Dissipation by flows

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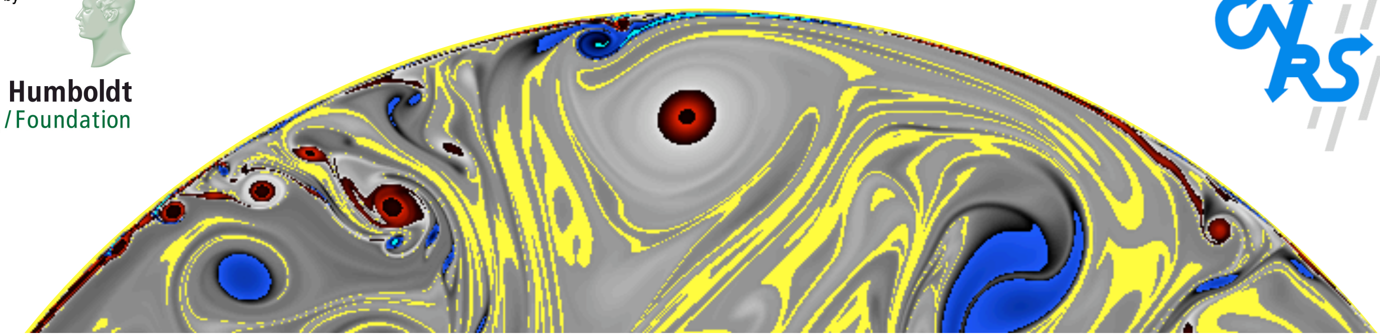

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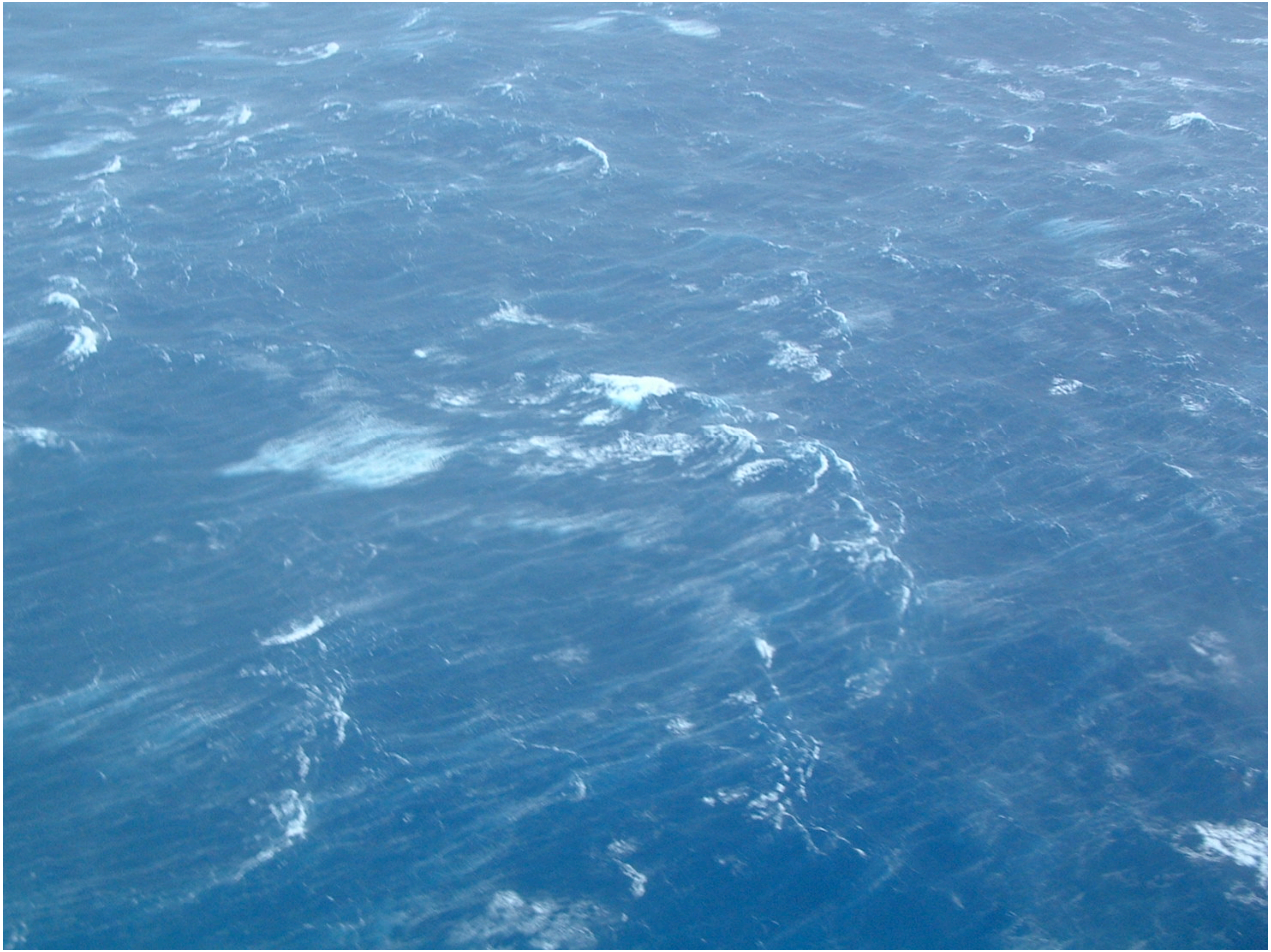
Outline

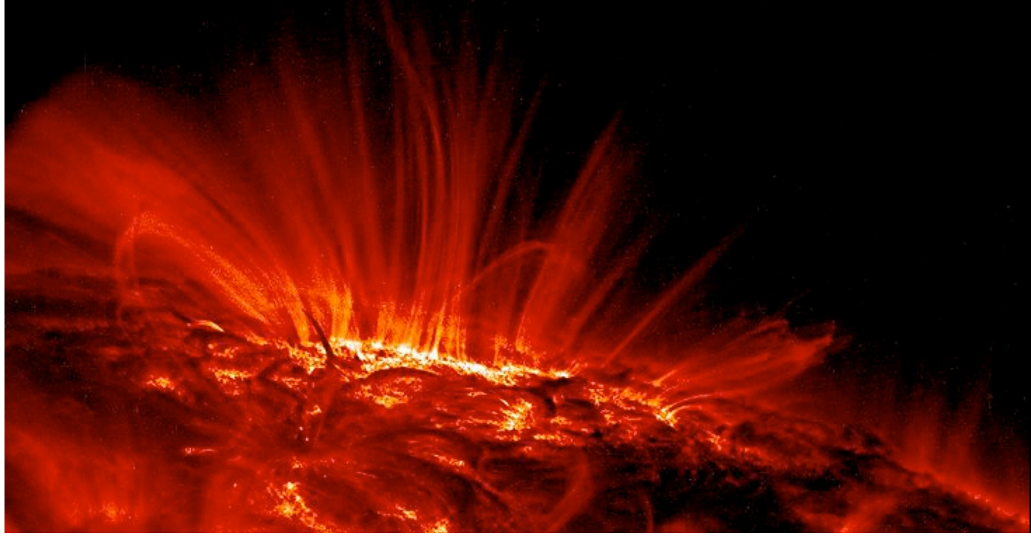
1. Introduction: dissipative singularities, macroscopic randomization, and wavelets.
2. 2D periodic Navier-Stokes turbulence
   - Methodology
   - Molecular dissipation
   - Wavelet-based macroscopic dissipation
3. 2D wall-bounded Navier-Stokes (1 slide)

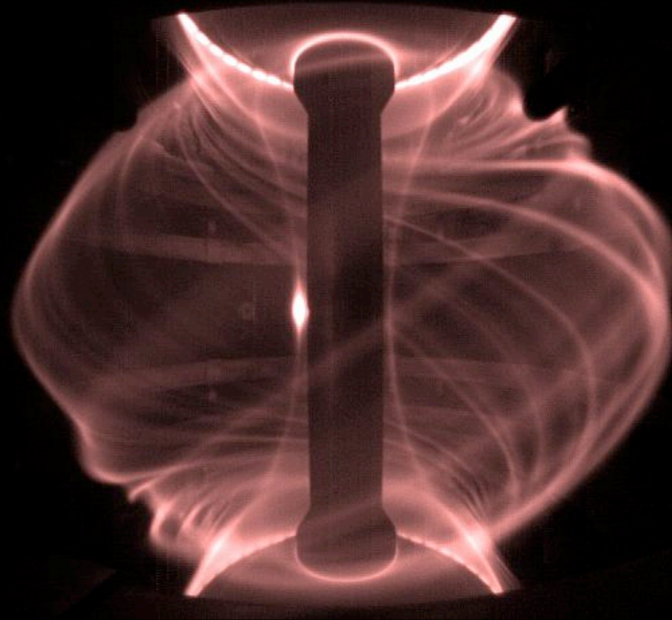



Biker in a wind tunnel


Earth (Apollo 17)

Ultraviolet sun (TRACE, NASA)




Mast tokamak (CCFE, UK)

A first attempt: molecular dissipation

- Flows = collective motions of many particles described macroscopically.
- Prediction usually impossible from principles of Lagrangian mechanics.
- But statistical assumptions are possible.
- The main statistical assumption is the closeness to a certain statistical equilibrium (local thermodynamic equilibrium, molecular chaos…),
- Global relaxation can usually be proved (growth of entropy),
- This phenomenon is called **molecular dissipation.**

R. Balian, *From microphyics to macrophysics*, Springer (2006)

Not the end of the story

- For example in incompressible Navier-Stokes this leads to the equation:

$$\frac{\mathrm{d}S_L}{\mathrm{d}t} = \boxed{\frac{\rho\nu}{T}} \int_\Omega |\nabla \mathbf{u}|^2$$

This coefficient may be very small !

- In practice relaxation often occurs on time-scales that are **independent on microscopic coupling coefficients**.

1. Singul… e coupling coeffic… ation

2. Micro… ion mecha… er (possi… equili…

3. A com…

Exampl… ropy

Exampl… systems.

