# Wavelet-based study of dissipation in plasma and fluid flows

Romain Nguyen van yen

Soutenance de thèse de doctorat de l'Université Paris 11

8 décembre 2010 - ENS Paris

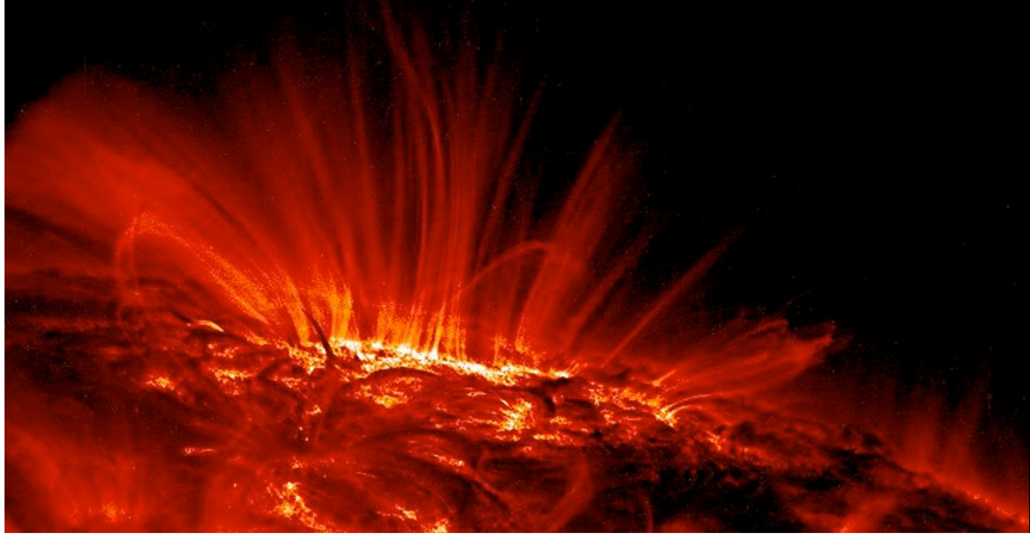
Thèse préparée au Laboratoire de Météorologie Dynamique,

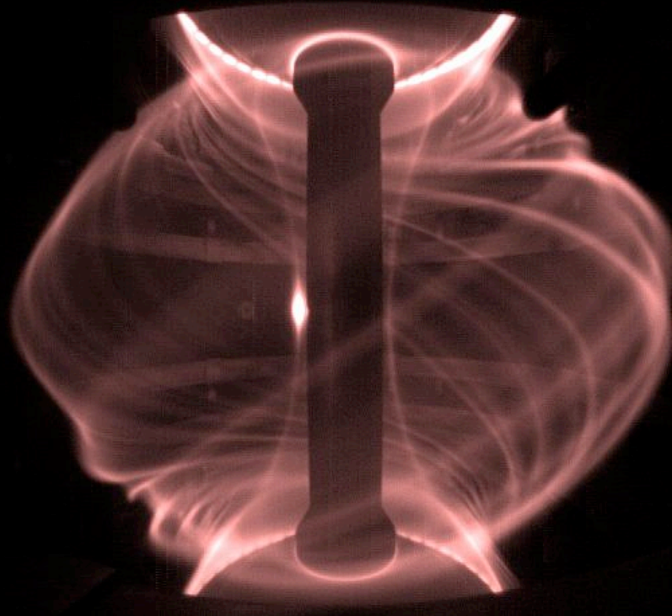
sous la direction de Marie Farge et Kai Schneider

Biker in a wind tunnel

Earth (Apollo 17)

Ultraviolet sun (TRACE, NASA)





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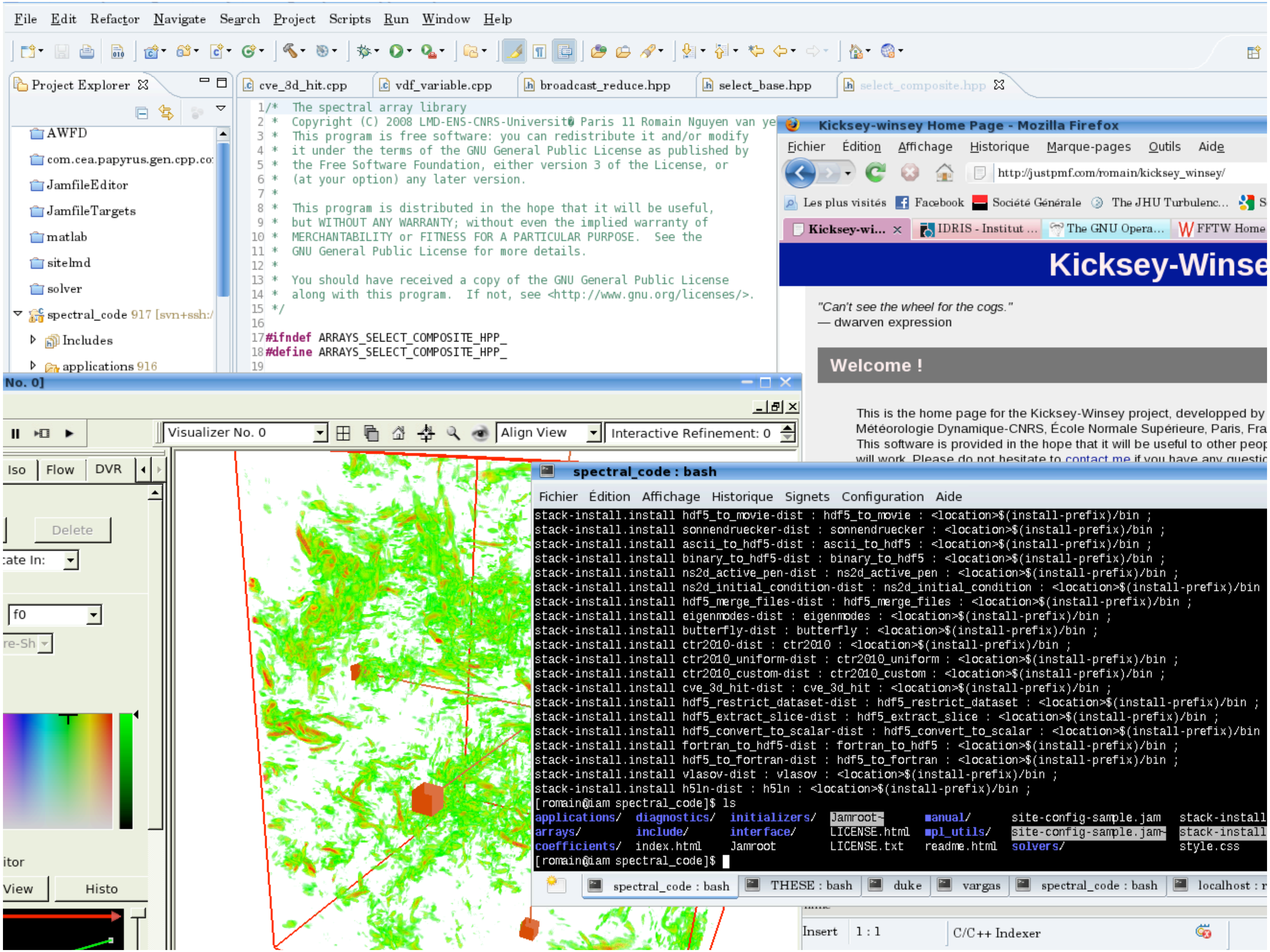
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cve_3d_hit.cpp
vdf_variable.cpp
broadcast_reduce.hpp
select_base.hpp
select_composite.hpp
/* The spectral array library
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"Can't see the wheel for the cogs."
— dwarven expression
Welcome !
This is the home page for the Kicksey-Winsey project, developped by
Météorologie Dynamique-CNRS, École Normale Supérieure, Paris, Fra
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coefficients/ index.html Jamroot LICENSE.txt readme.html solvers/ style.css
[romain@iam spectral_code]$
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localhost
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C/C++ Indexer

# I
# Flows in plasmas and fluids

# II
# Dissipation by fluid flows

# III
# Wavelet-based study of dissipation by fluid flows

# I
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# Kinetic theory of gases and plasmas

- By definition, **flows** are collective motions in **systems of many interacting particles**.
- They are best described using **reduced models** which focus on macroscopic quantities.
- **Kinetic theory** takes as main quantity the probability $f(\mathbf{x},\mathbf{v},t)d\mathbf{x}d\mathbf{v}$ of finding a particle at position $\mathbf{x}$ with velocity $\mathbf{v}$ at time t,
- in general, **it is not possible to obtain a closed equation** on $f$ using only Hamiltonian mechanics,
- only in the limiting case of weak interactions (**collisionless** gas or plasma).

# Fluids

- Sometimes an even more reduced description is preferable → **fluid description**

$$n(\mathbf{x},t) = \int f(\mathbf{x},\mathbf{v},t)\mathrm{d}\mathbf{v}\,, \quad \mathbf{u}(\mathbf{x},t) = \frac{1}{n}\int \mathbf{v} f(\mathbf{x},\mathbf{v},t)\mathrm{d}\mathbf{v}\,, \text{ etc.}$$

density fluid velocity

- As before, no closed equation follows only from conservation principles, except in the case of a **perfect fluid**.

- In particular, the viscosity of such a perfect fluid vanishes. It is called **inviscid**.

# Dissipative models

- To get closed equations in more general cases (for example, **collisional** plasmas and **viscous** fluids), **dissipative terms have to be added**, yielding:
  - in kinetic theory, the **Boltzmann equation**:

$$\partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} f = C(f)$$

  - in fluid theory, the **Navier-Stokes equations**:

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \frac{1}{\rho}\nabla p = \nu \Delta \mathbf{u} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

pressure

**Here we focus on 2D incompressible Navier-Stokes flows**

# Boundary conditions

- Most of the time, flowing systems are not isolated, and boundary conditions must be specified,

- For a fluid in contact with a solid, we impose **non-penetration**:

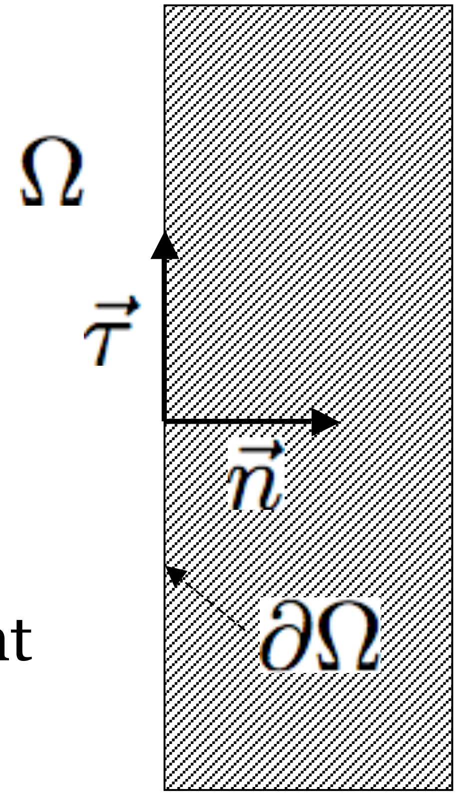
$$u_n = 0$$

- Navier (1823):

$$u_\tau = -\alpha \partial_n u_\tau$$

slip length

- The special case $\alpha = 0$ (**no-slip**) is relevant to a lot of situations.



