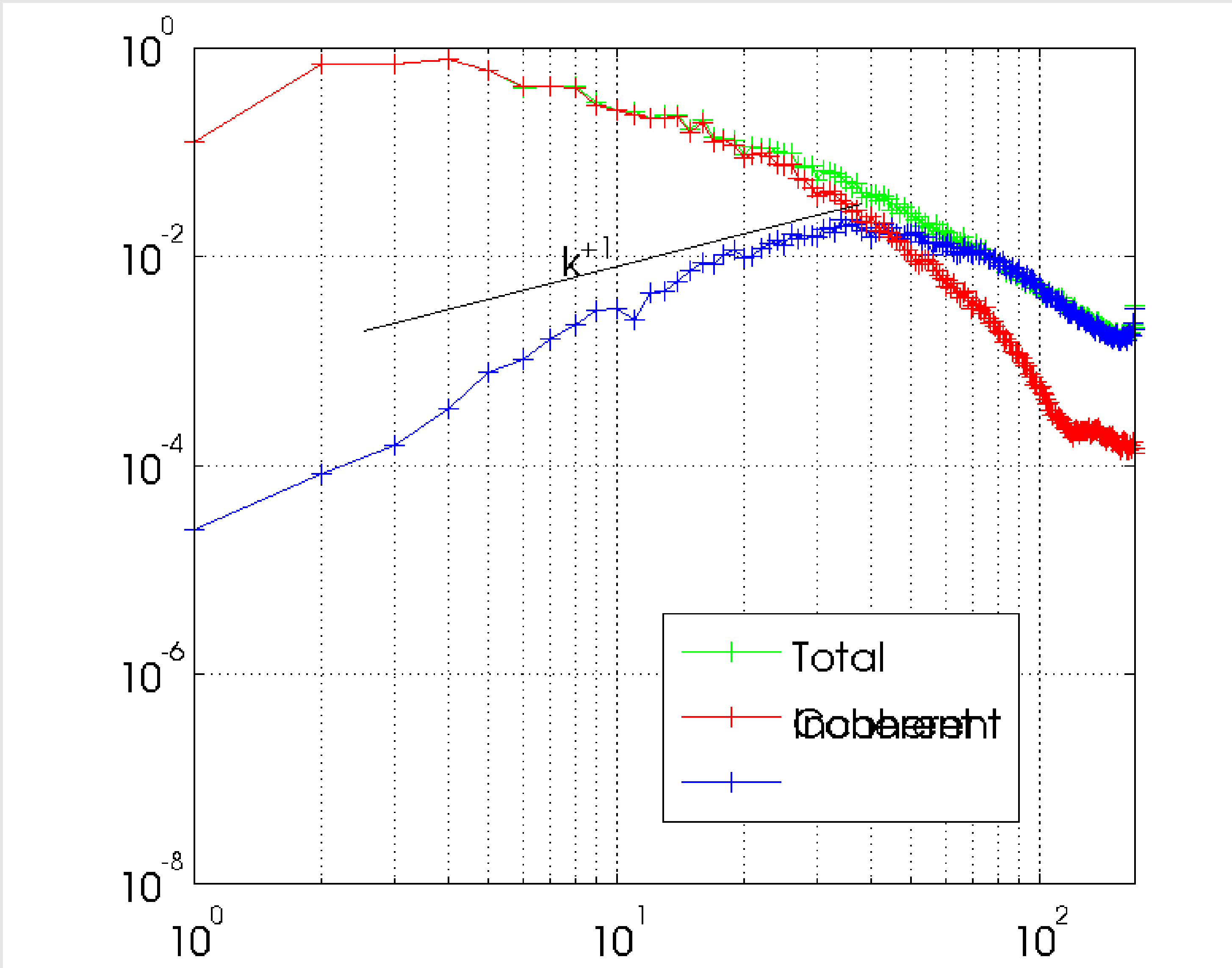
Additional slide (spectra with DWT)



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Definition of coherent structures

No tractable, widely accepted definition

Minimal hypothesis : coherent structures are not noise

Extracting coherent structures amounts to removing the noise

Hypotheses need (only) be made concerning the noise

Extraction of coherent structures

- Goals :
  - ➔ Understand the role played by coherent structures in the nonlinear dynamics of fully developped turbulent flows
  - ➔ Isolate as few degrees of freedom as possible while keeping all the relevant information necessary to numerically simulate these flows
- Means :
  - ➔ wavelet-based denoising algorithms

Role of wavelets

- **Compression**: we expect coherent structures to have some components at all scales, but to be highly intermittent. This means that they will be <u>sparse</u> iin wavelet space.
- **Efficiency**: we rely on the fast wavelet transform to be able to compute the decomposition at every time step.
- **Fixed basis**: for numerical simulations it is important that the basis is known in advance

Now, we are going to compare two wavelet families.