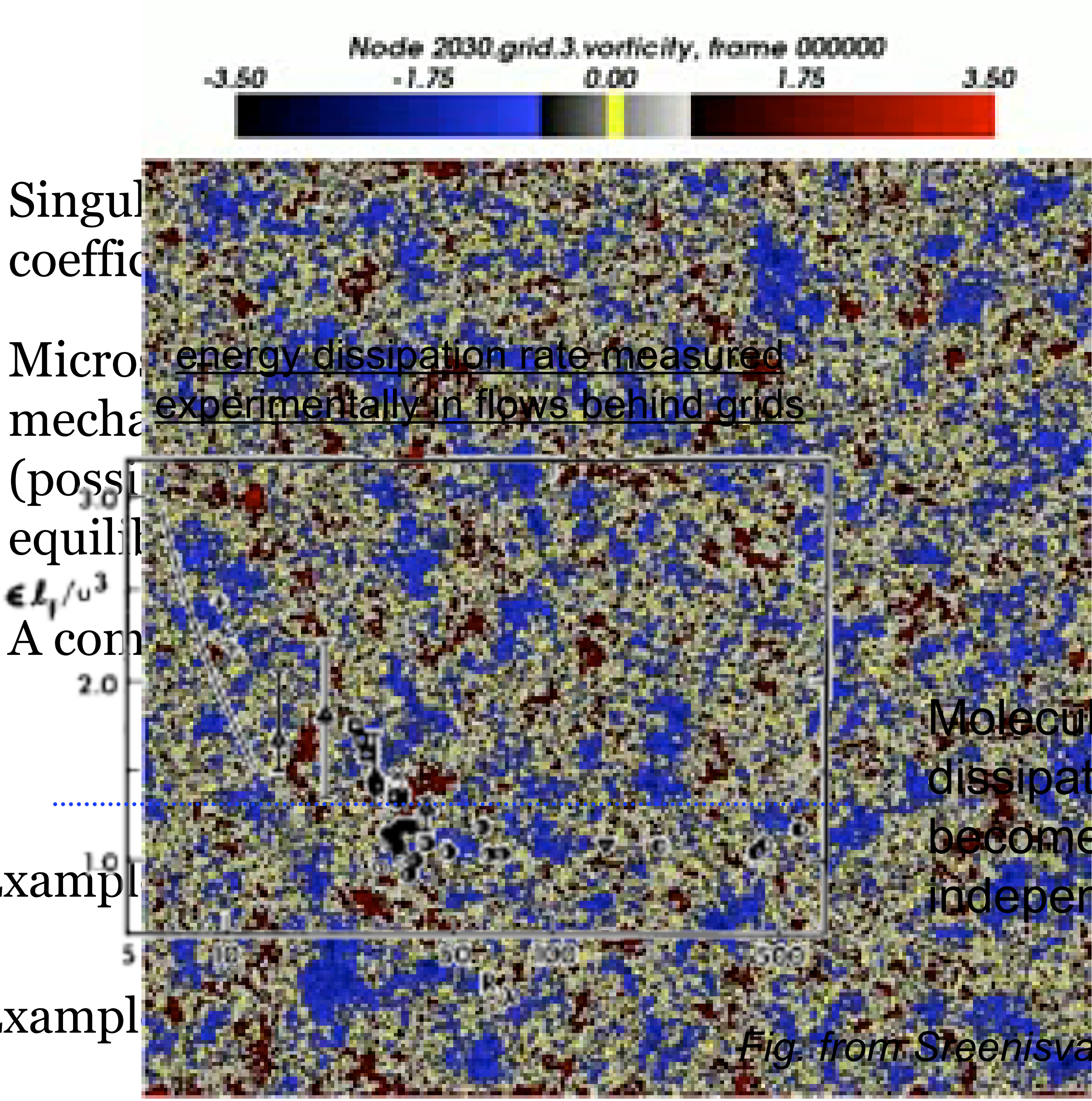
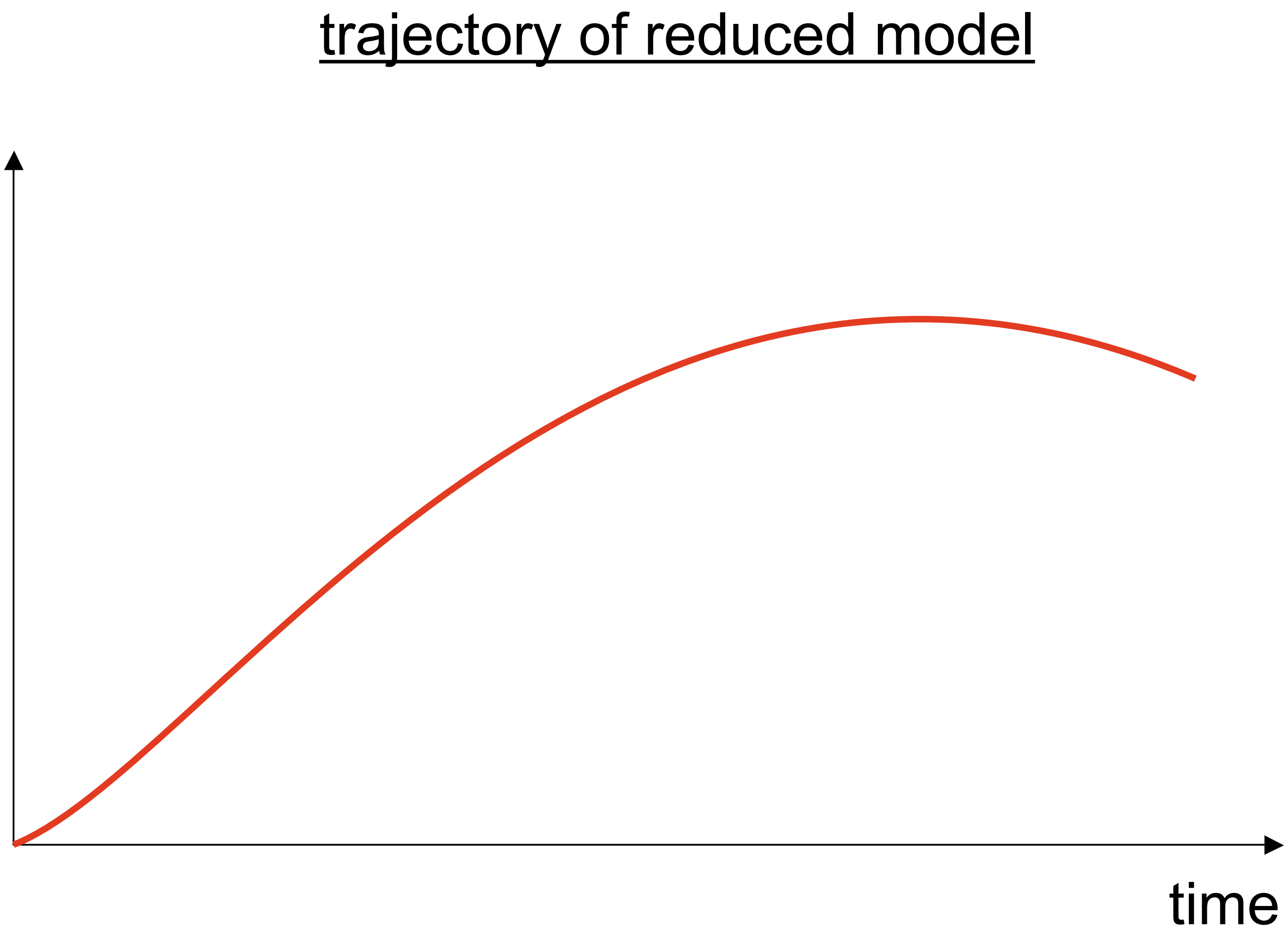


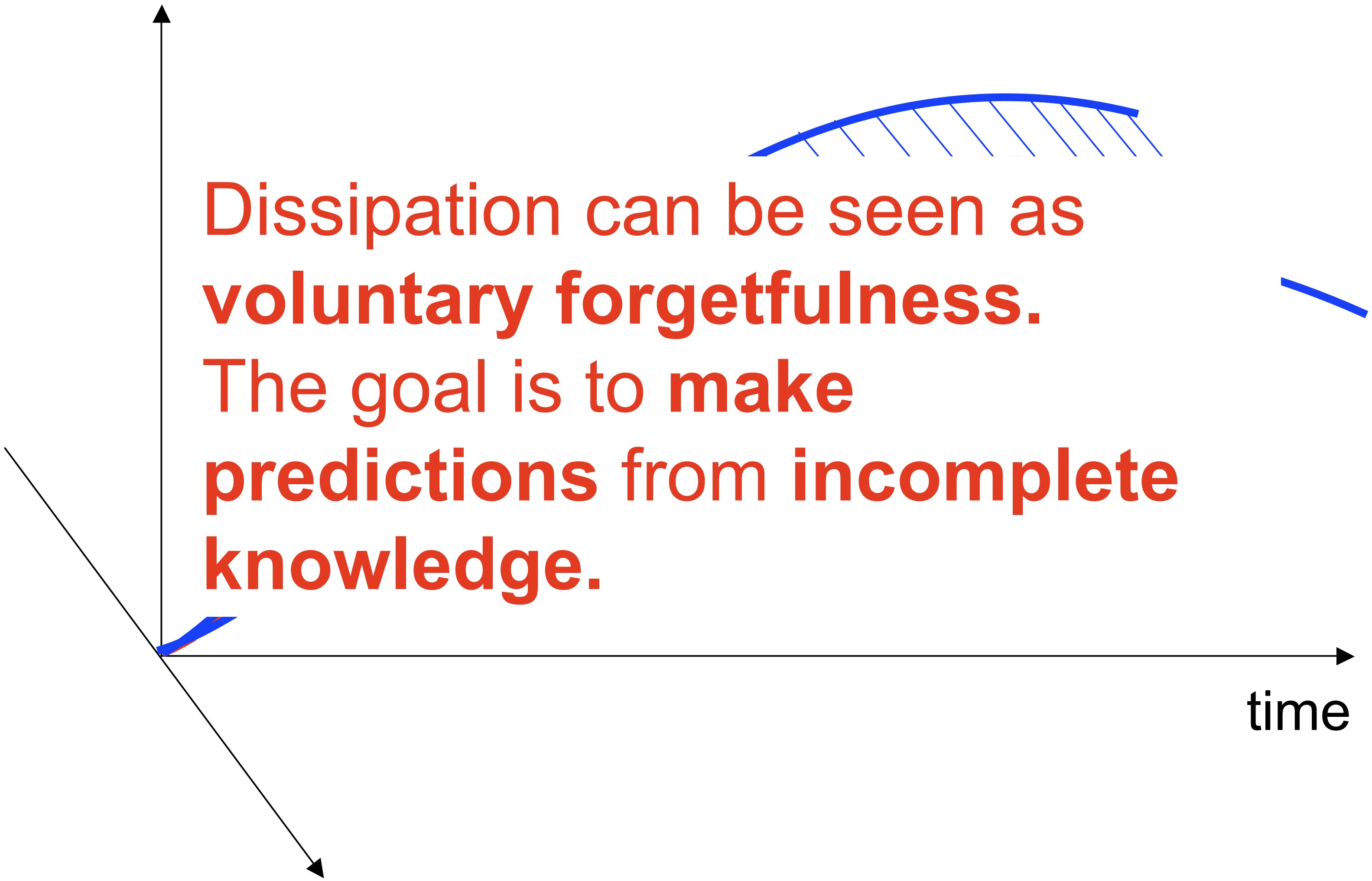
Fig. from Sreenisvasan (1984)

Dissipation as randomization



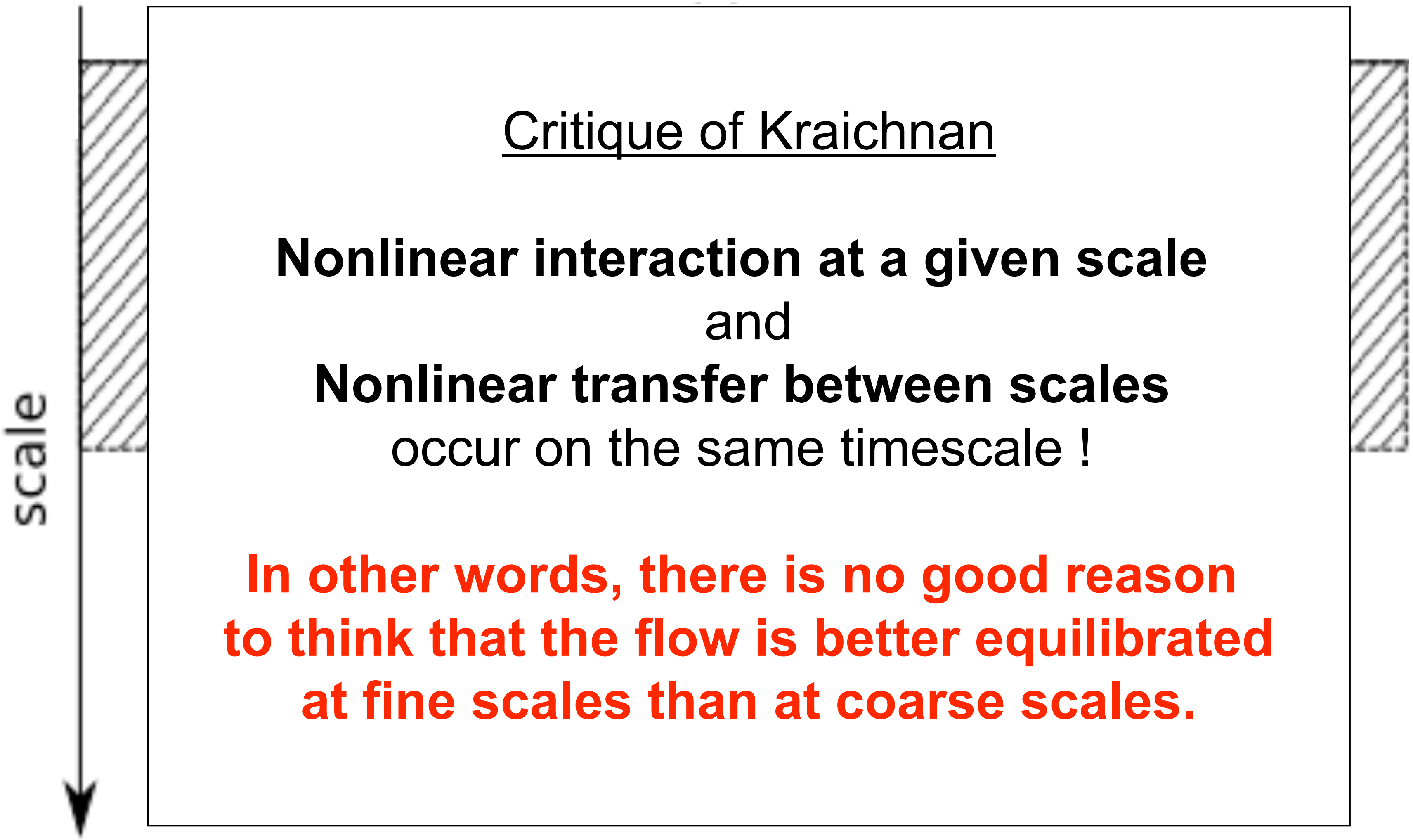
Dissipation as randomization

trajectories of a more complete model



Dissipation can be seen as **voluntary forgetfulness.** The goal is to **make predictions** from **incomplete knowledge.**