# Navier-Stokes initial-boundary value problem

Equation

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla p + \frac{1}{\mathrm{Re}}\Delta \mathbf{u} \\ \nabla \cdot \mathbf{u} = 0 \end{cases}$$

$$\mathrm{Re} = \frac{UL}{\nu}$$

(no body forces → decaying flow)

Boundary conditions

$$u_n = 0 \quad \text{and} \quad u_\tau = -\alpha \partial_n u_\tau \quad \text{on} \quad \partial\Omega$$

Initial conditions

$$\mathbf{u}(\mathbf{x}, t = 0) = \mathbf{u}_0$$

**In 2D this is a well-posed problem**

I

Flows
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# Dissipation by flows

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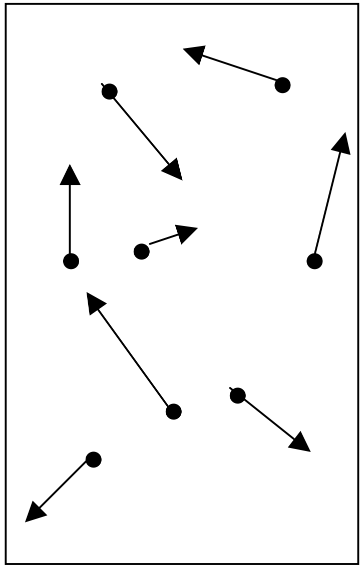
# Definition of dissipation

- Lagrangian mechanics are **insufficient** to predict the behavior of the majority of many-particles systems.
- But **equilibrium** implies equivalence between a (very) **large number** of microscopic configurations.
- This **principle of equivalence** is sufficient to predict macroscopic properties at equilibrium!
- Statistical distributions compatible with this principle are **entropy maxima**.
- The phenomenon by which these equilibria are attained is called **dissipation**.

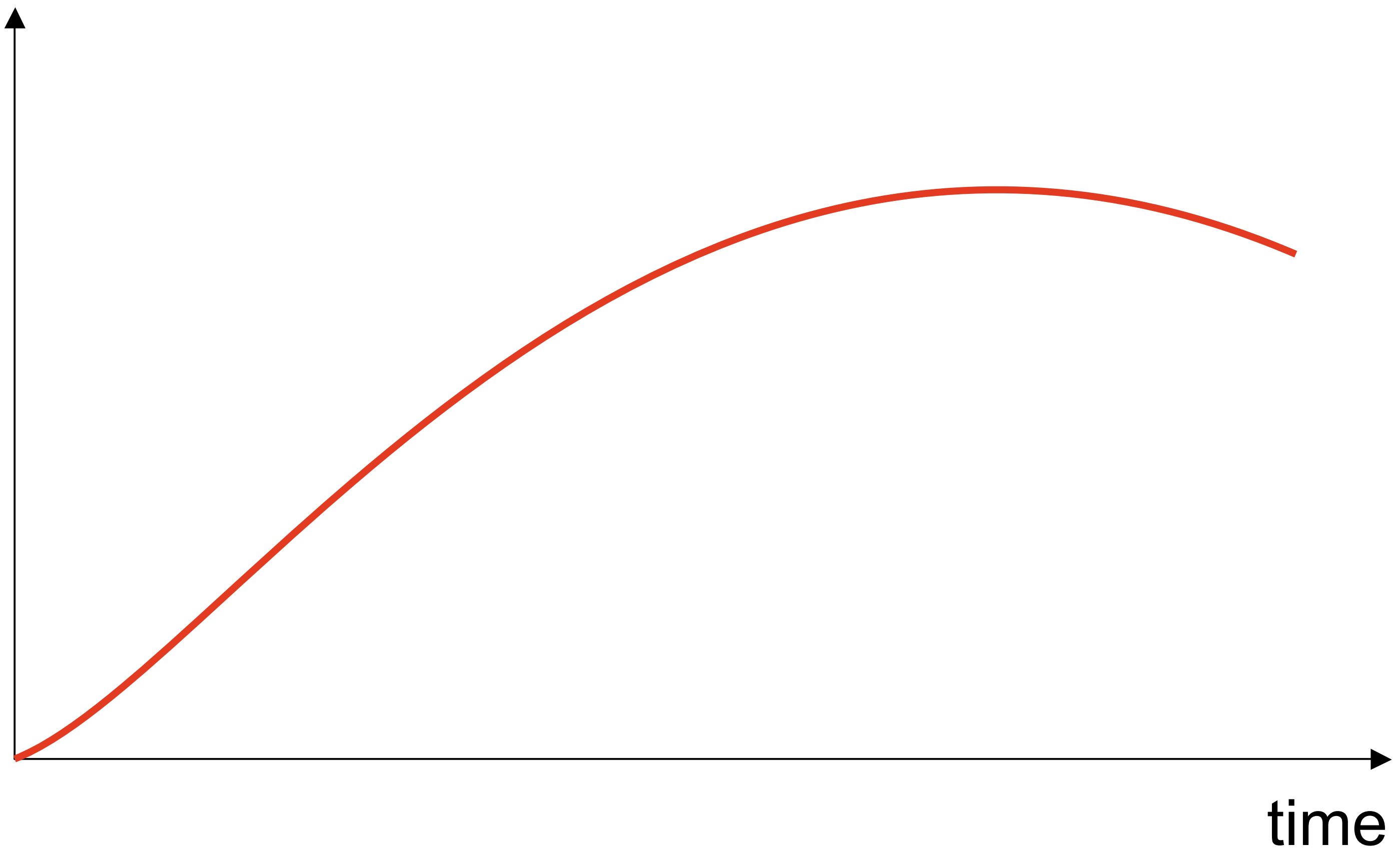


# Dissipation is a matter of choice!

- The definition of dissipation **depends** on the definitions of equilibrium:
  - return to local equilibrium → collisional dissipation,
  - return to global equilibrium → fluid dissipation.
- Close to equilibrium, dissipation can be predicted by a linear theory (Onsager relations, etc).
- **But far from equilibrium, open questions:**
  - **does the standard dissipation still play a role?**
  - **is there another relevant dissipation?**

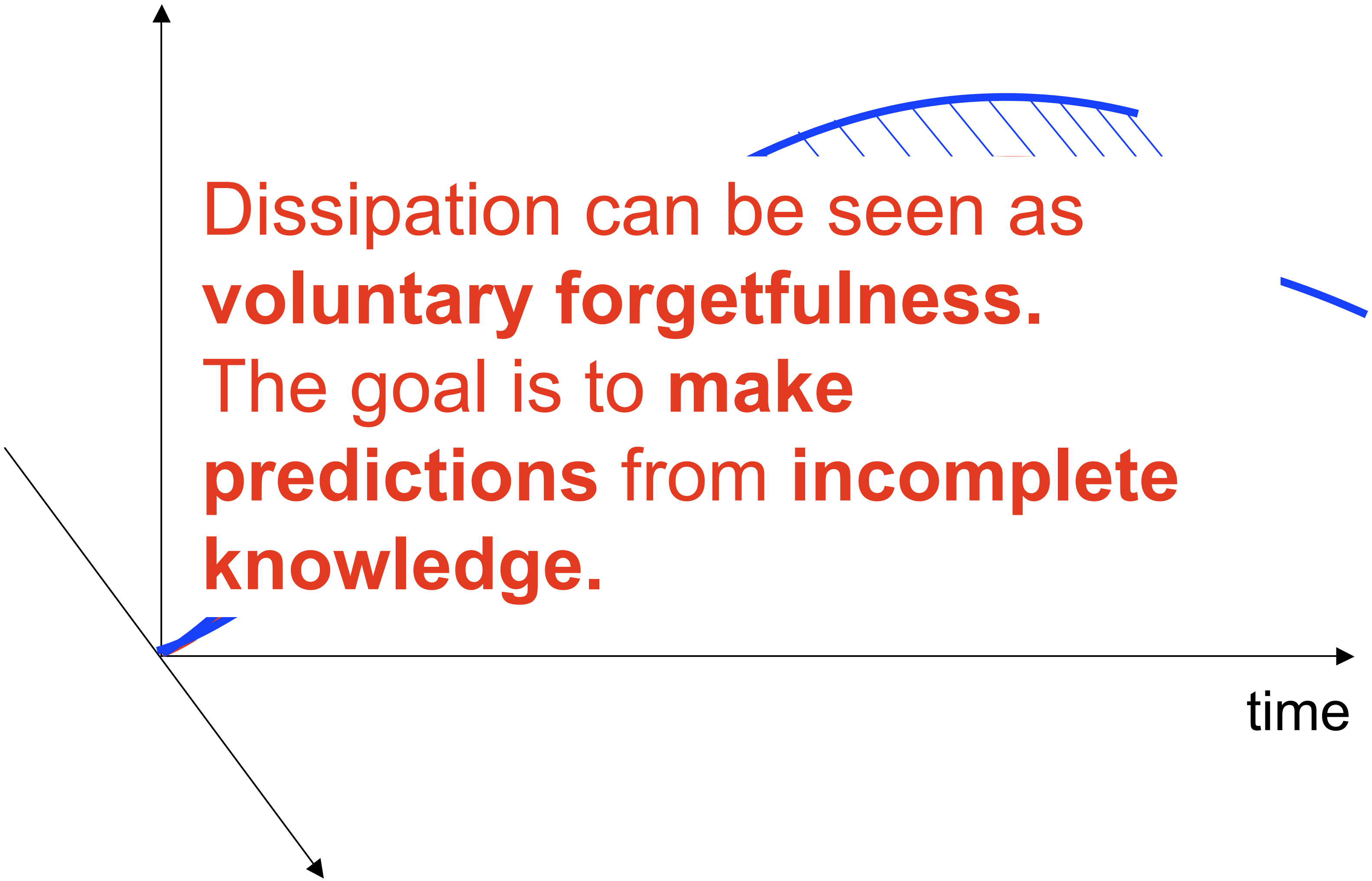
# Dissipation as randomization

<u>trajectory of reduced model</u>



# Dissipation as randomization

trajectories of more complete model




Dissipation can be seen as **voluntary forgetfulness.** The goal is to **make predictions** from **incomplete knowledge.**

# Definition of "dissipation by flows"

- Particle systems subject to flows are **out of local thermodynamic equilibrium**.
- There is a dissipation associated to this departure, which we will call **molecular dissipation**.
- Corresponding coupling coefficient: **viscosity** or **collisionality.**
- To understand the effects that are **due to the flow**, we study the following regimes:
  - **vanishing viscosity limit**,
  - vanishing collisionality limit.