Flow dissipation seen in Fourier space



<u>Critique of Kraichnan</u>

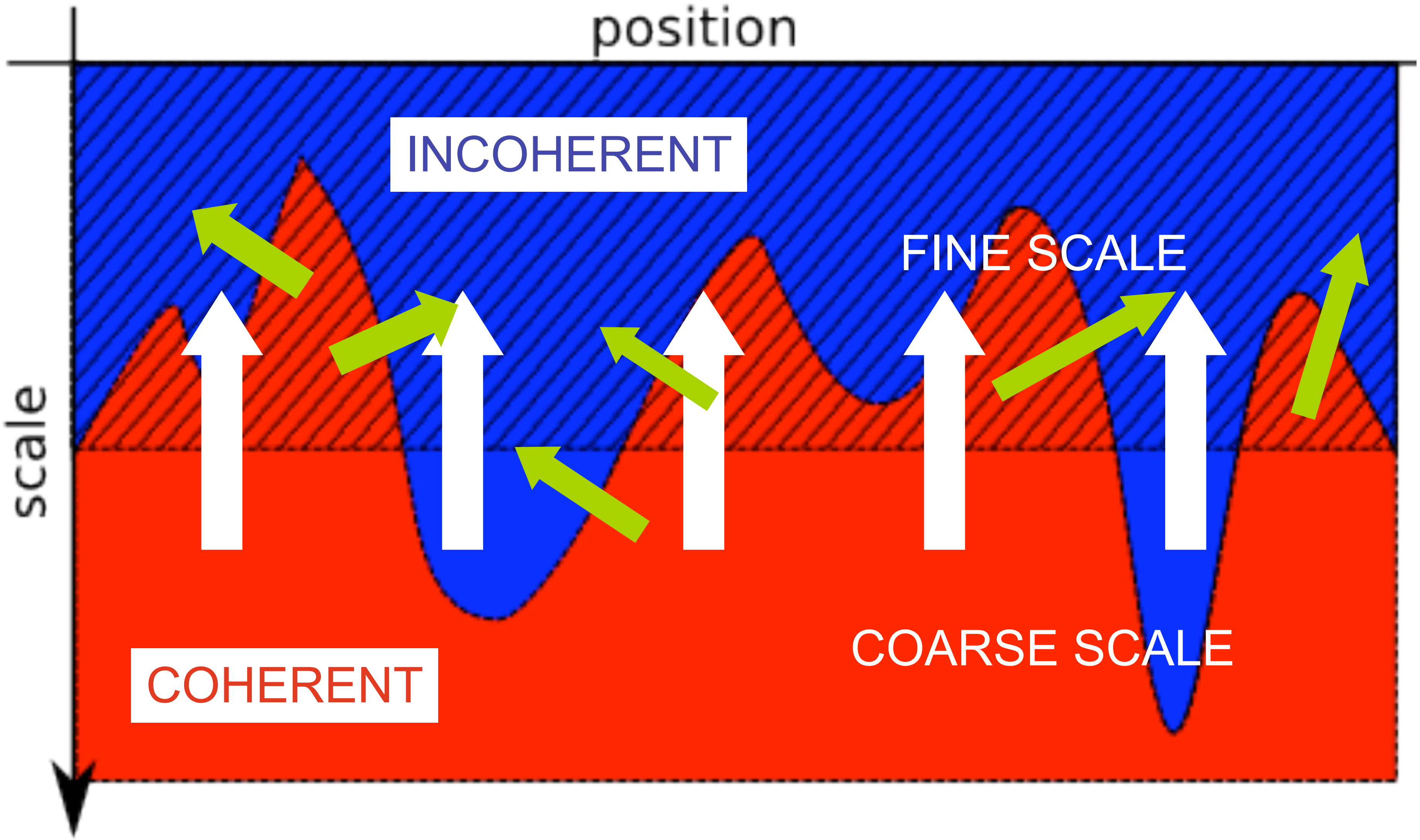
Nonlinear interaction at a given scale
and
Nonlinear transfer between scales
occur on the same timescale !

In other words, there is no good reason to think that the flow is better equilibrated at fine scales than at coarse scales.

L.F. Richardson, *Diffusion regarded as a compensation for smoothing* (1930)

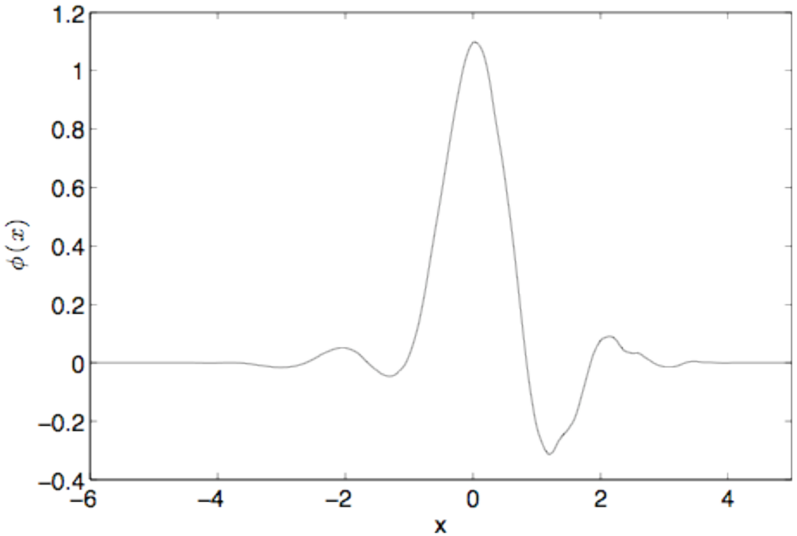
R.H. Kraichnan, *On Kolmogorov's inertial range theories* (1974)

Flow dissipation seen in wavelet space

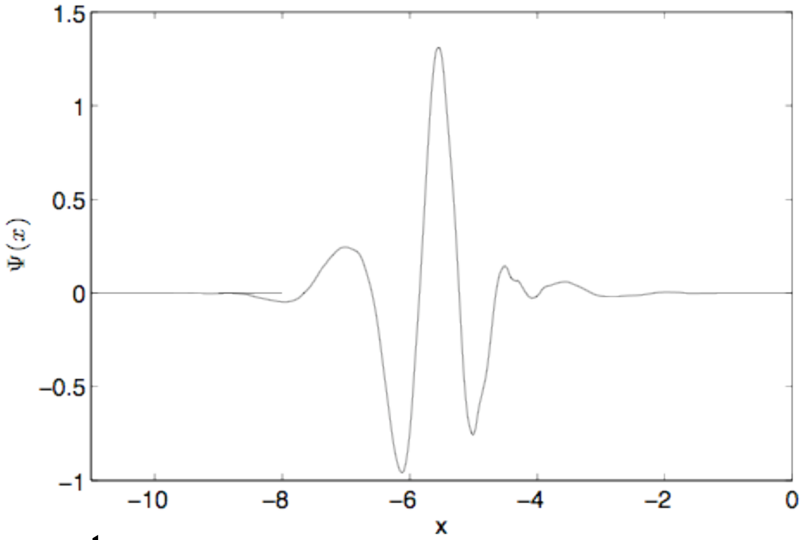


Orthogonal wavelet bases

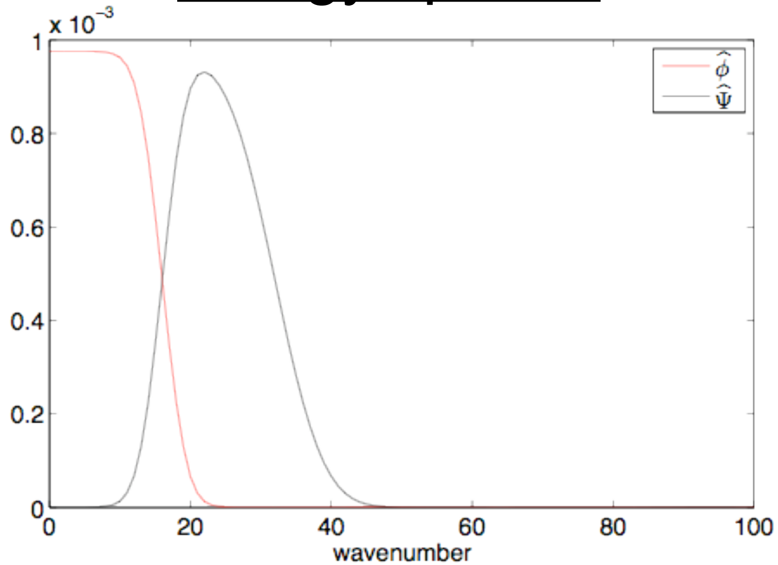
scaling function



wavelet



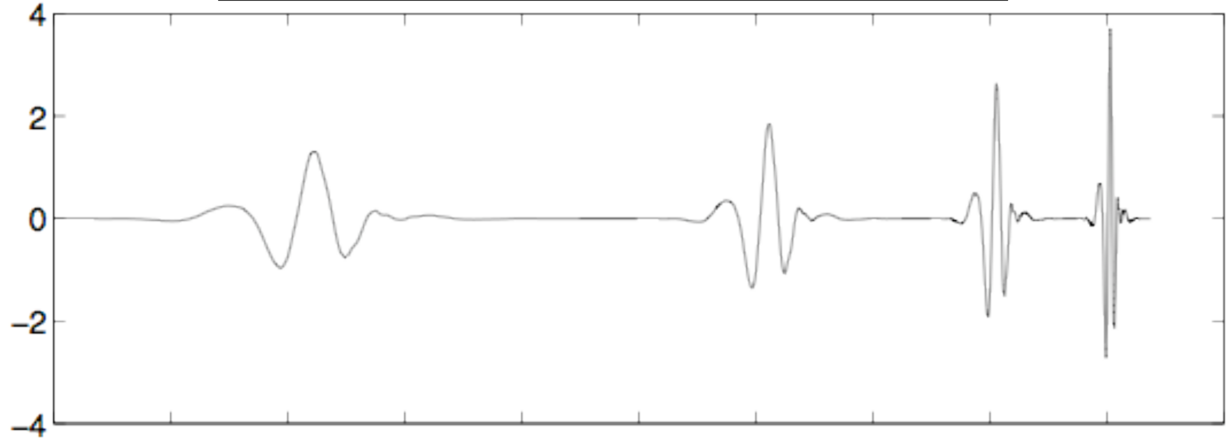
energy spectra



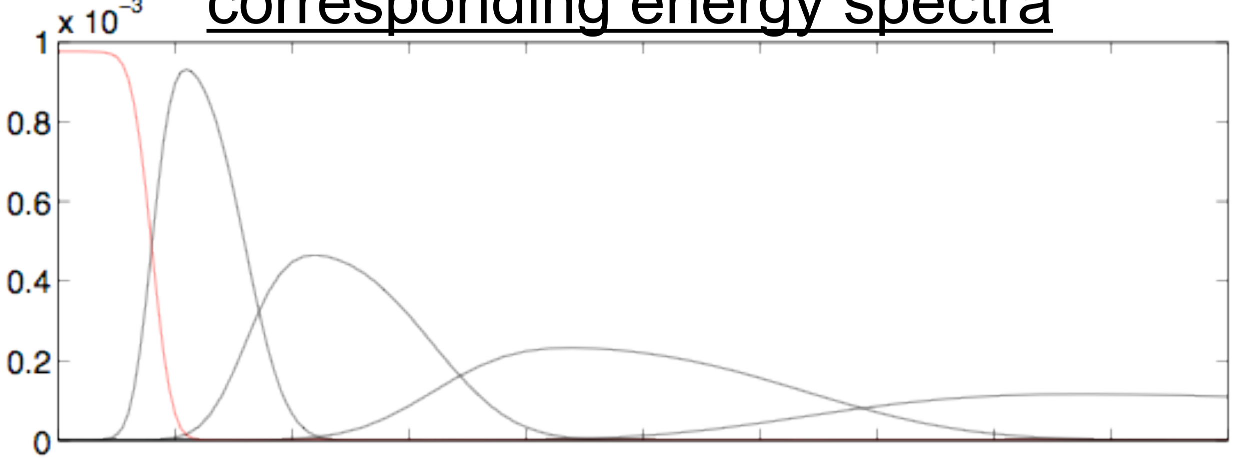
Orthogonal wavelet bases

$$\psi_\lambda(x) = 2^{j/2}\psi(2^j x - i)$$

dilated / translated wavelets

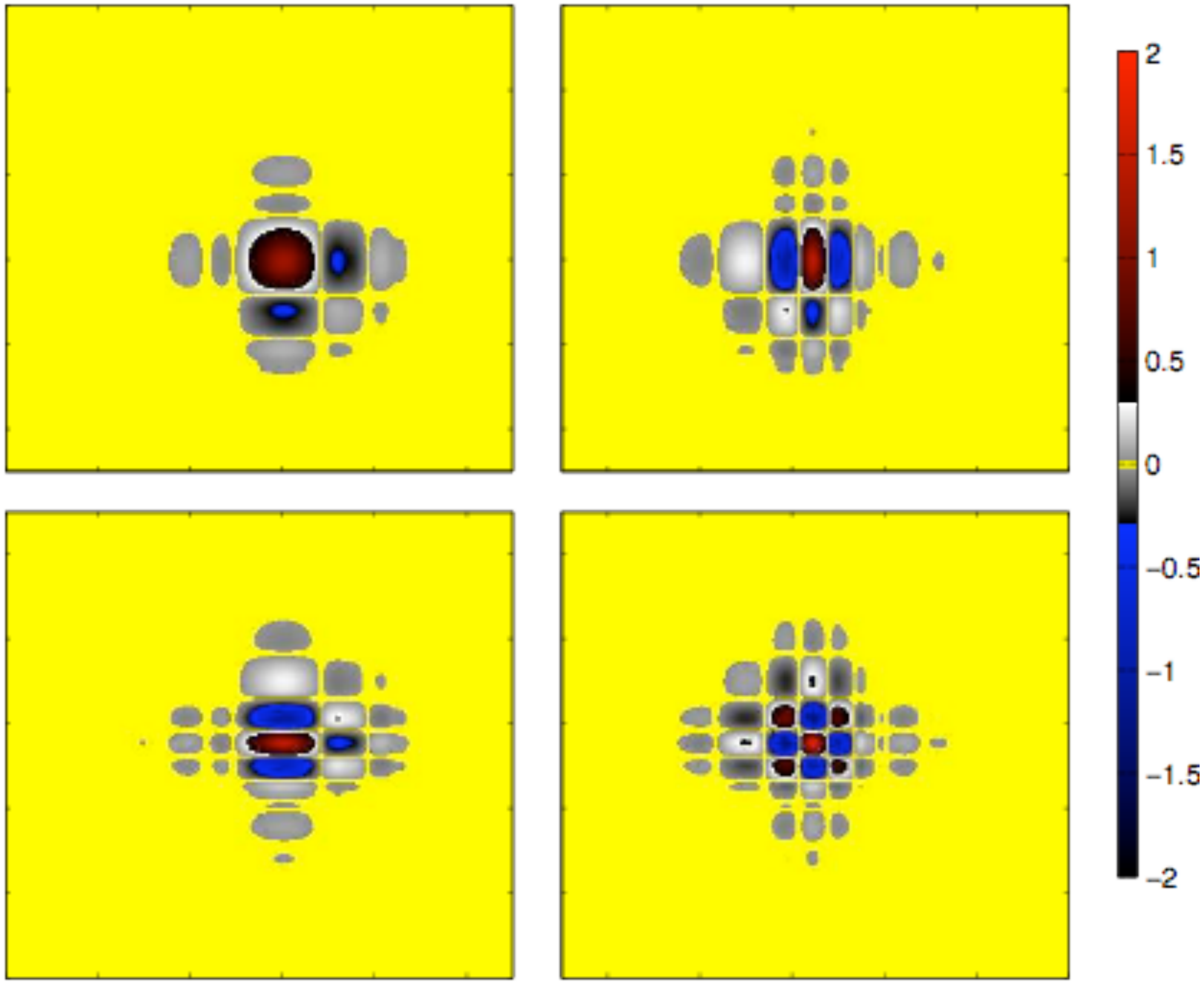


corresponding energy spectra



Orthogonal wavelet bases

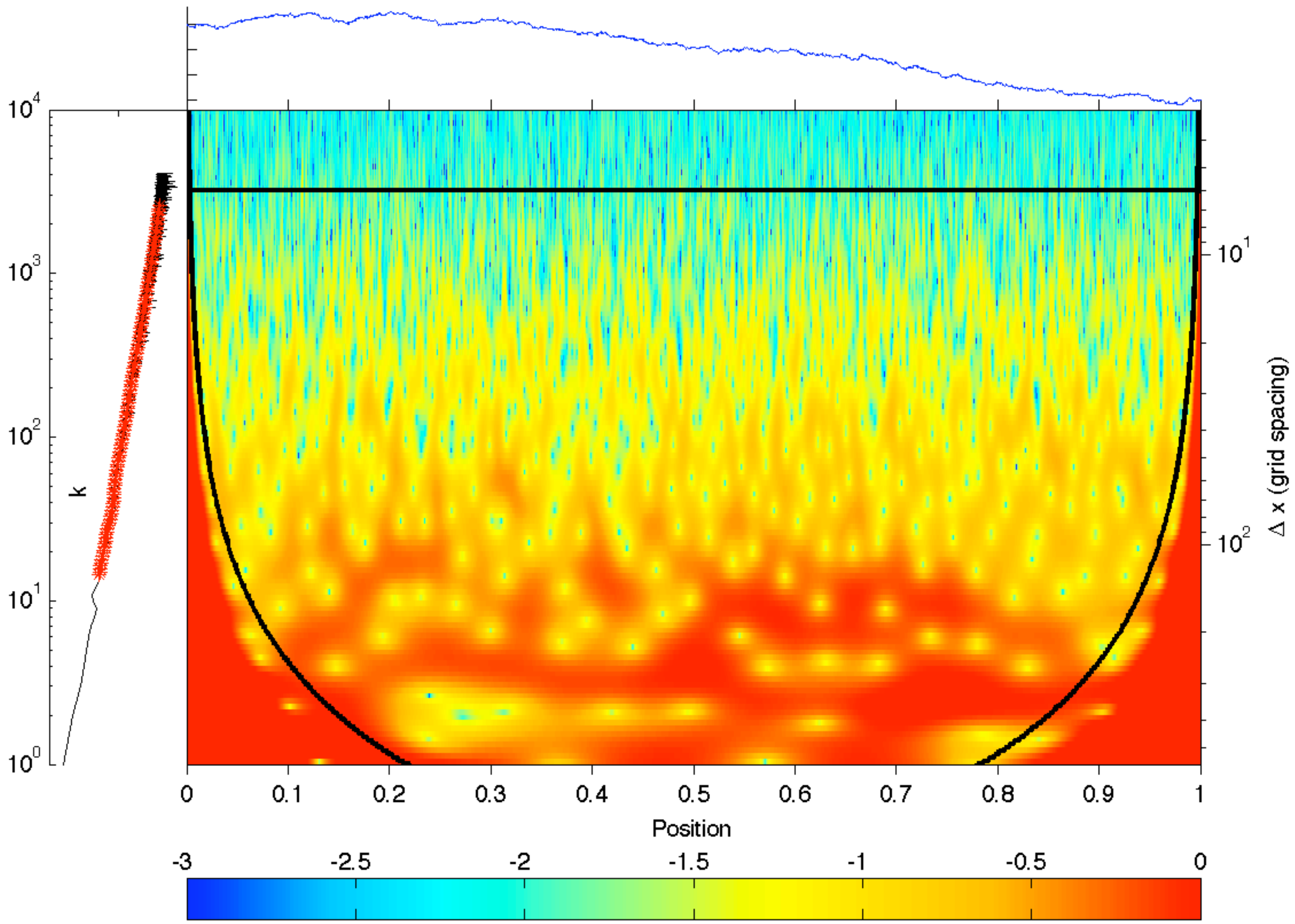
2d scaling function and wavelets



Wavelets and spatial localization

Brownian motion

Two-step function



Wavelet nonlinear thresholding

- Orthogonal wavelet decomposition:

$$f = \sum_{\lambda \in \Lambda} \widetilde{f}_\lambda \psi_\lambda \text{ , where } \widetilde{f}_\lambda = \int_{\mathbb{T}^2} f \psi_\lambda$$

- Idea: split wavelet coefficients between two sets, "**large** coefficients" and "**small** coefficients":

$$f = \sum_{\lambda \in \Lambda_C} \widetilde{f}_\lambda \psi_\lambda + \sum_{\lambda \in \Lambda_I} \widetilde{f}_\lambda \psi_\lambda$$

Where large and small are defined with respect to a certain threshold: $|\widetilde{f}_\lambda| > \Theta_\lambda$ or $|\widetilde{f}_\lambda| \leq \Theta_\lambda$

D. Donoho & I. Johnstone (1992), M. Farge, N. Kevlahan, K. Schneider (1999)

Navier-Stokes initial-boundary value problem

<u>Equation</u>

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}}\Delta\mathbf{u} \\ \nabla\cdot\mathbf{u} = 0 \end{cases}$$

$$\mathrm{Re} = \frac{UL}{\nu}$$

(no body forces → decaying flow)

<u>Boundary conditions</u> : periodic

<u>Initial conditions</u>

$$\mathbf{u}(\mathbf{x}, t=0) = \mathbf{u}_0$$

In 2D this is a well-posed problem

Molecular dissipation in 2D Navier-Stokes

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p + \frac{1}{\mathrm{Re}} \Delta \mathbf{u} \\ \boldsymbol{\nabla} \cdot \mathbf{u} = 0 \end{cases}$$

energy $E(t) = \dfrac{1}{2} \displaystyle\int_\Omega \mathbf{u^2}$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -2\nu Z$$

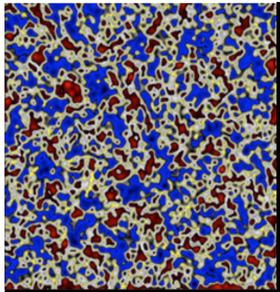
enstrophy $Z(t) = \dfrac{1}{2} \displaystyle\int_\Omega \omega^2$

$\omega = \boldsymbol{\nabla} \times \mathbf{u}$

vorticity

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = -2\nu P + \frac{\nu}{2} \oint_{\partial\Omega} \boldsymbol{\nabla} \boldsymbol{\omega}^2 \cdot \mathbf{n}$$