# Molecular dissipation in 2D Navier-Stokes

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p + \frac{1}{\mathrm{Re}}\Delta\mathbf{u} \\ \boldsymbol{\nabla}\cdot\mathbf{u} = 0 \end{cases}$$

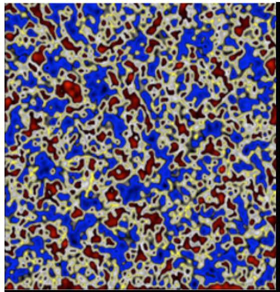
<u>energy</u> $E(t) = \frac{1}{2}\int_\Omega \mathbf{u}^2$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -2\nu Z$$

<u>enstrophy</u> $Z(t) = \frac{1}{2}\int_\Omega \omega^2$ $\omega = \boldsymbol{\nabla}\times\mathbf{u}$

vorticity

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = -2\nu P + \frac{\nu}{2}\oint_{\partial\Omega} \boldsymbol{\nabla}\omega^2\cdot\mathbf{n} \qquad P(t) = \frac{1}{2}\int_\Omega |\boldsymbol{\nabla}\omega|^2$$



Re ≈ 1

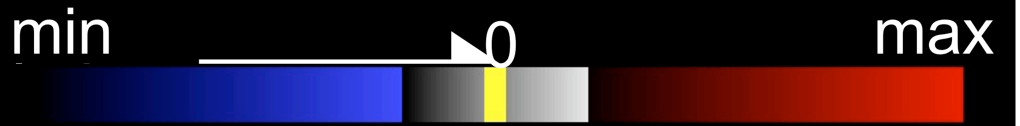
Re ≈ 6

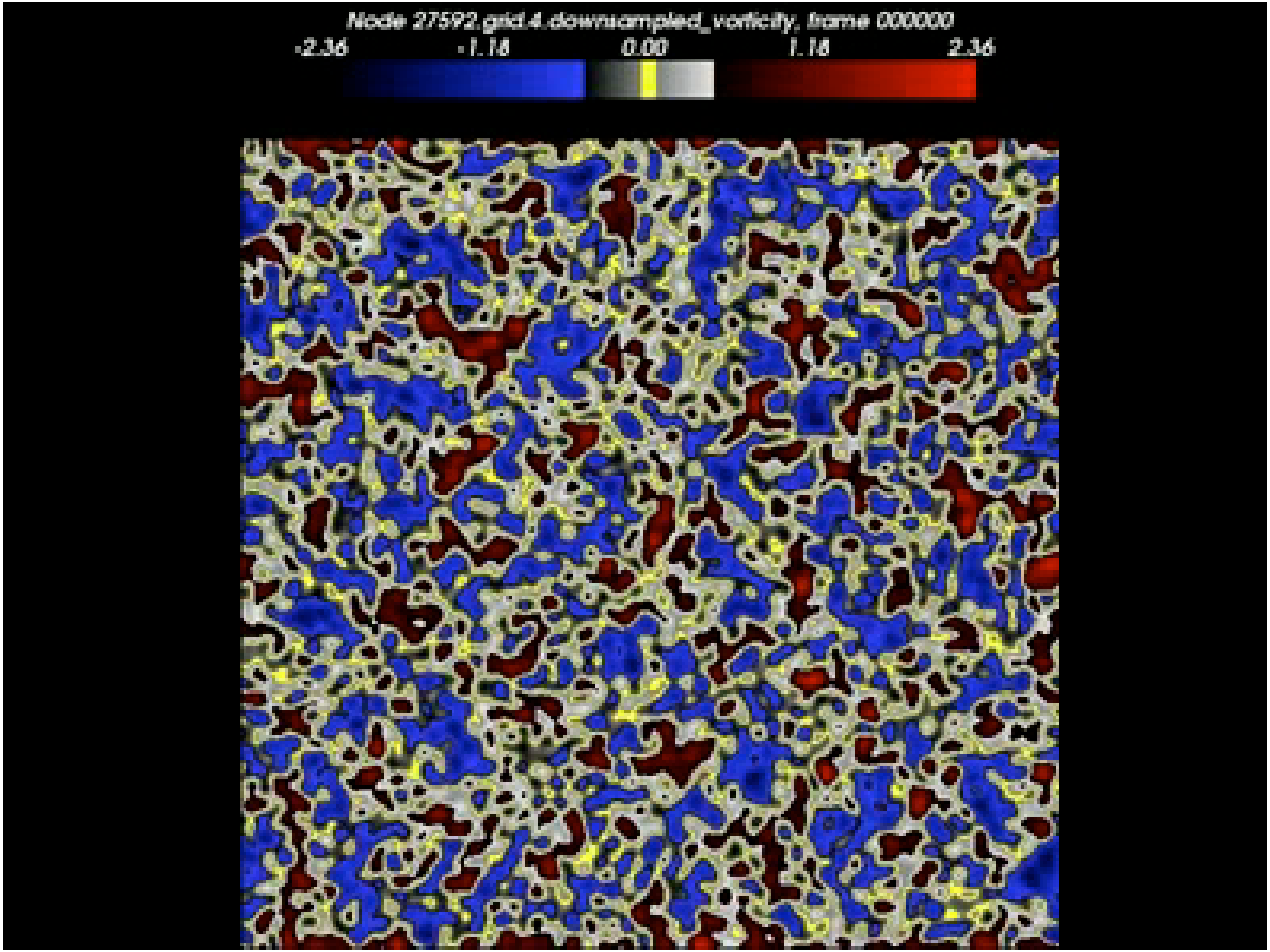
Re ≈ 2

Re ≈ 1

Re ≈ 4

time

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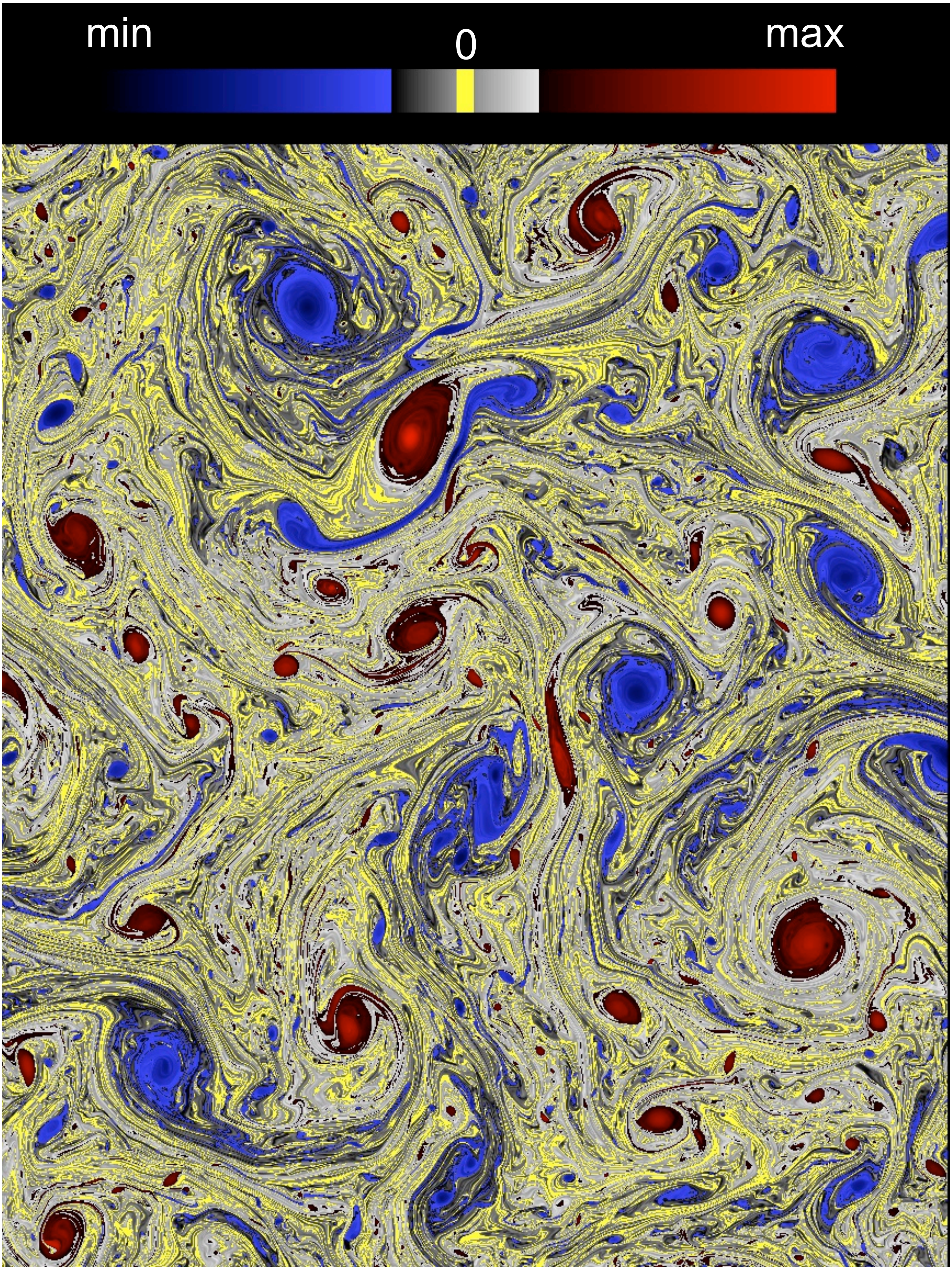
min
0
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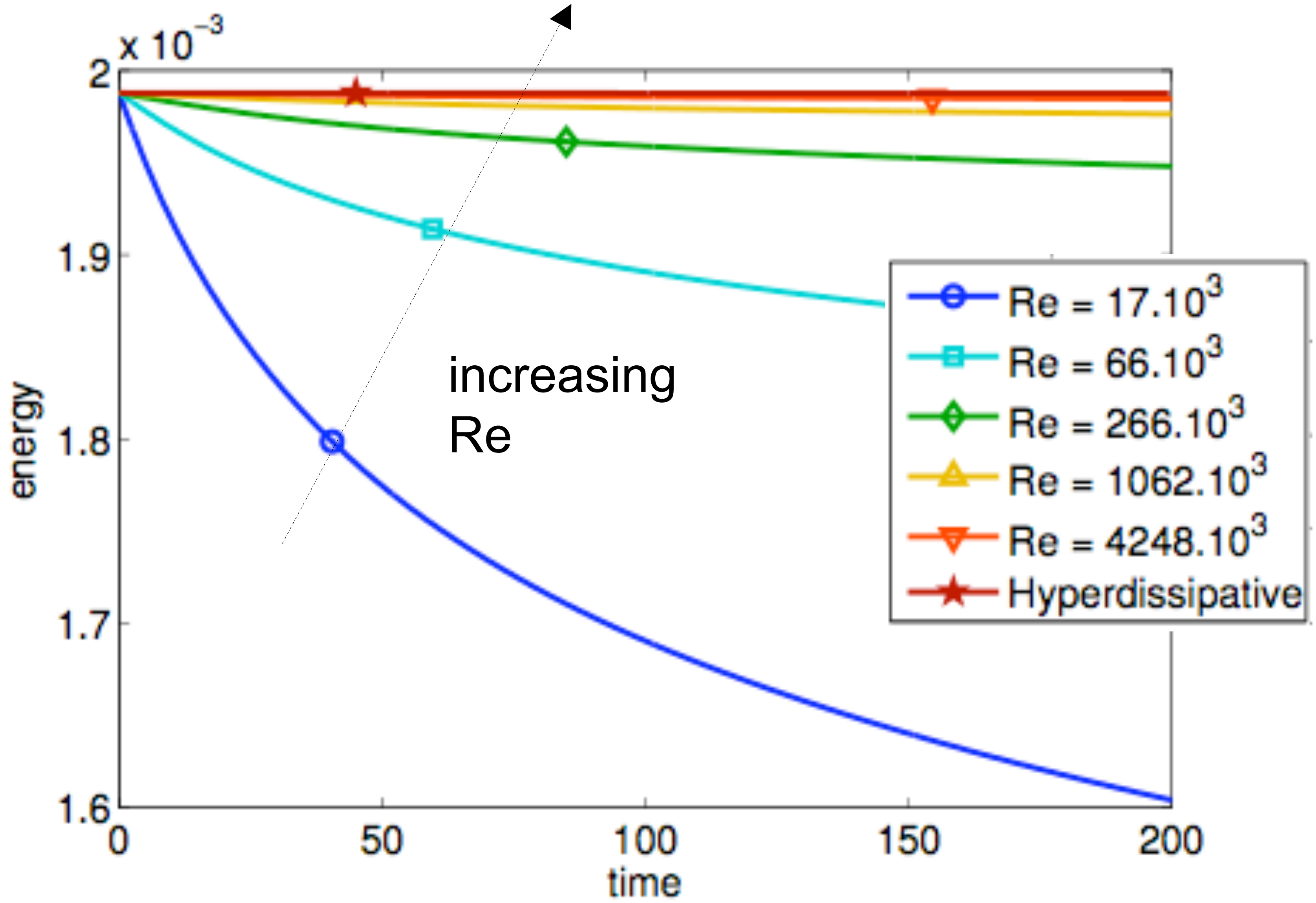
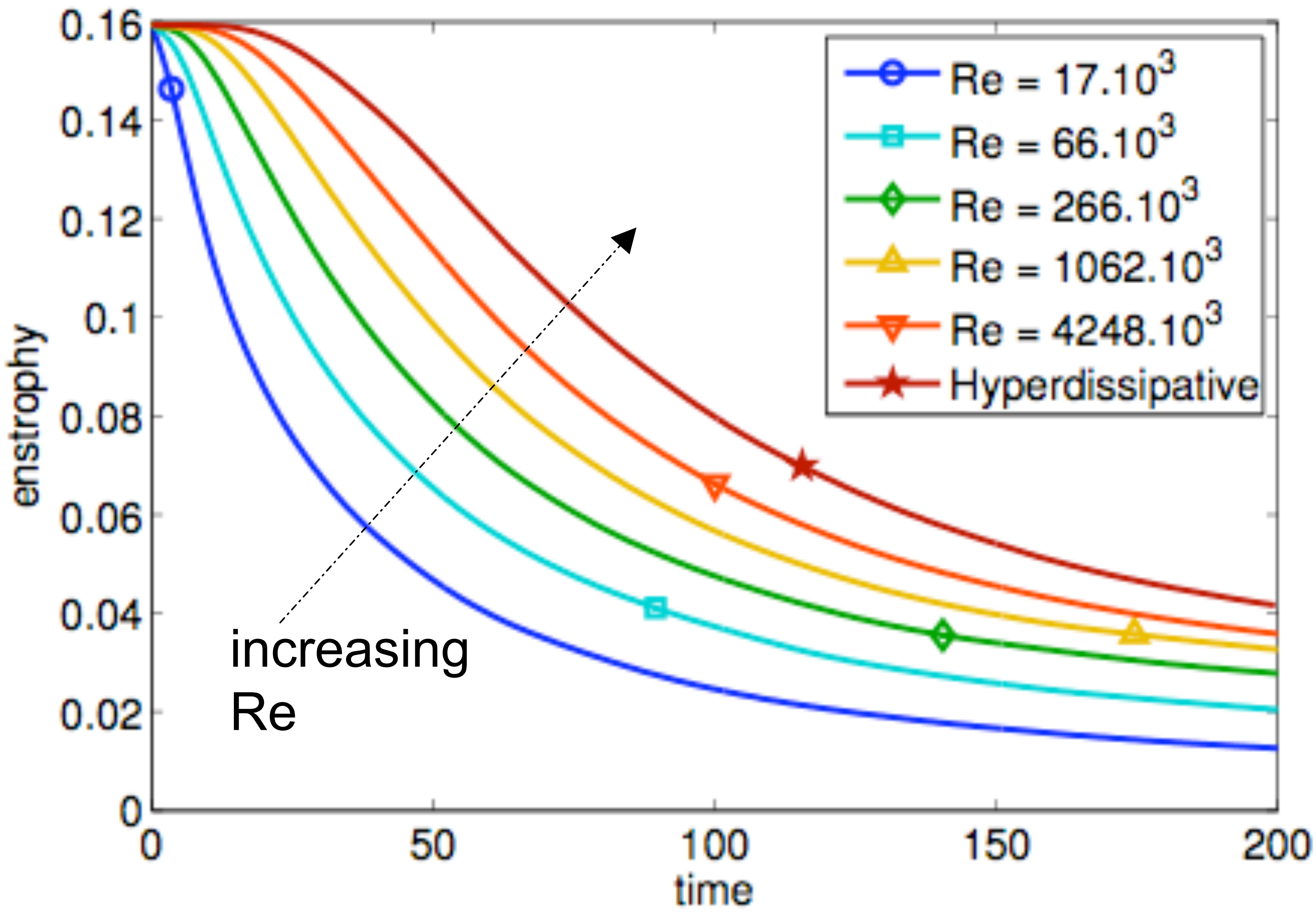
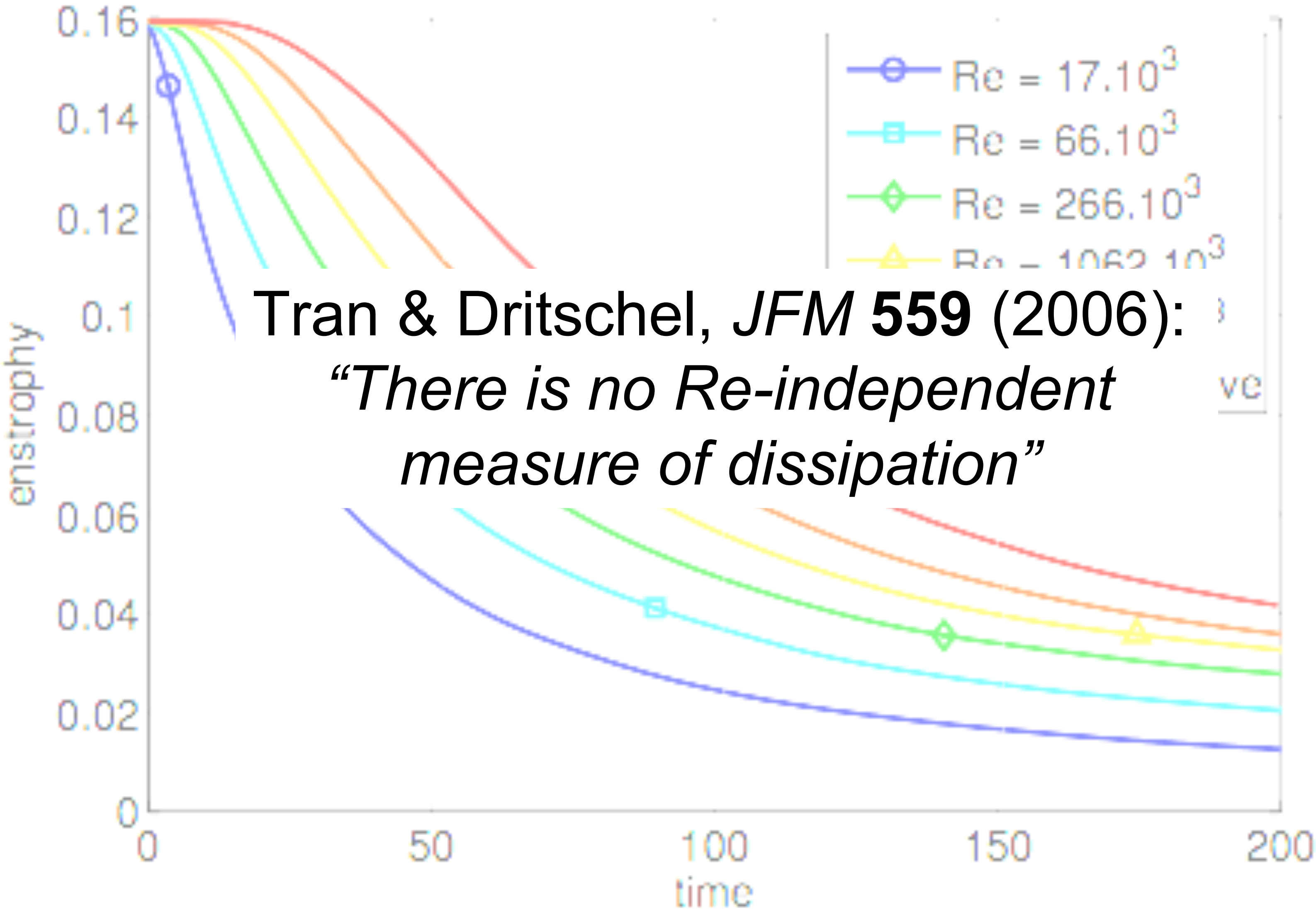
# Energy dissipation at vanishing viscosity?