$P(t) = \frac{1}{2} \int_\Omega |\boldsymbol{\nabla} \omega|^2$

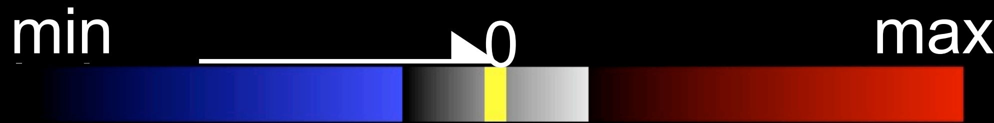


Re ≈ 1

Re ≈ 6

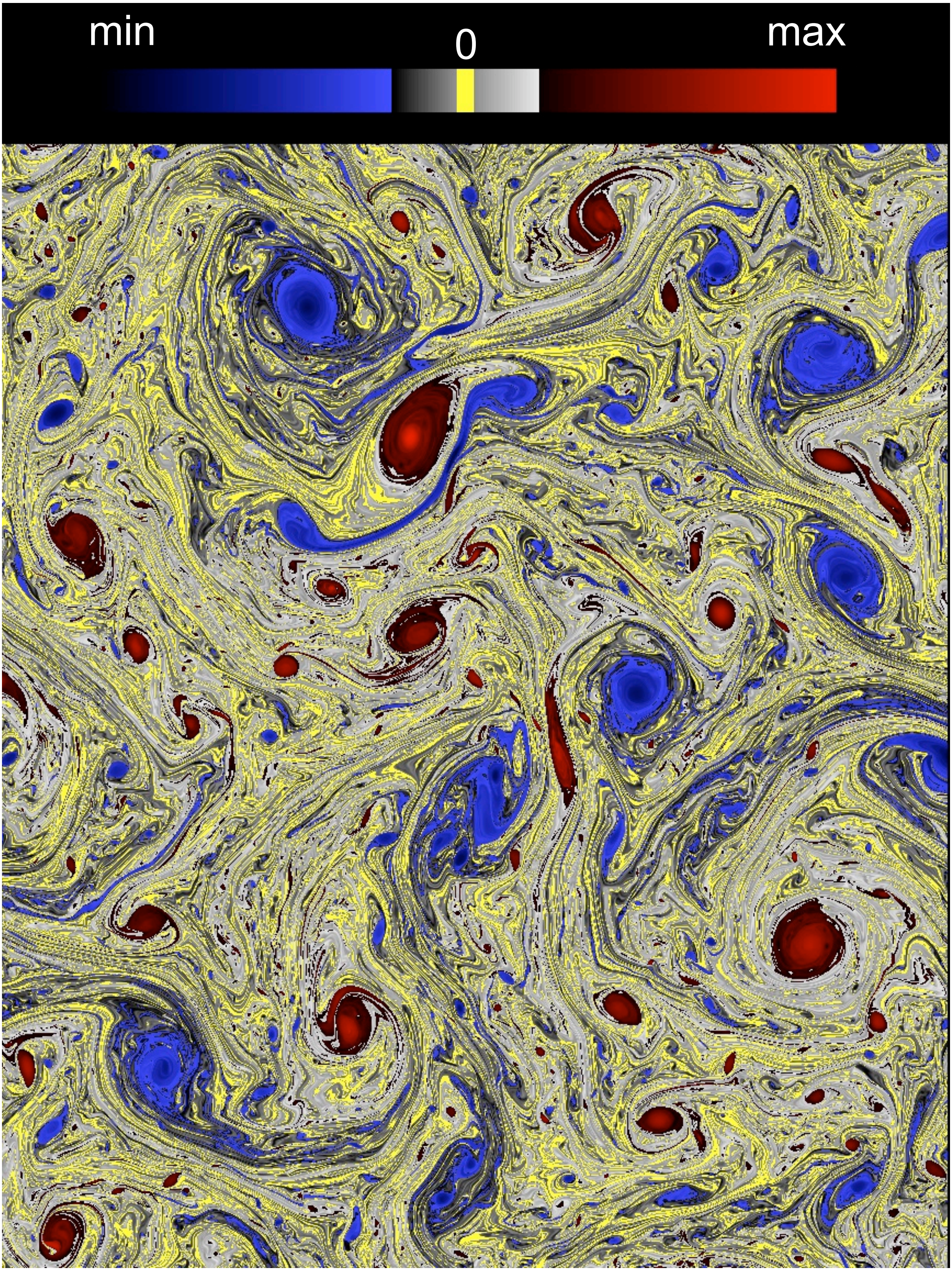
Re ≈ 2

time

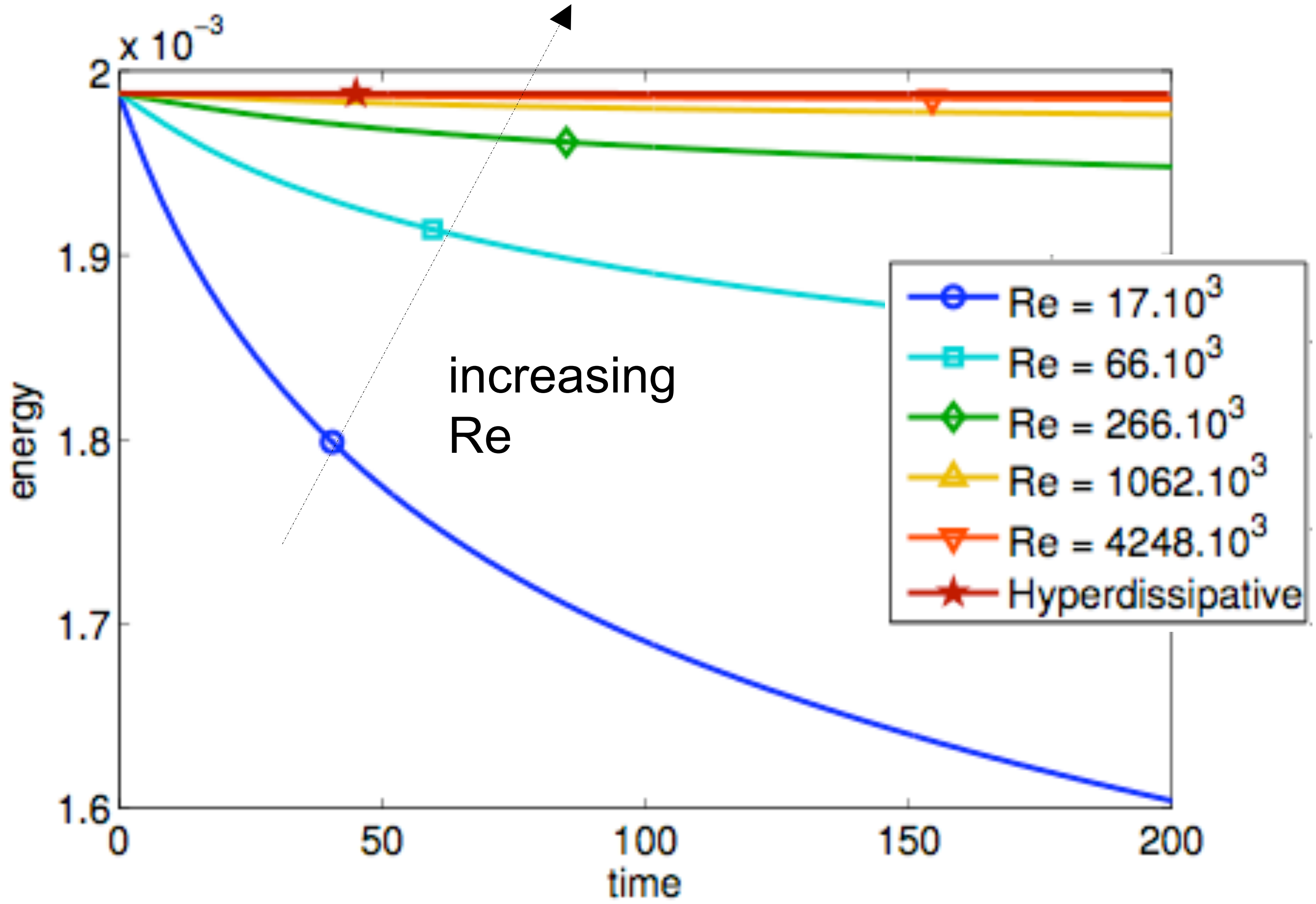


Re ≈ 1

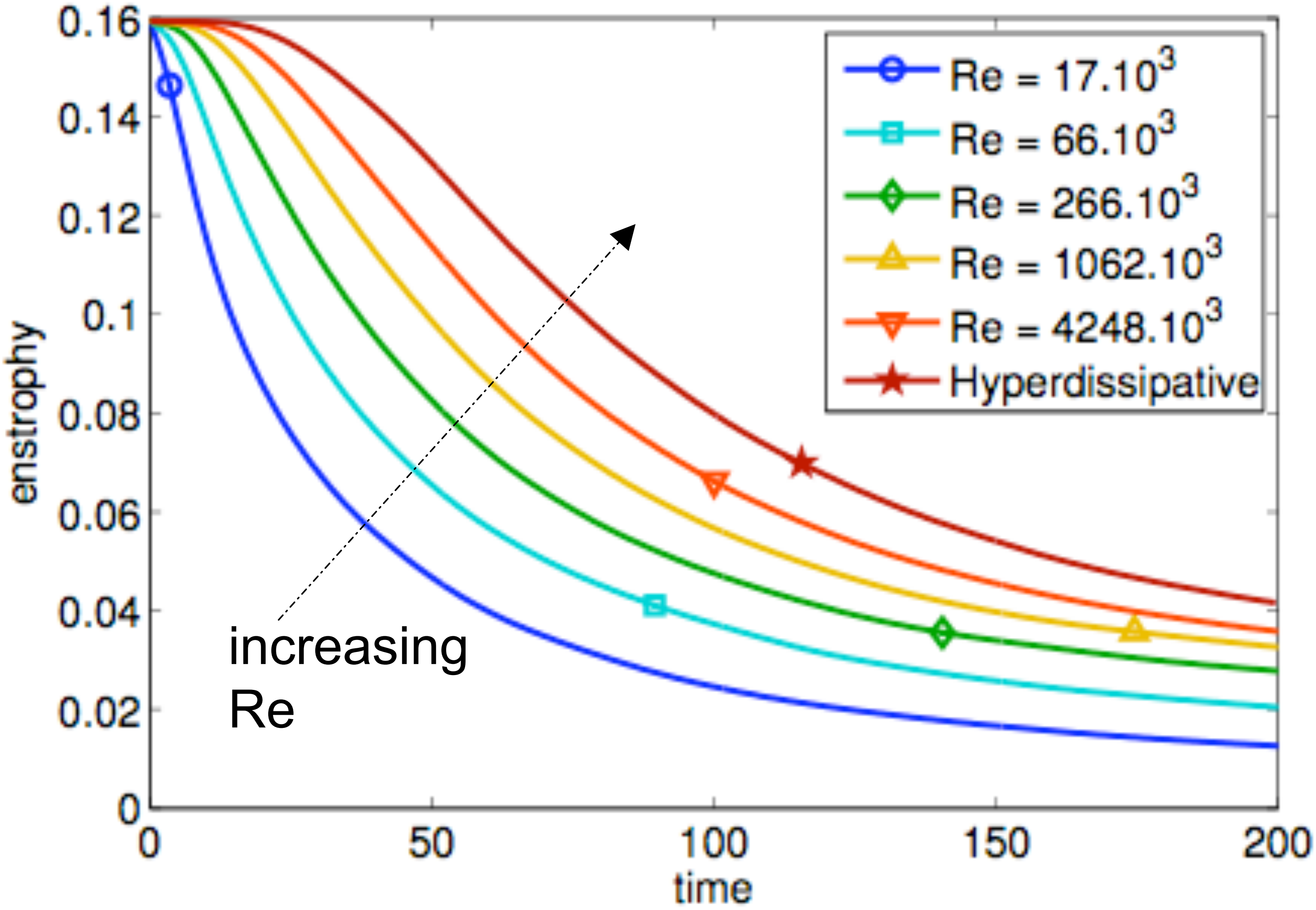
Re ≈ 4


min
0
max

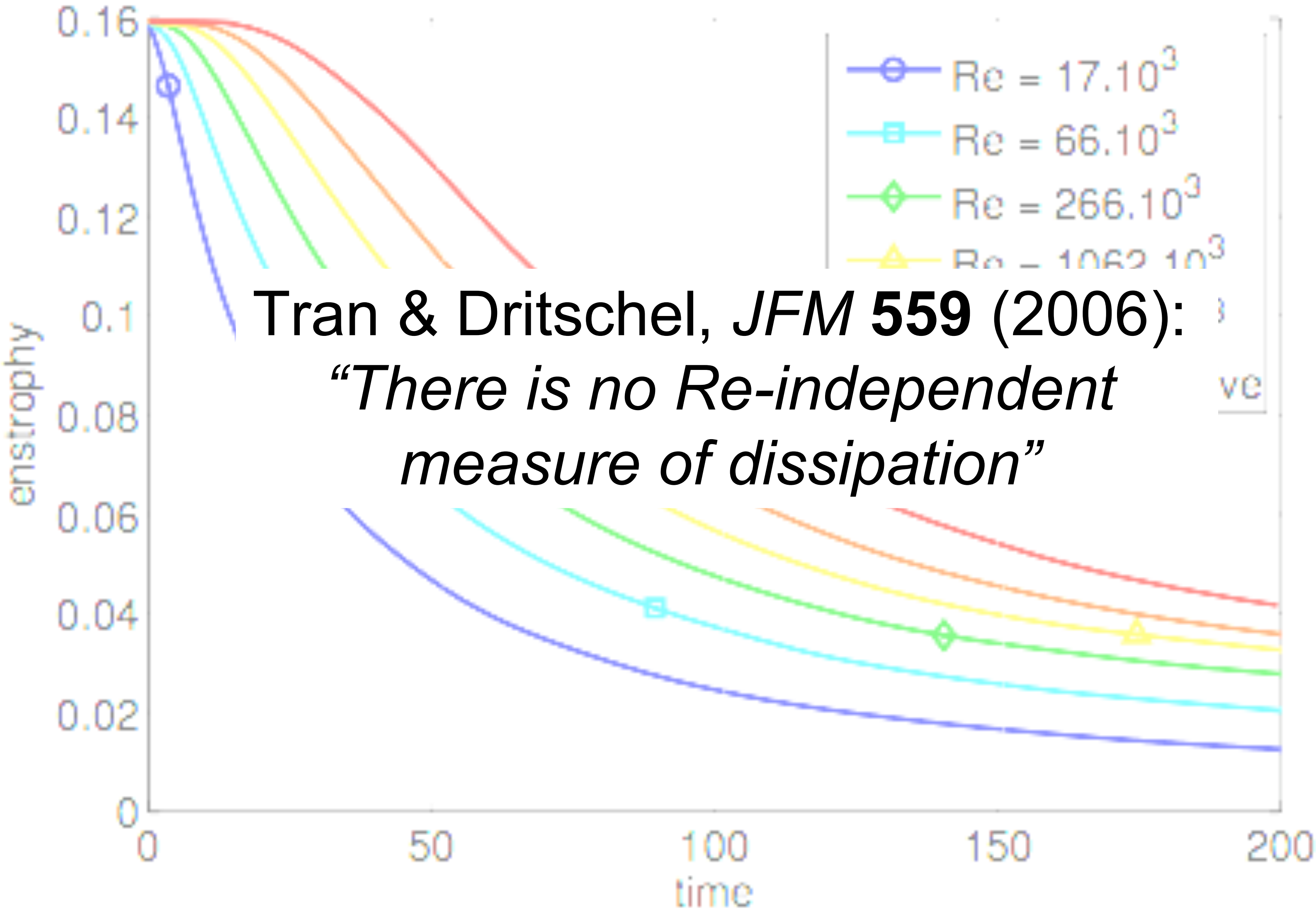
Energy dissipation at vanishing viscosity?



Enstrophy dissipation at vanishing viscosity?

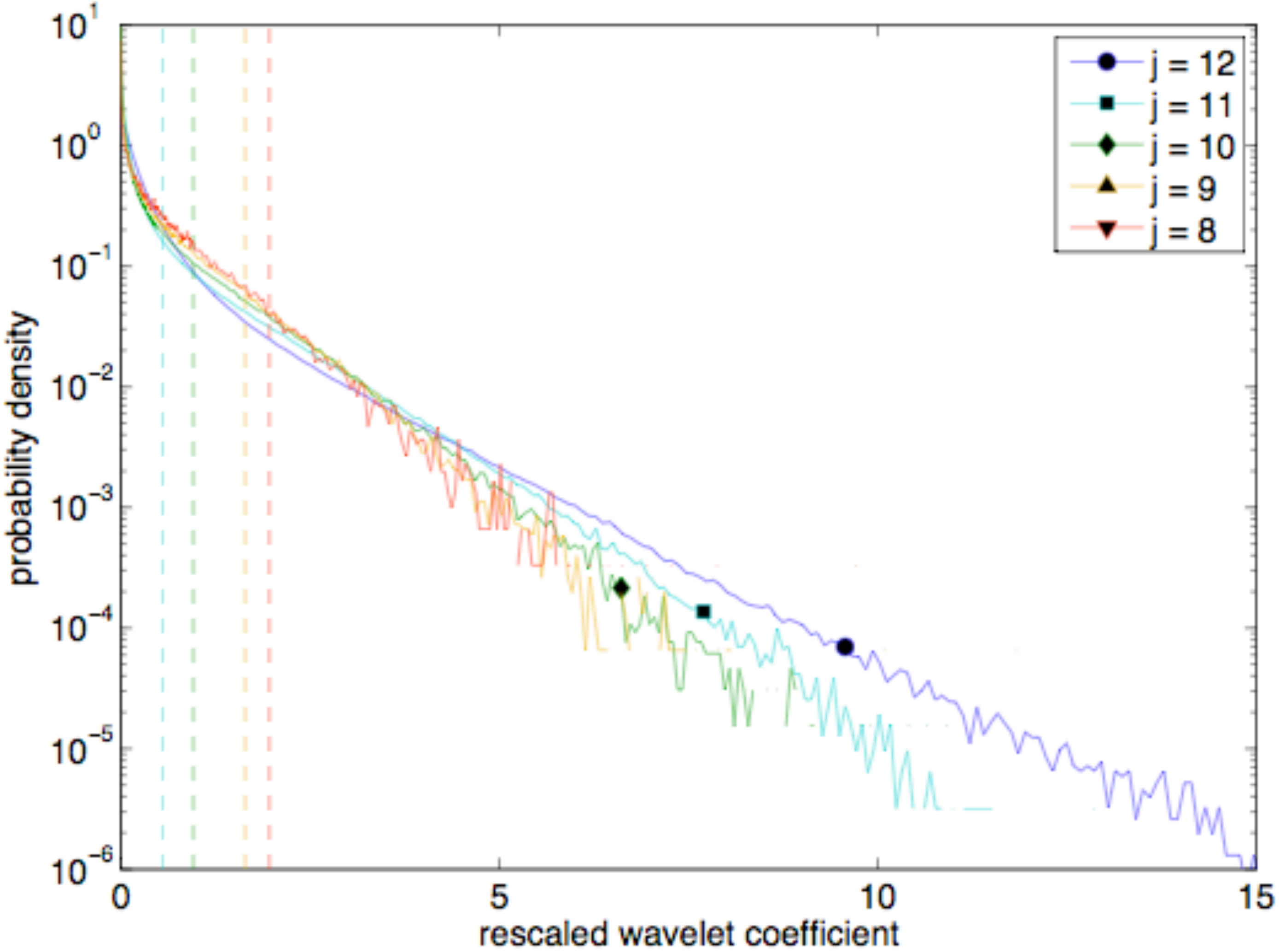


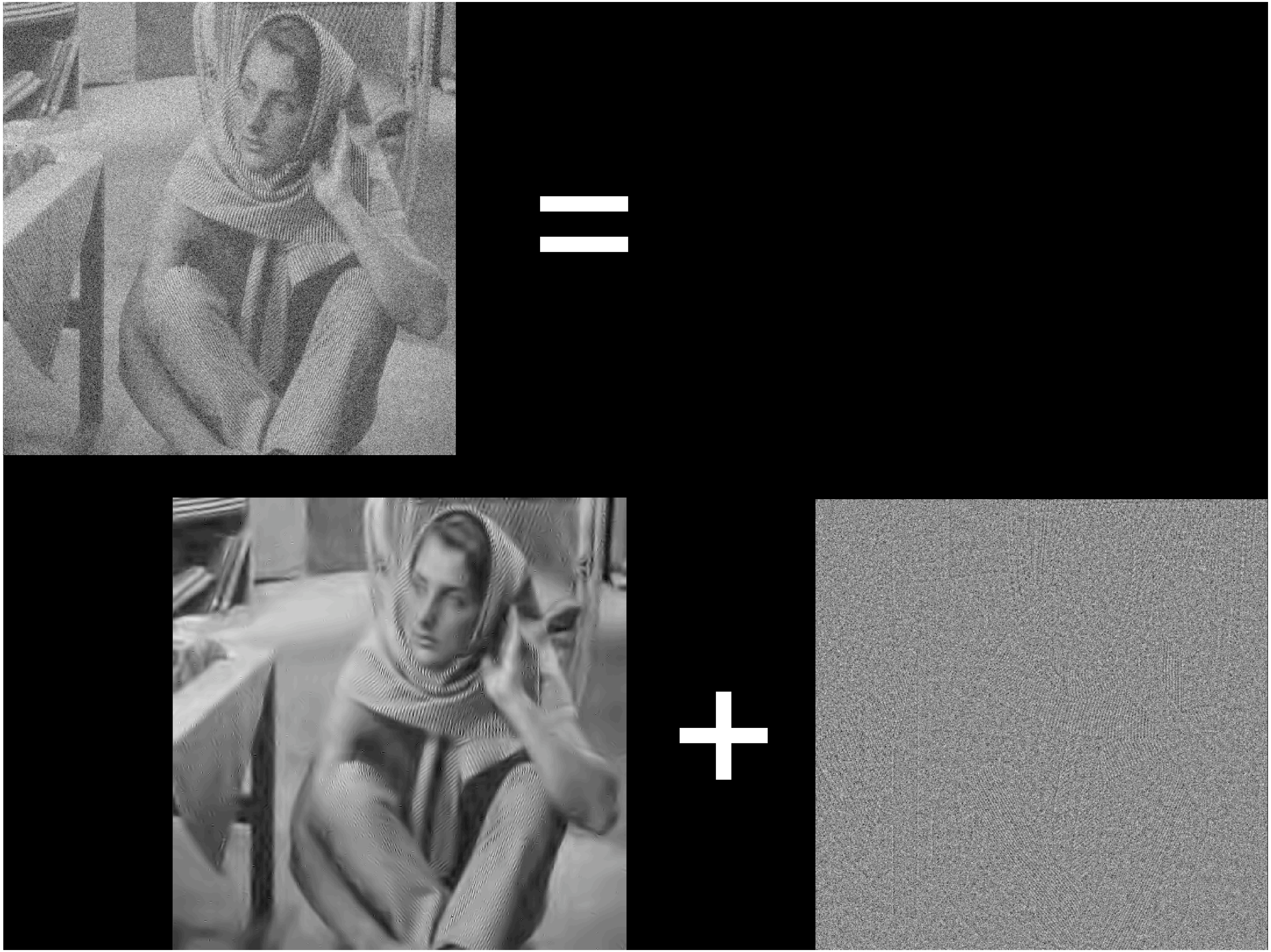
Enstrophy dissipation at vanishing viscosity?



Tran & Dritschel, *JFM* **559** (2006):
"There is no Re-independent measure of dissipation"

Scale-wise statistics





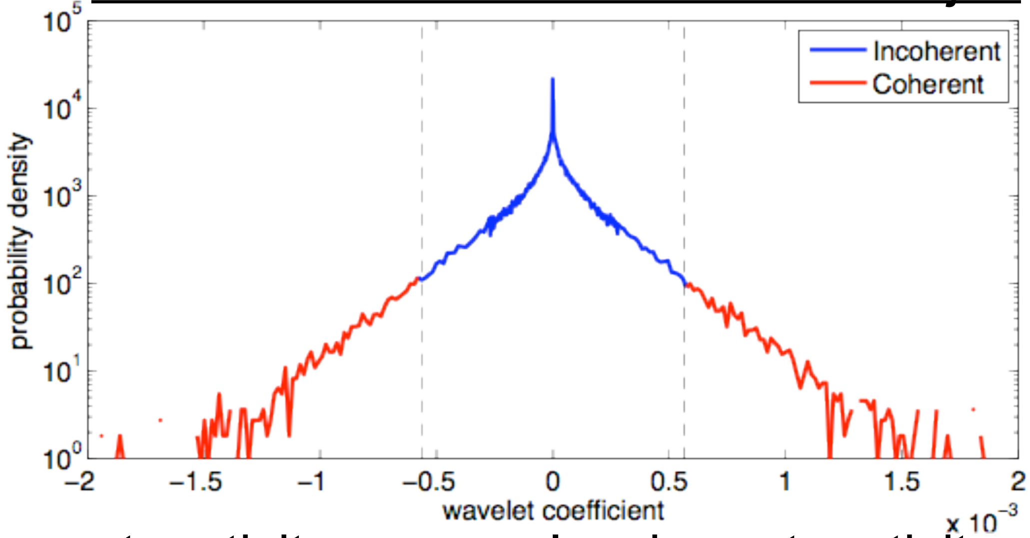
Scale-wise coherent vorticity extraction

The threshold is defined at each scale by:

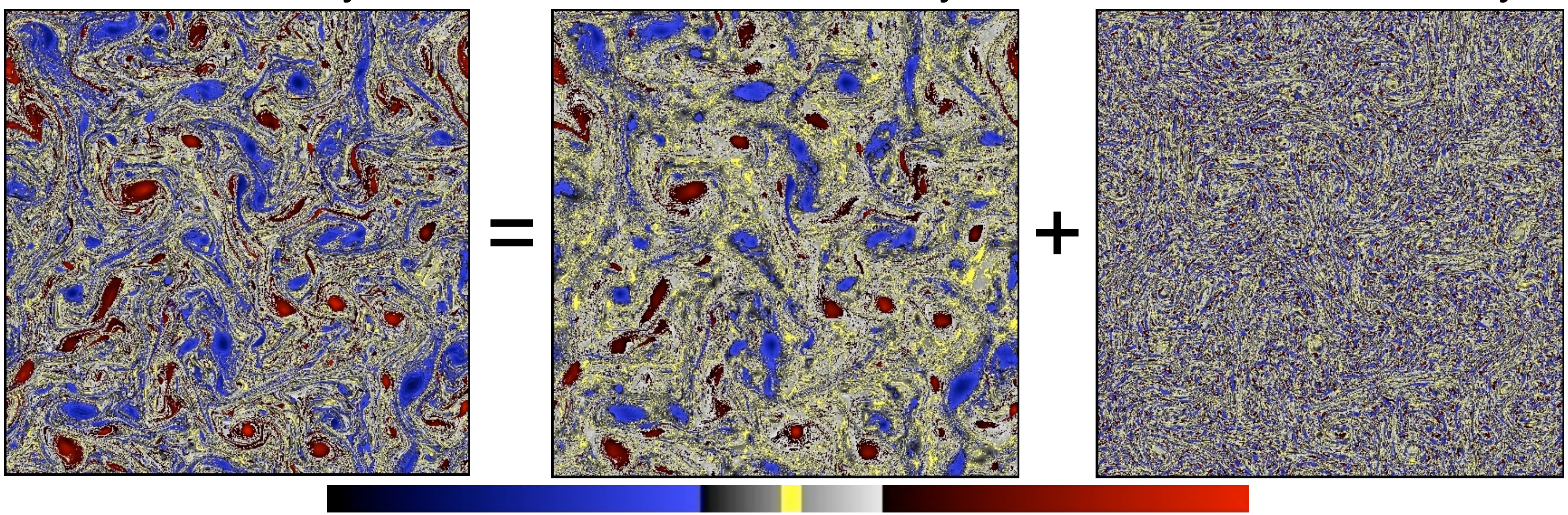
$$\Theta = q\sigma$$

constant parameter → q; standard deviation → σ

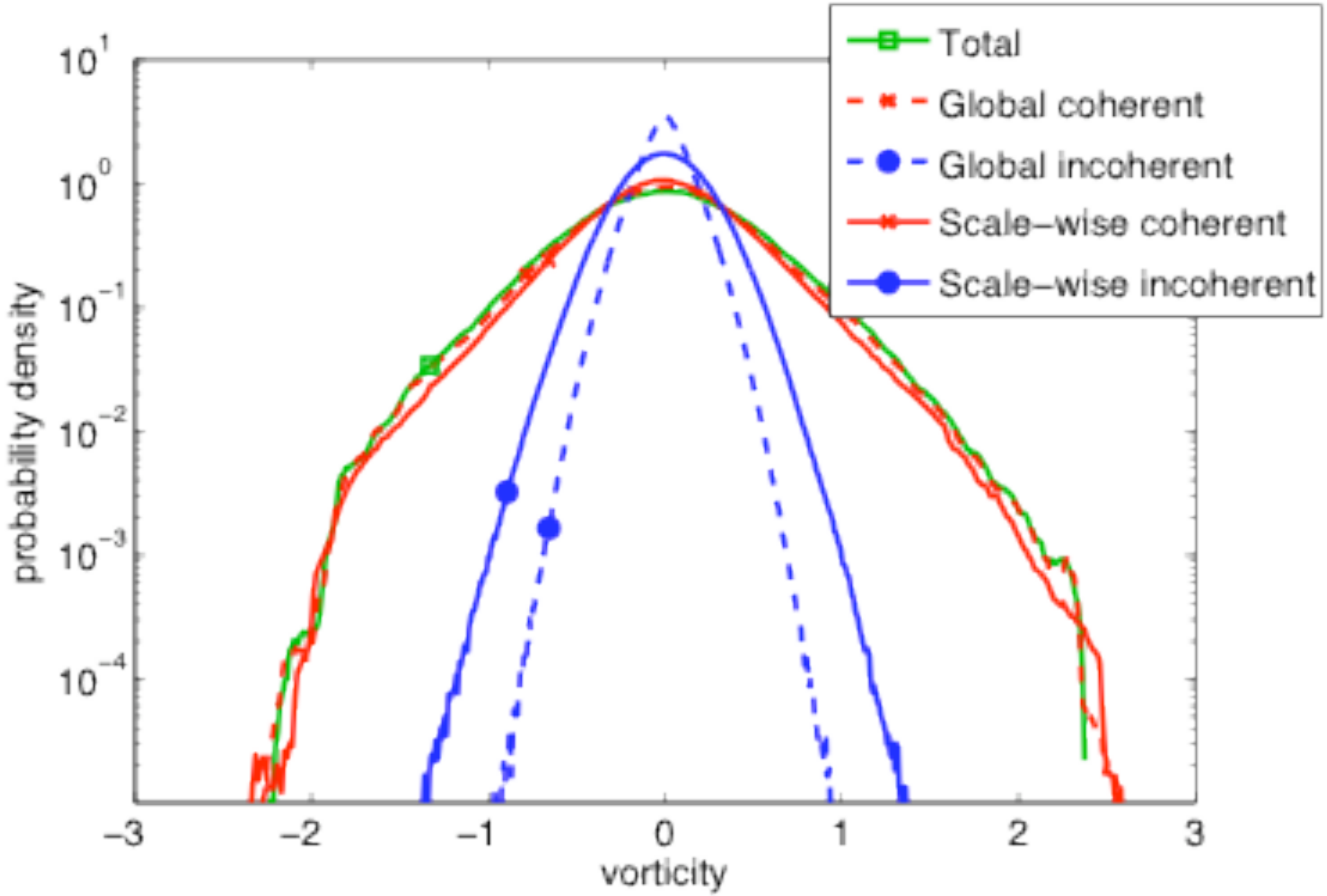
PDF of wavelet coefficients at scale j=8



Total vorticity = Coherent vorticity + Incoherent vorticity

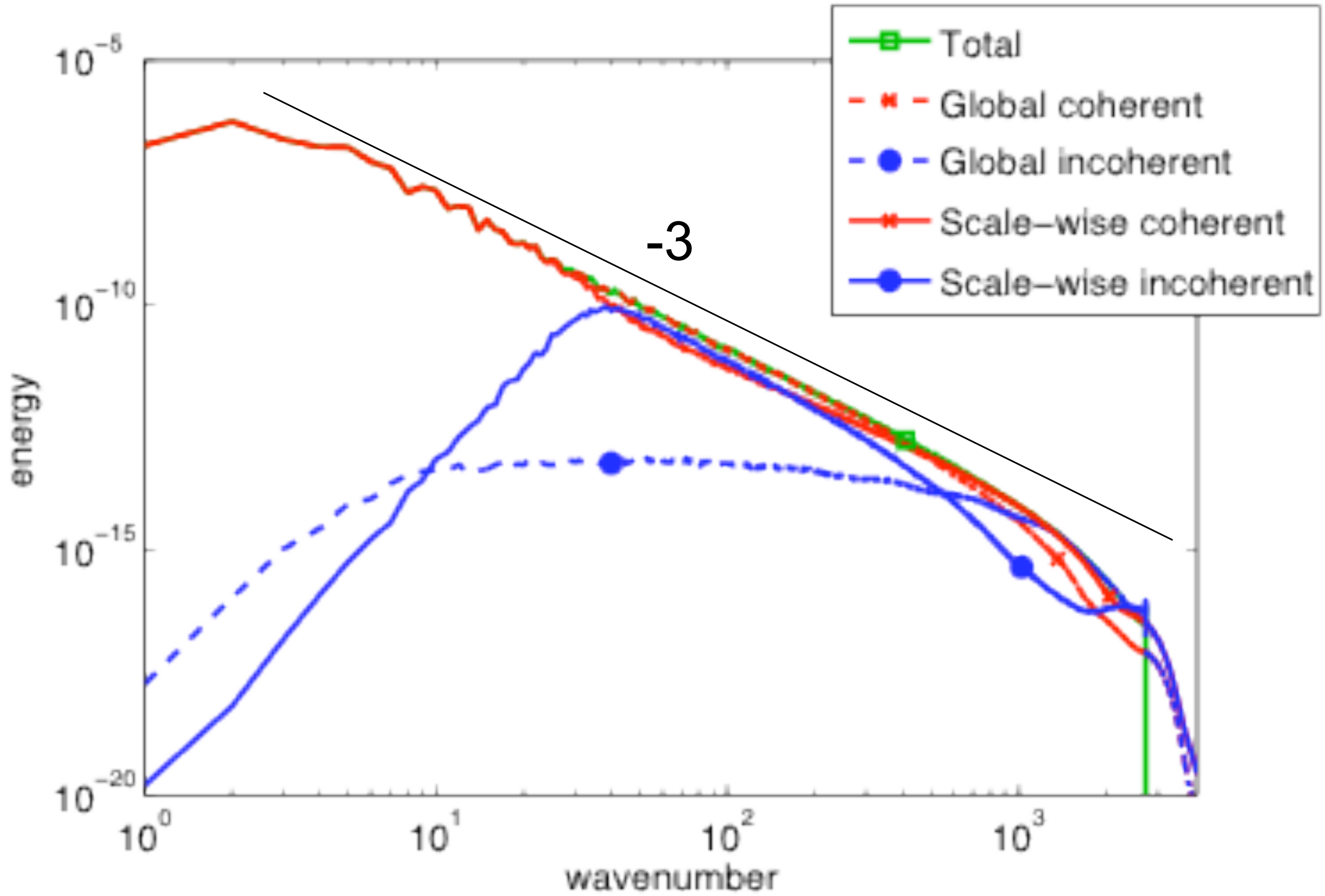


Statistical properties of the split



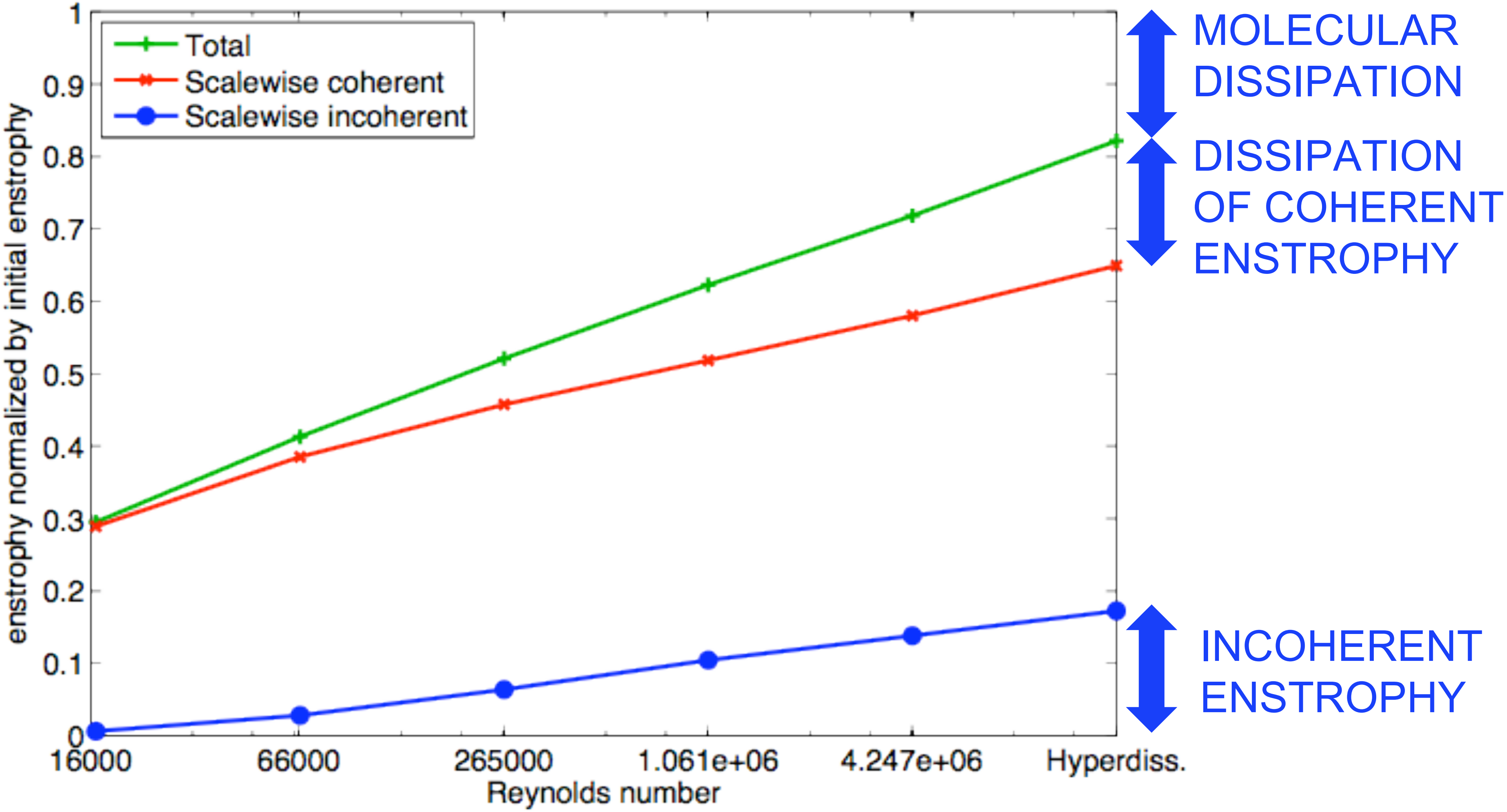
- The global PDF of the incoherent part is close to a Gaussian

Statistical properties of the split

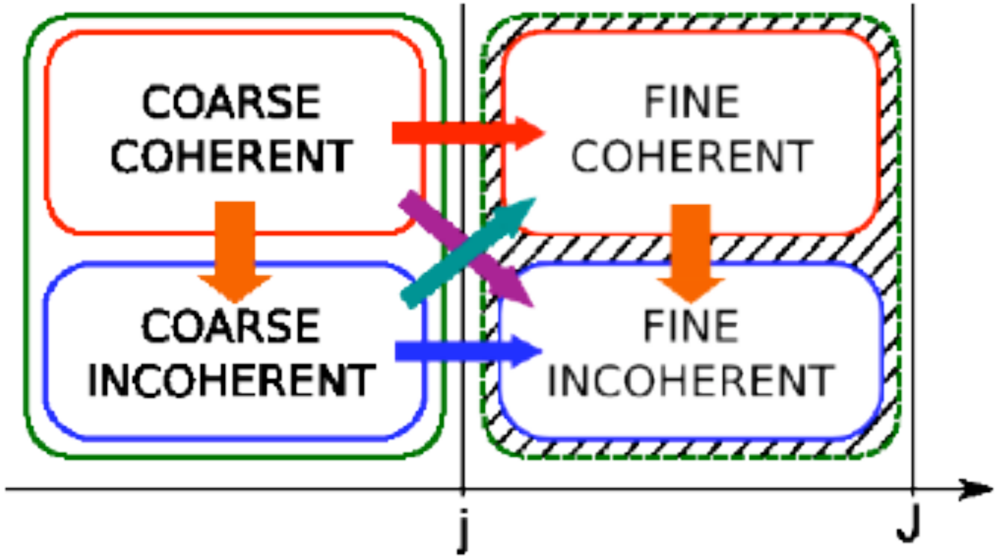


Dissipation of coherent enstrophy

$$\frac{Z(t_1)}{Z(0)}$$



Inter-scale and intra-scale transfers



	Global	Scale-wise
$C_{\leq} \to C_{>}$	92.3%	48.4%
$C_{\leq} \to I_{>}$	17.1%	15.9%
$C_{>} \to I_{\leq}$	-1.58%	-20.2%
$I_{\leq} \to I_{>}$	1.39%	24.8%
$\leq \to >$	100%	100%
$C_{\leq} \to I_{\leq}$	-4.4%	51.3%