# Enstrophy dissipation at vanishing viscosity?



# Enstrophy dissipation at vanishing viscosity?

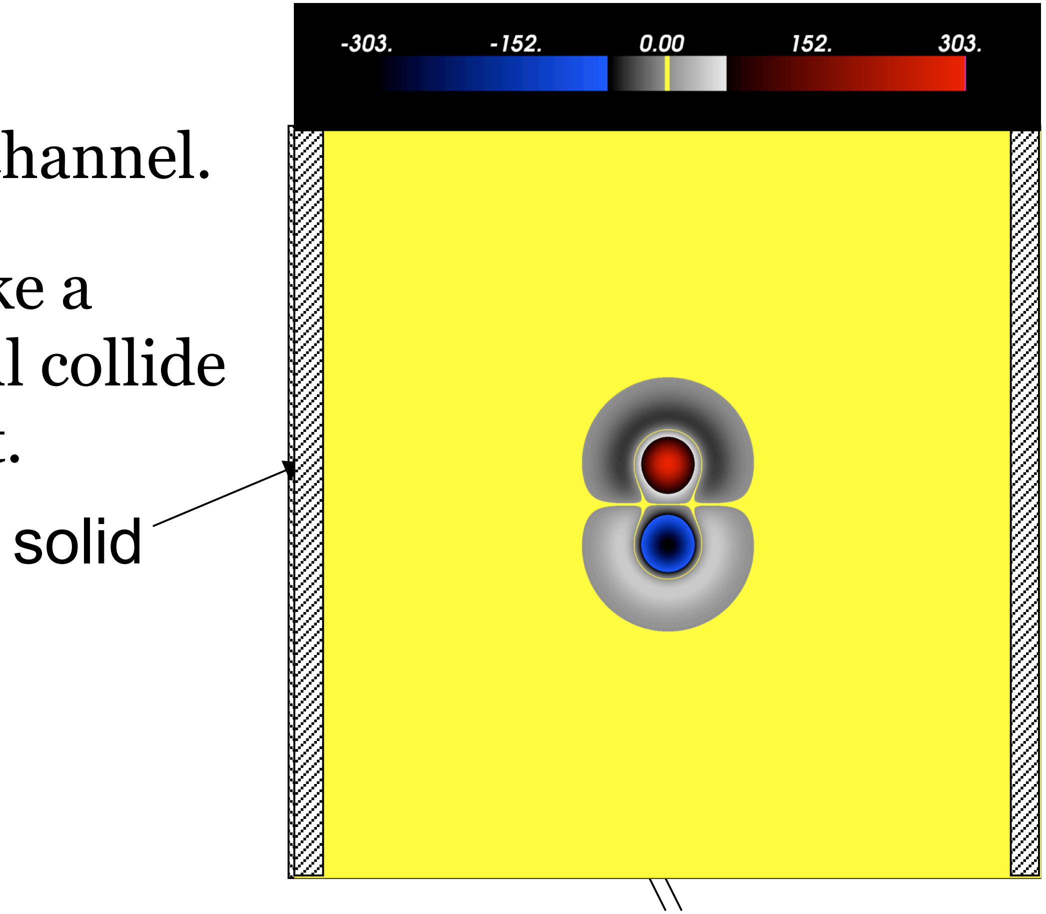



Tran & Dritschel, *JFM* **559** (2006):

*“There is no Re-independent measure of dissipation”*

# The effect of a wall: dipole-wall collision

- We now show an example where the molecular dissipation of **energy** does not vanish at vanishing viscosity.
- In 2D this effect can be observed only in the presence of a solid boundary!
- We consider a periodic channel.
- As intial condition we take a vorticity dipole which will collide with the wall on the right.



# Numerical method

- We use the volume penalization method to approximate no-slip boundary conditions at the walls:

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} p + \frac{1}{\mathrm{Re}} \Delta \mathbf{u} - \frac{1}{\eta} \chi \mathbf{u}$$

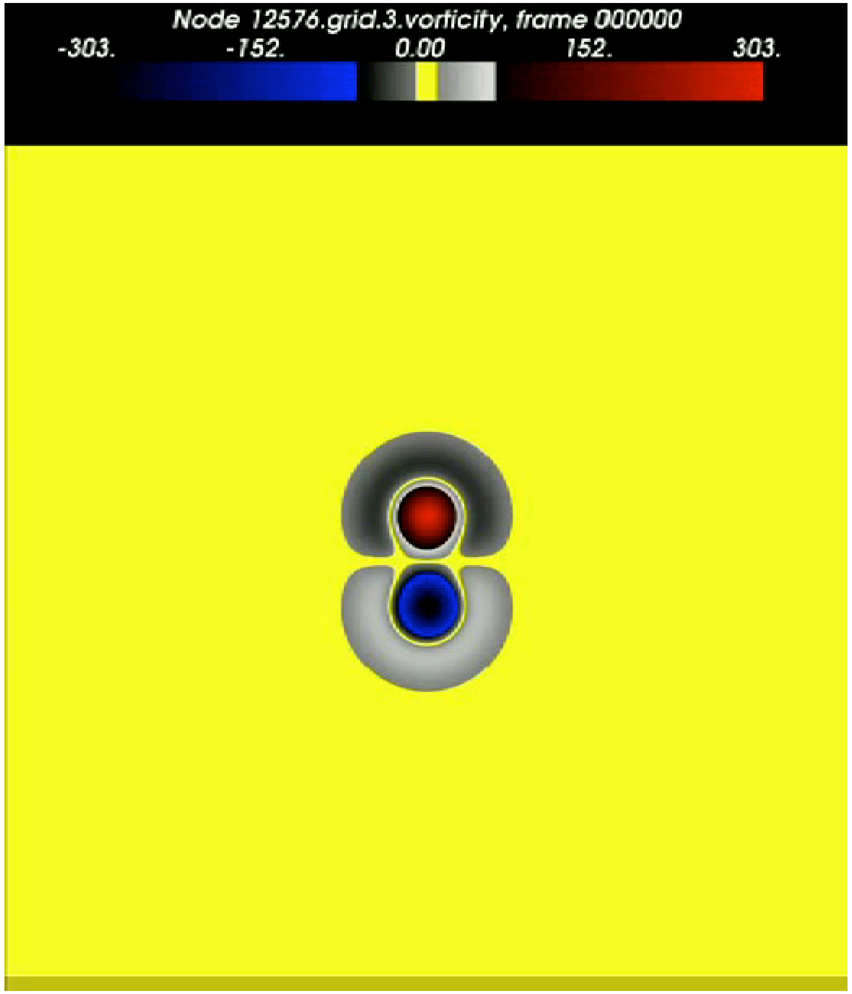
where $\eta \propto \mathrm{Re}^{-1}$

- Then, pseudo-spectral discretization with grid resolution such that :

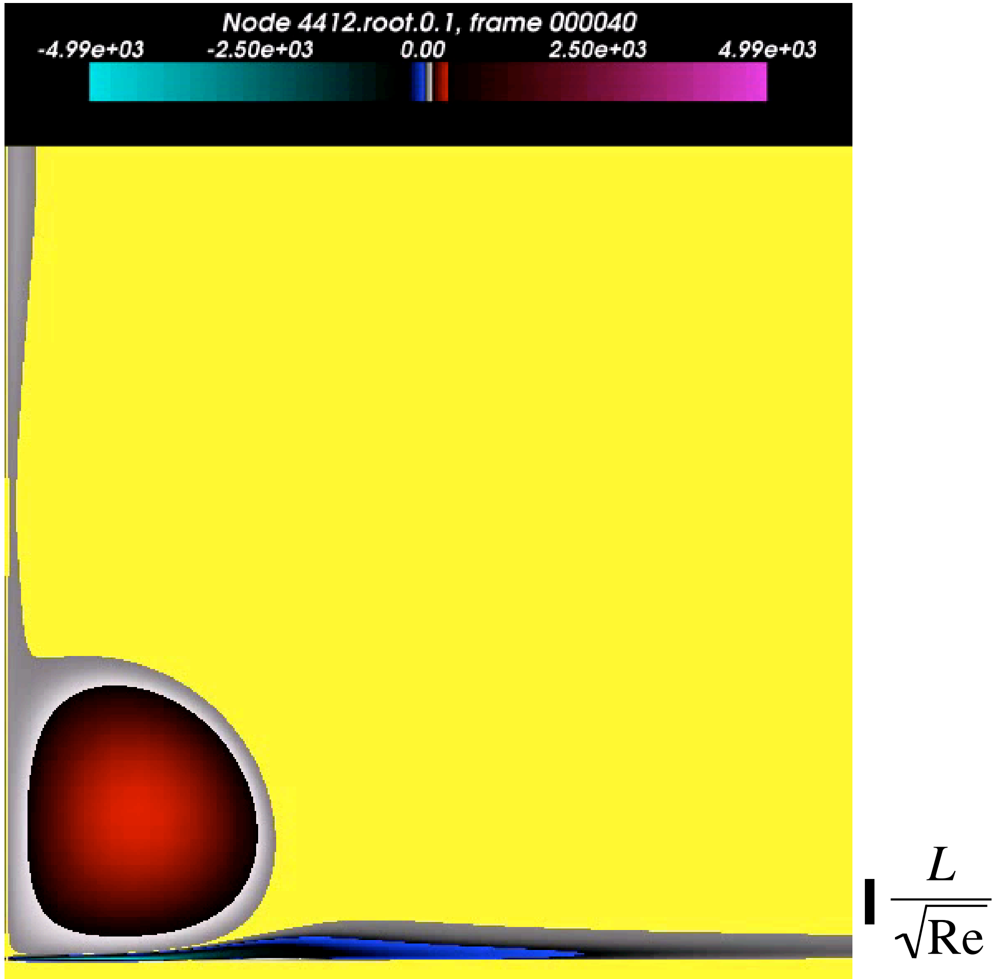
$$N \propto \mathrm{Re}$$

- Perform computations for Re up to **7980**.

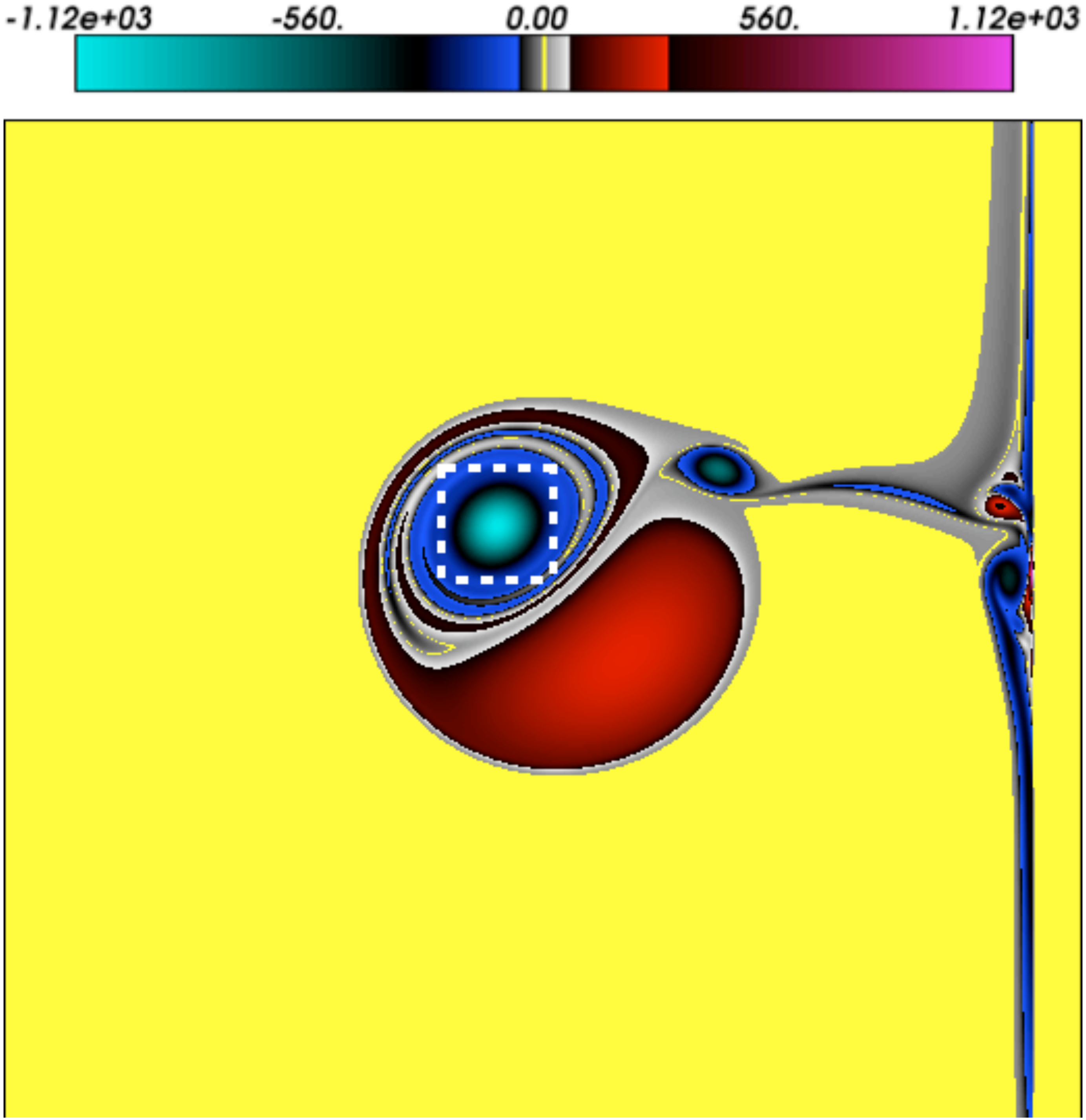
# Results




vorticity movie for Re = 3940




zoom on collision for Re = 7980


-1.12e+03
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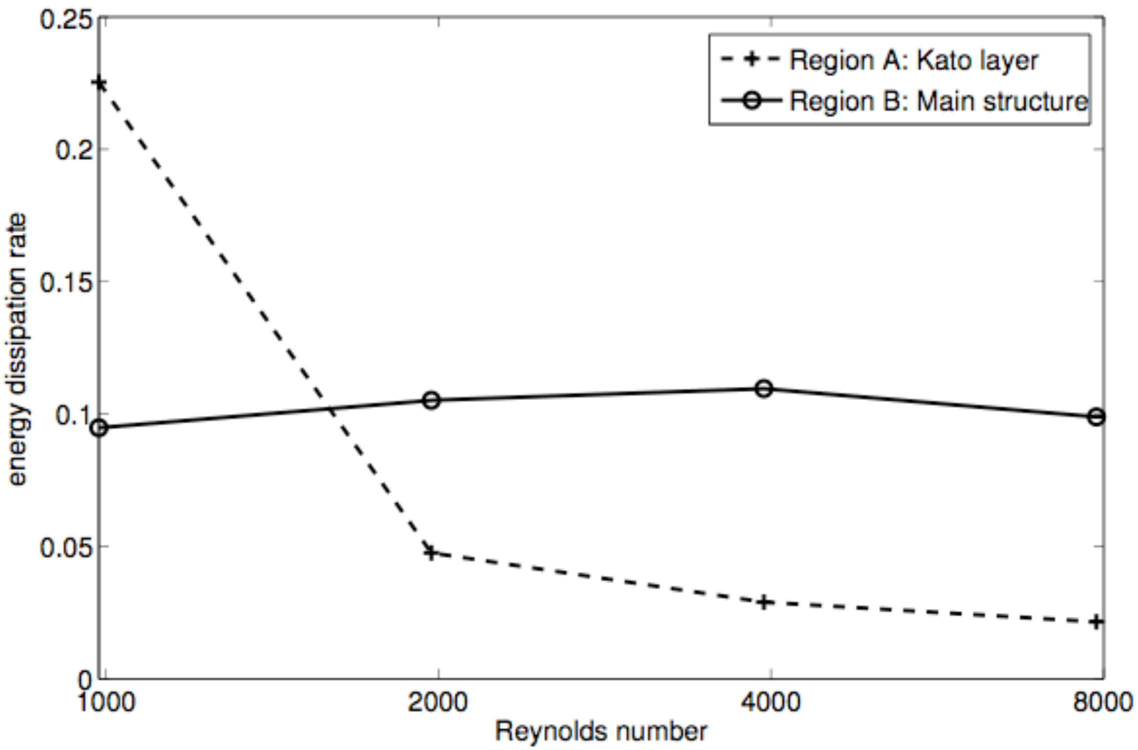
# Subregions

Define two subregions of interest in the flow :

- **region A** : vertical slab of width $10N^{-1}$ along the wall,
- **region B** : square box of side length 0.025 around the center of the main structure.