Ex: j = 9

negative dissipation (-20.2%)

intra-scale transfer (51.3%)

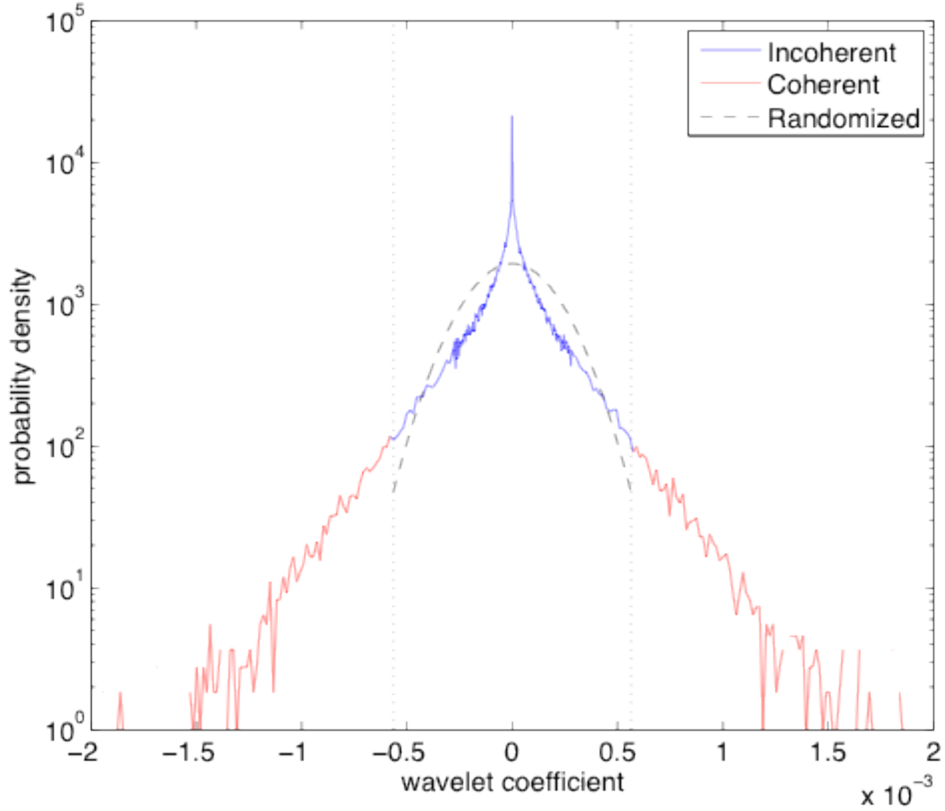
Retroaction of the dissipated flow

- Model the incoherent wavelet coefficients by random variables,
- Maximum entropy distribution, with constraints:

(1) $\widetilde{W}_\lambda \in [-\Theta_\lambda, \Theta_\lambda]$

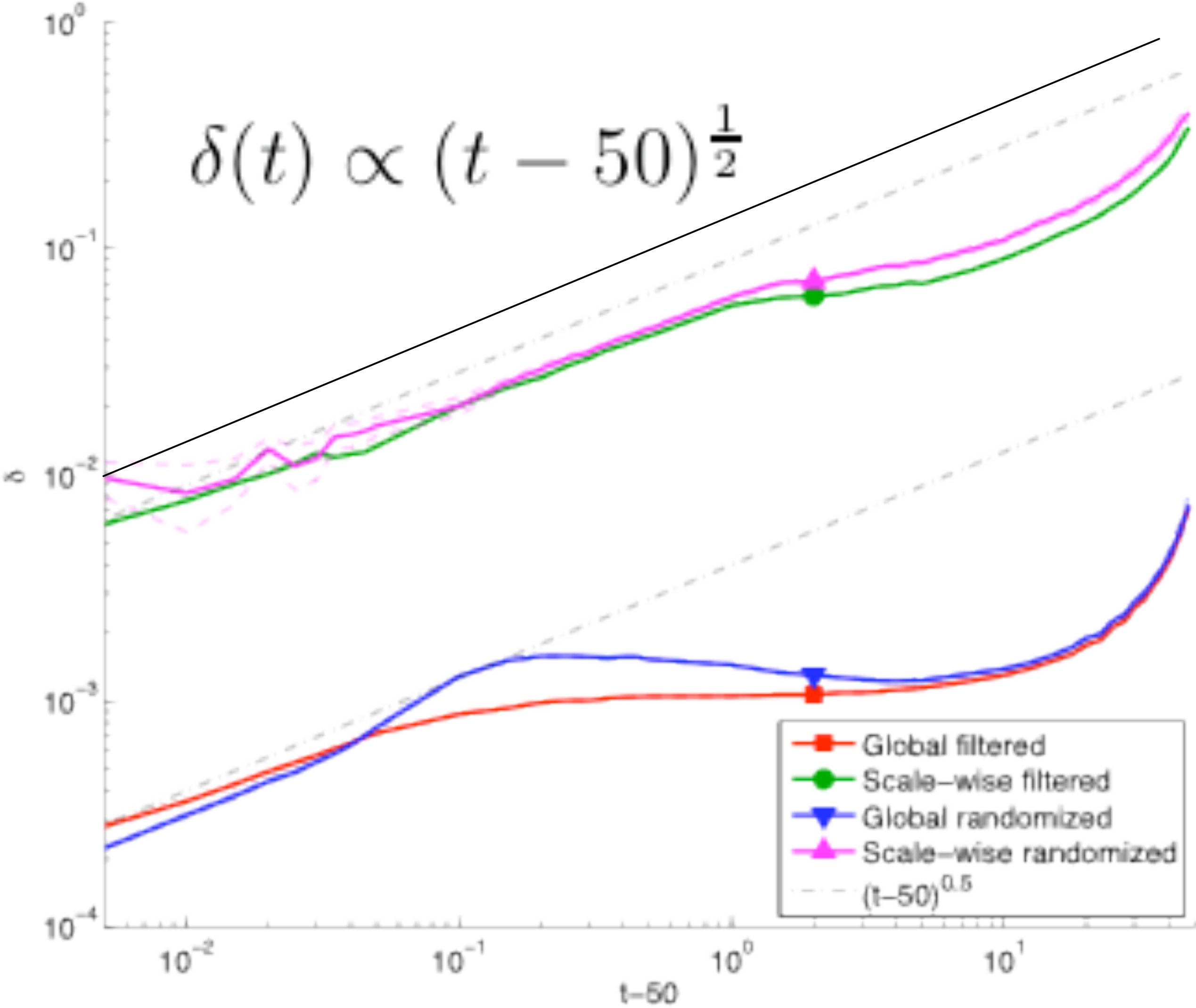
(2) $\mathbb{E}(\widetilde{W}_\lambda) = 0$

(3) $\mathbb{E}\left(\widetilde{W}_\lambda^2\right) = \left(\frac{\Theta_\lambda}{q}\right)^2$



It doesn't fit,
but we proceed anyway.

Retroaction of the dissipated flow



Summary

- We have introduced a "dissipation mechanism" for 2D turbulence based on a split of the flow between **explicit** and **dissipated** components.
- The associated enstrophy dissipation rate does not vanish at vanishing viscosity.
- The dissipation rate can be directly related to the nonlinear transfers and studied quantitatively scale-wise.
- Negative dissipation is allowed.
- We have shown that the retroaction of the dissipated flow has a diffusive effect on the explicit flow at short times.

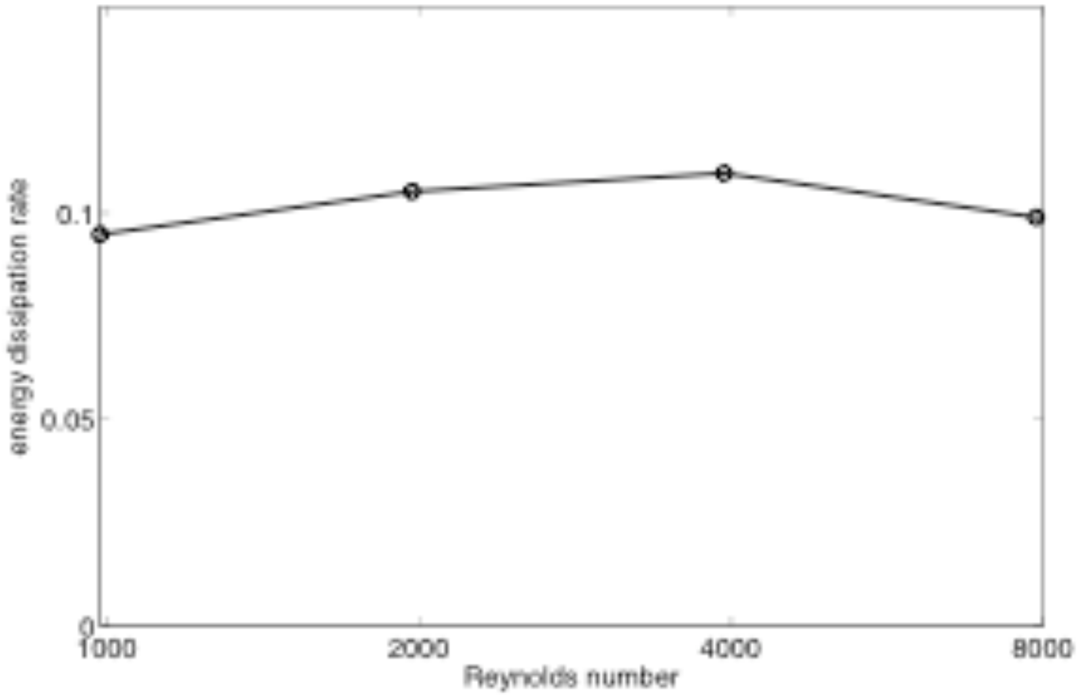
R.H. Kraichnan, *Reduced descriptions of hydrodynamic turbulence* (1988)

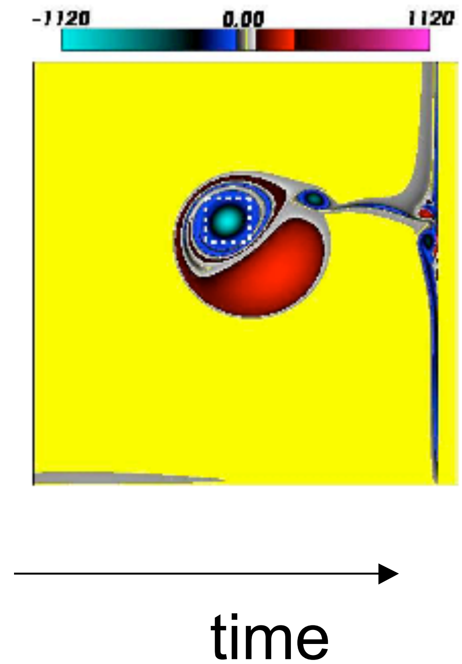
R.H. Kraichnan & S. Chen *Is there a statistical mechanics of turbulence?* (1989)

RNVY, M. Farge, K. Schneider, doi: 10.1016/j.physd.2011.05.022

A result on dissipative singularities

- There are no dissipative singularities in 2D Navier-Stokes in the absence of walls and for bounded vorticity fields.
- However this happens in the presence of boundaries.





RNVY, M. Farge, K. Schneider, *PRL* **106**, 184502

Acknowledgements

- Thanks to Claude Bardos, Dmitry Kolomenskiy, Xavier Garbet, Greg Hammett, any many others.

- The Kicksey-Winsey code can be downloaded at :

  http://justpmf.com/romain

- Papers are available on :

  http://wavelets.ens.fr/

Thank you!

Le temps efface tout comme effacent les vagues

Les travaux des enfants sur le sable aplani

Nous oublierons ces mots si précis et si vagues

Derrière qui chacun nous sentions l'infini.

Marcel Proust