Integrate over A and B the **local** energy dissipation rate:

$$\varepsilon = \nu \left|\nabla u\right|^2$$



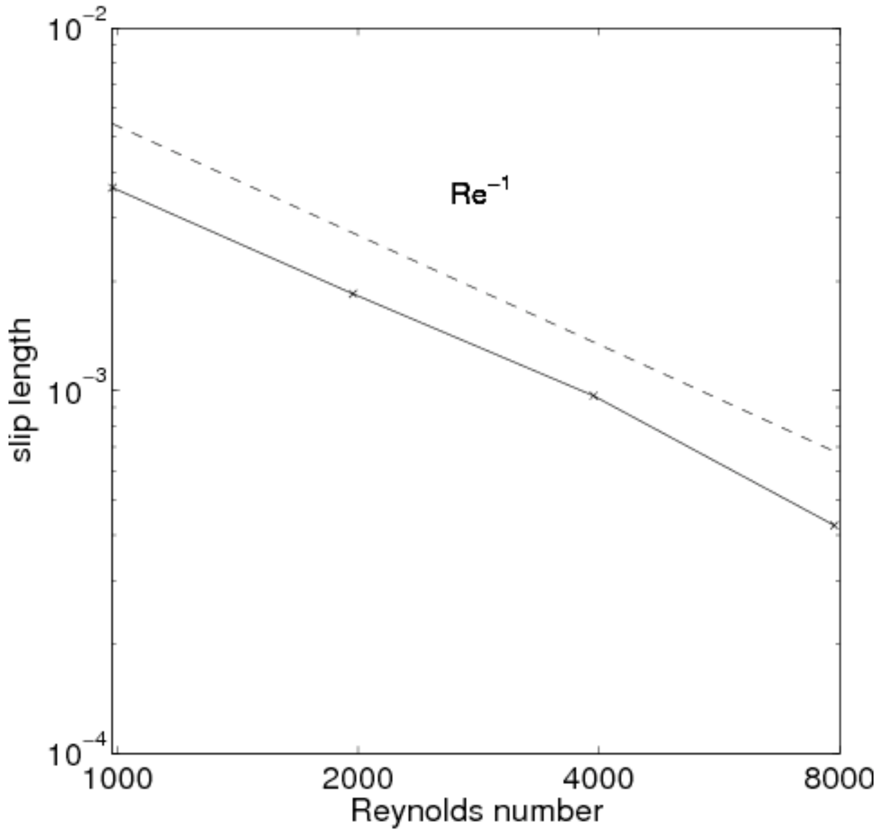
# Boundary conditions

- A posteriori, check what were the boundary conditions seen by the flow.
- The **normal velocity** is smaller than $10^{-3}$ (to be compared with the initial RMS velocity 0.443).
- The **tangential velocity** approximately satisfies a **Navier** boundary condition:

$$u_y + \alpha(\mathrm{Re})\partial_x u_y = 0$$

- With a slip length scaling like

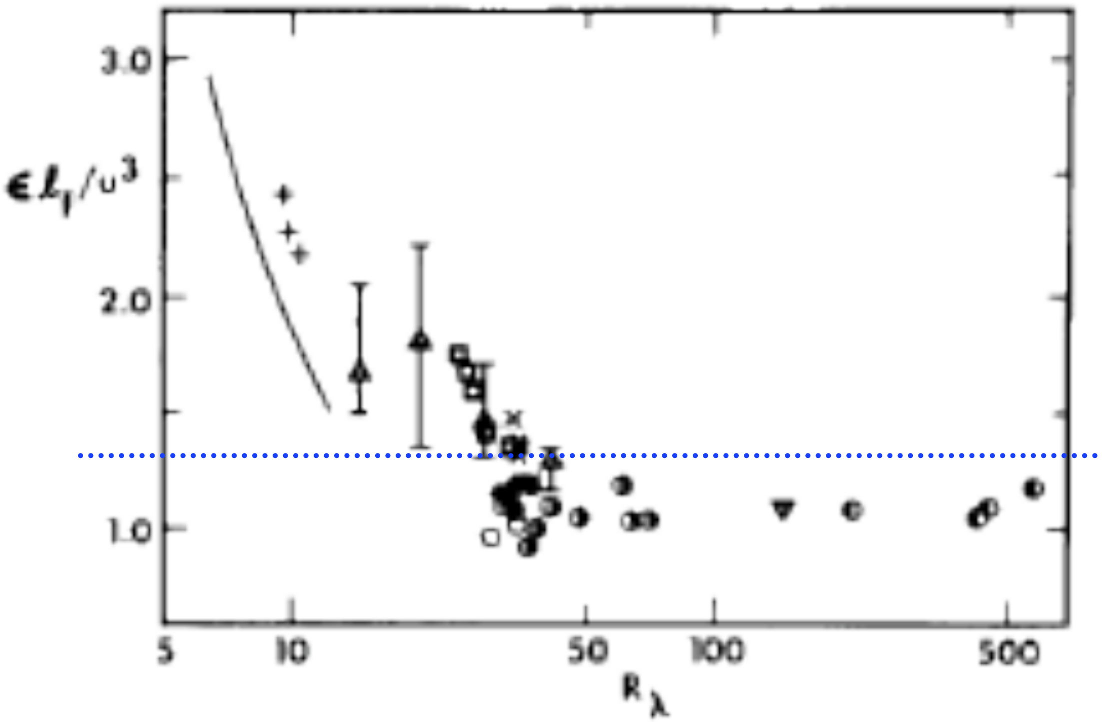
$$\alpha(\mathrm{Re}) \propto \mathrm{Re}^{-1}$$



# What about 3D flows?

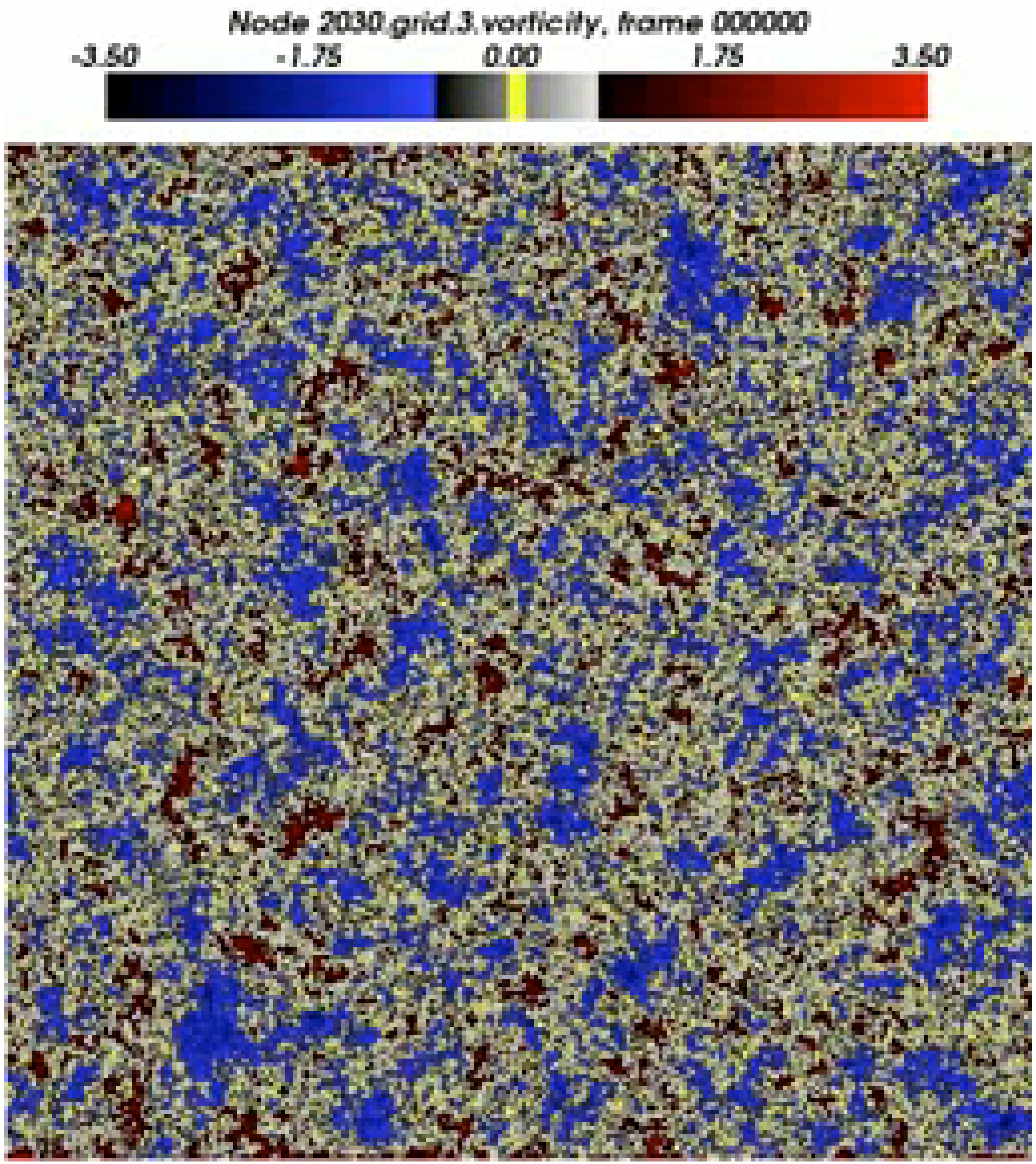
- The same phenomena are likely to play a role in 3D flows.

energy dissipation rate measured experimentally in flows behind grids




Molecular dissipation becomes Re-independent

*Fig. from Sreenisvasan (1984)*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I

Flows
in plasmas
and fluids

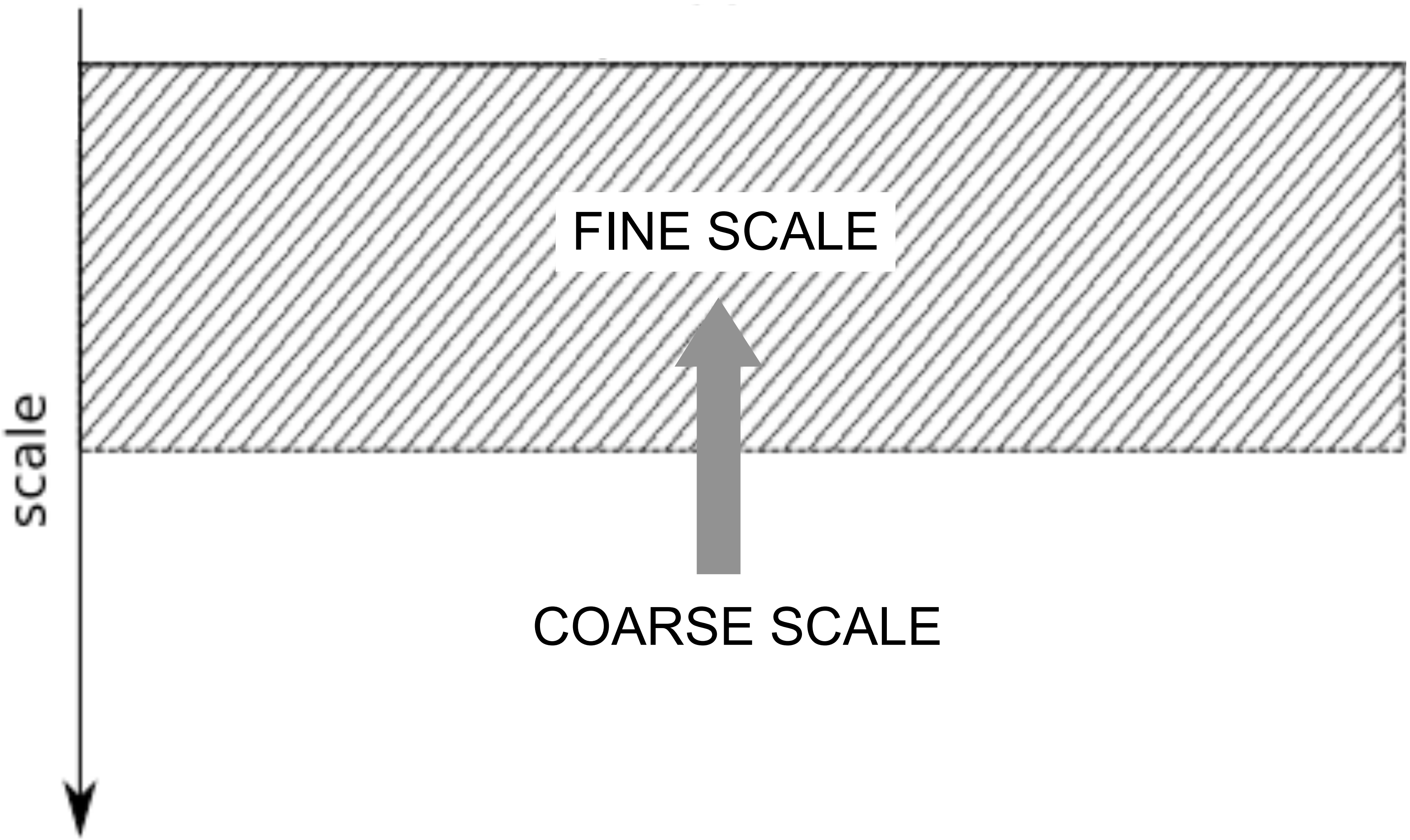
II

Dissipation
by flows

# III

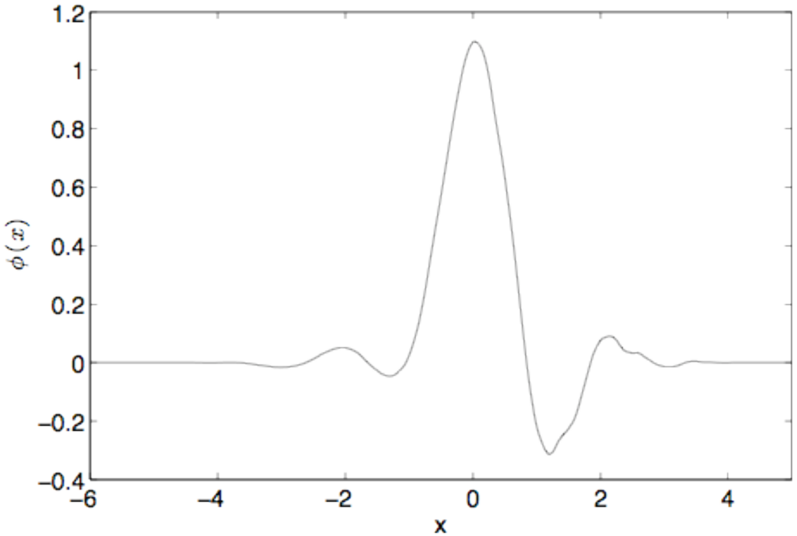
# Wavelet-based study of dissipation by flows

# Flow dissipation seen in Fourier space

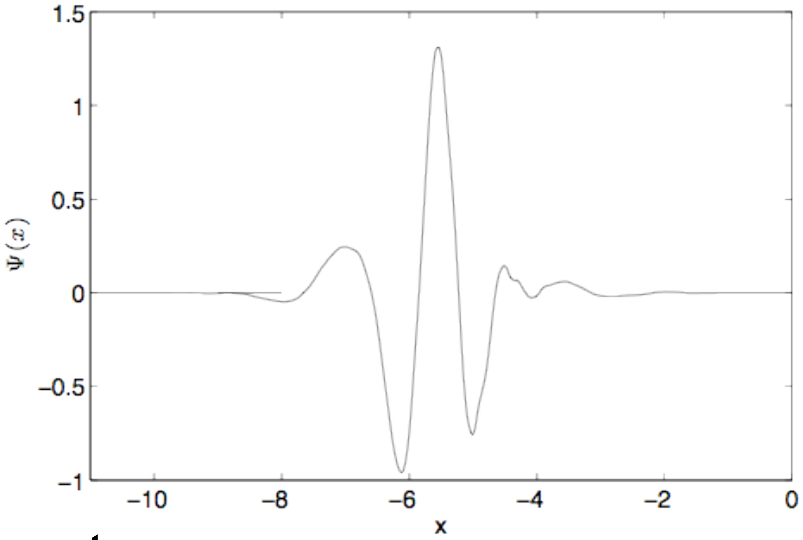


# Orthogonal wavelet bases

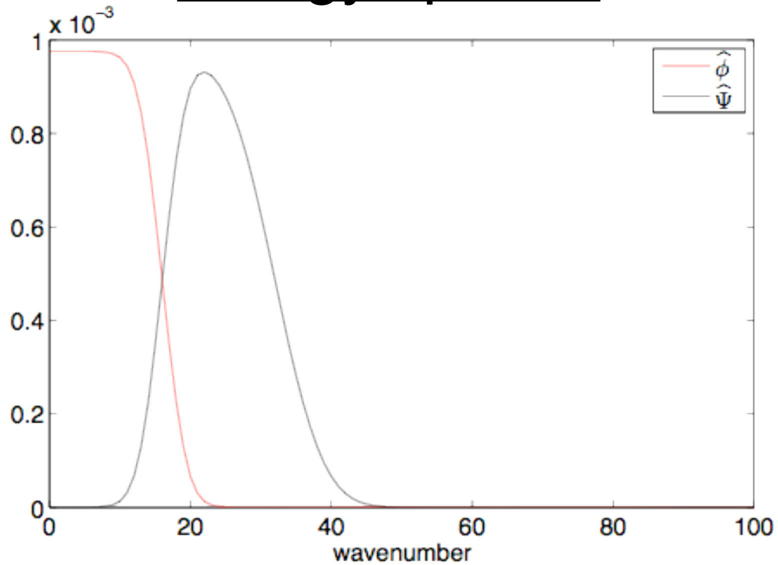
scaling function




wavelet



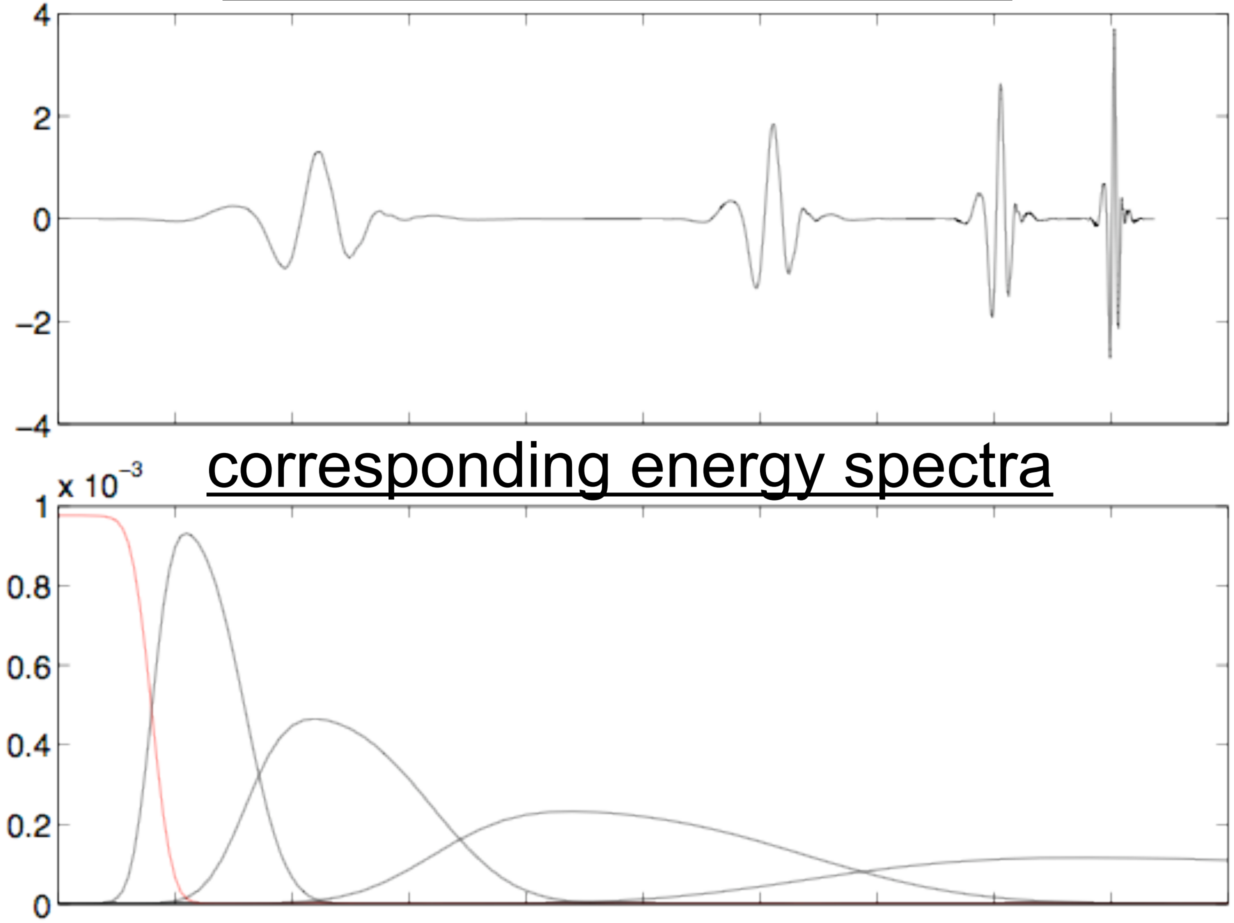

energy spectra



# Orthogonal wavelet bases

$$\psi_\lambda(x) = 2^{j/2}\psi(2^j x - i)$$

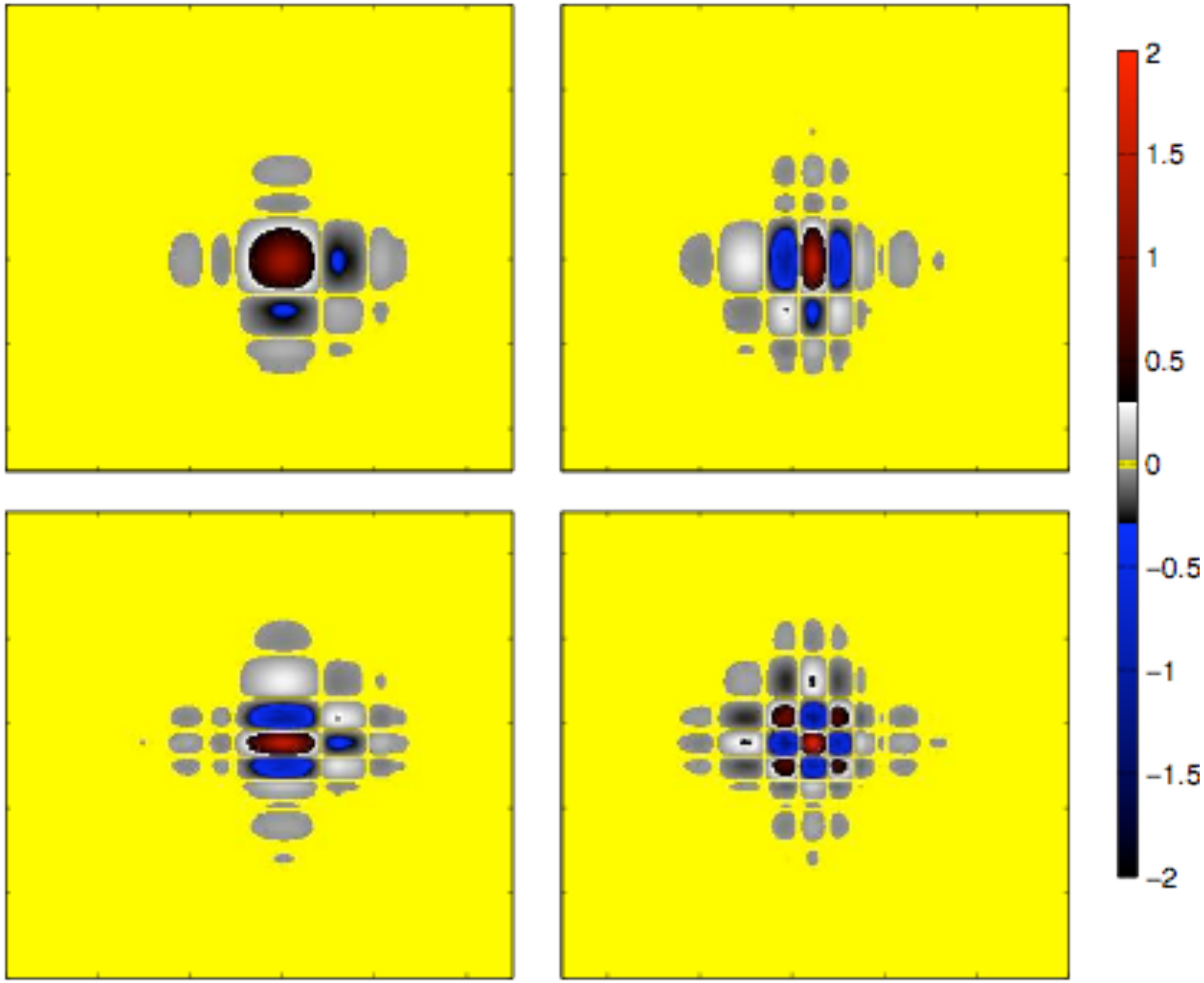
dilated / translated wavelets




corresponding energy spectra

# Orthogonal wavelet bases

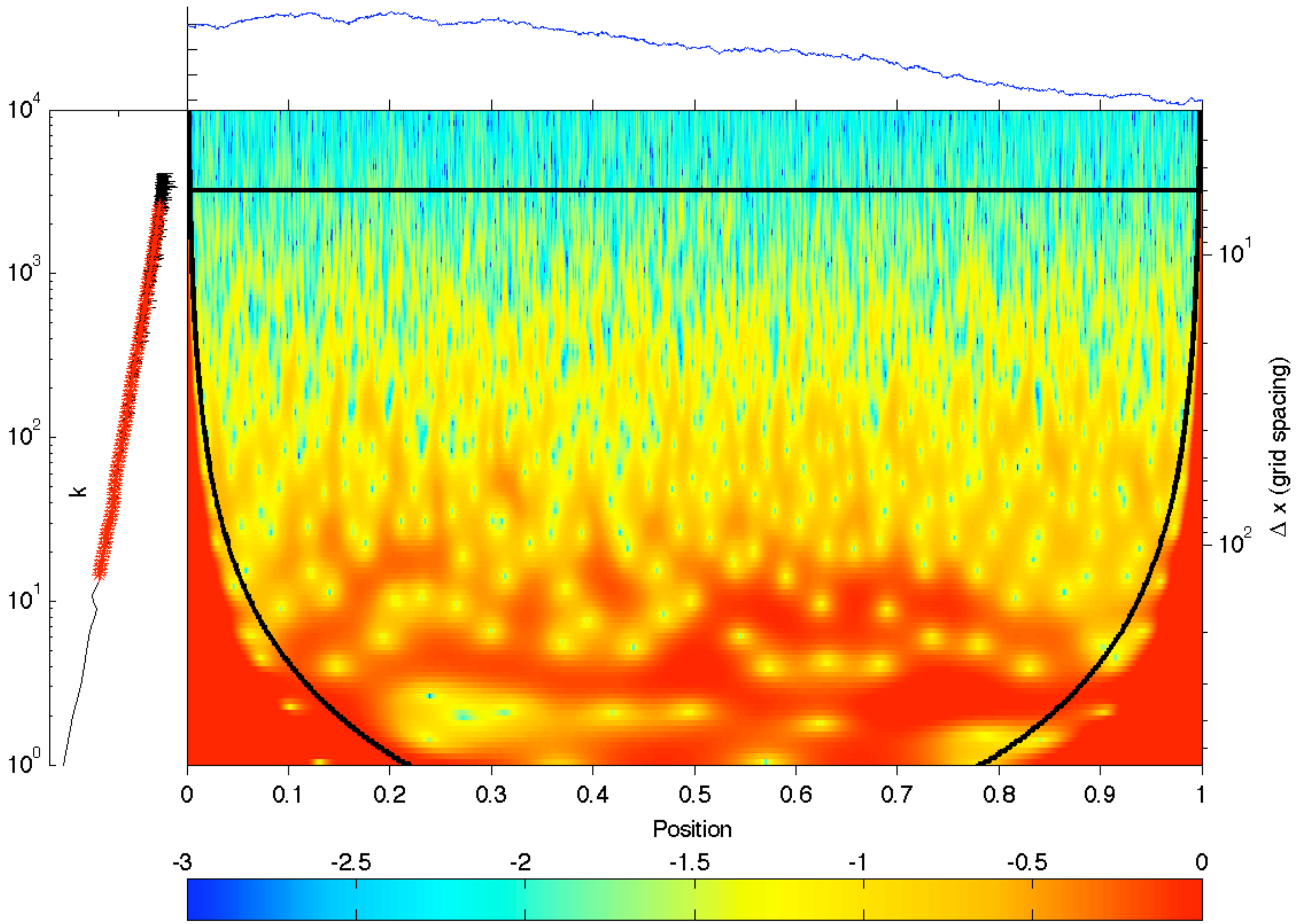
2d scaling function and wavelets



# Wavelets and spatial localization

Brownian motion

Two-step function



# Wavelet thresholding

- Orthogonal wavelet decomposition:

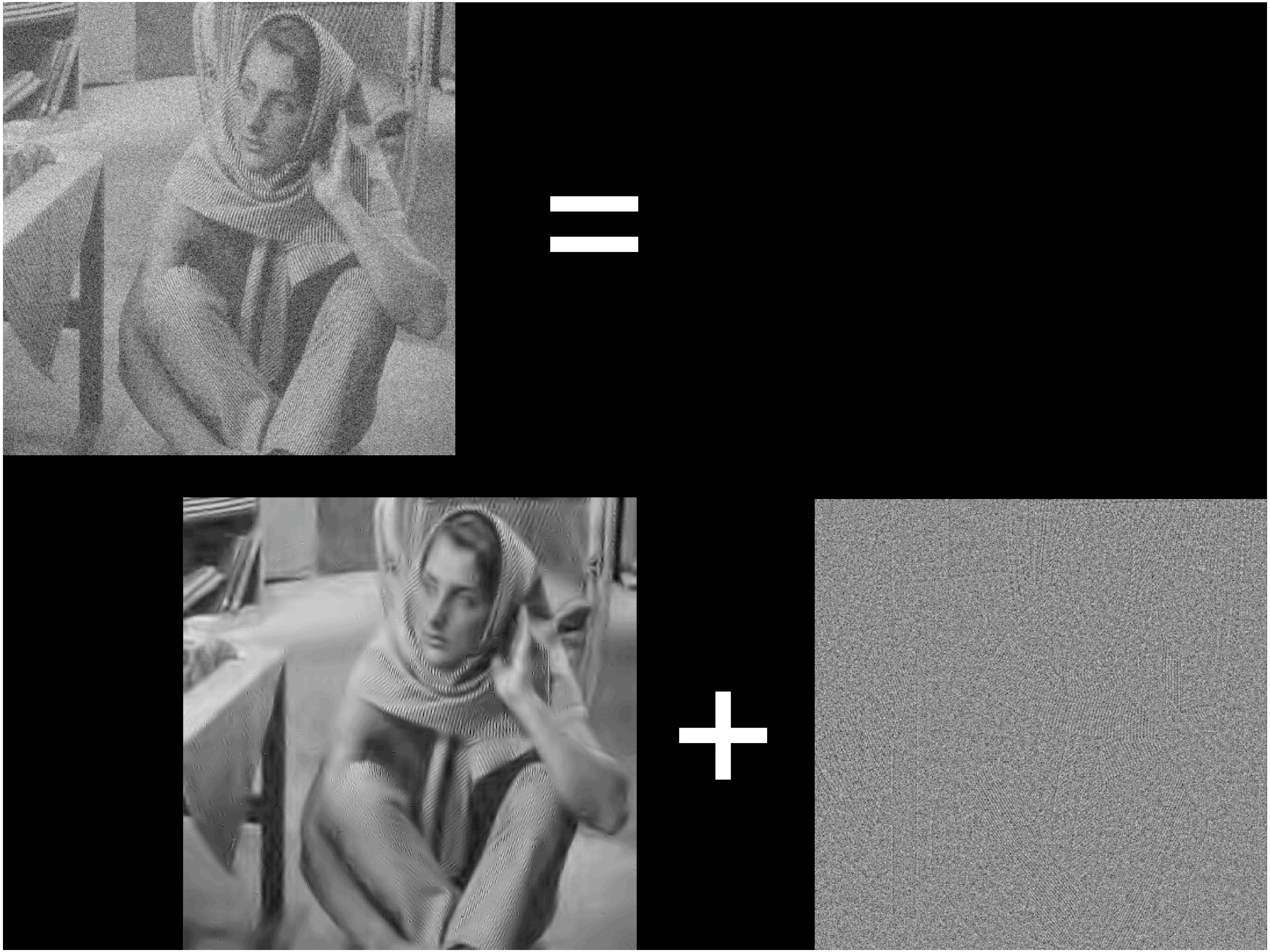
$$f = \sum_{\lambda \in \Lambda} \widetilde{f}_\lambda \psi_\lambda \text{ , where } \widetilde{f}_\lambda = \int_{\mathbb{T}^2} f \psi_\lambda$$

- Idea: split wavelet coefficients between two sets, "**large** coefficients" and "**small** coefficients":

$$f = \sum_{\lambda \in \Lambda_C} \widetilde{f}_\lambda \psi_\lambda + \sum_{\lambda \in \Lambda_I} \widetilde{f}_\lambda \psi_\lambda$$

Where large and small are defined with respect to a certain threshold:

$$|\widetilde{f}_\lambda| > \Theta_\lambda \text{ or } |\widetilde{f}_\lambda| \leq \Theta_\lambda$$



# Scale-wise coherent vorticity extraction

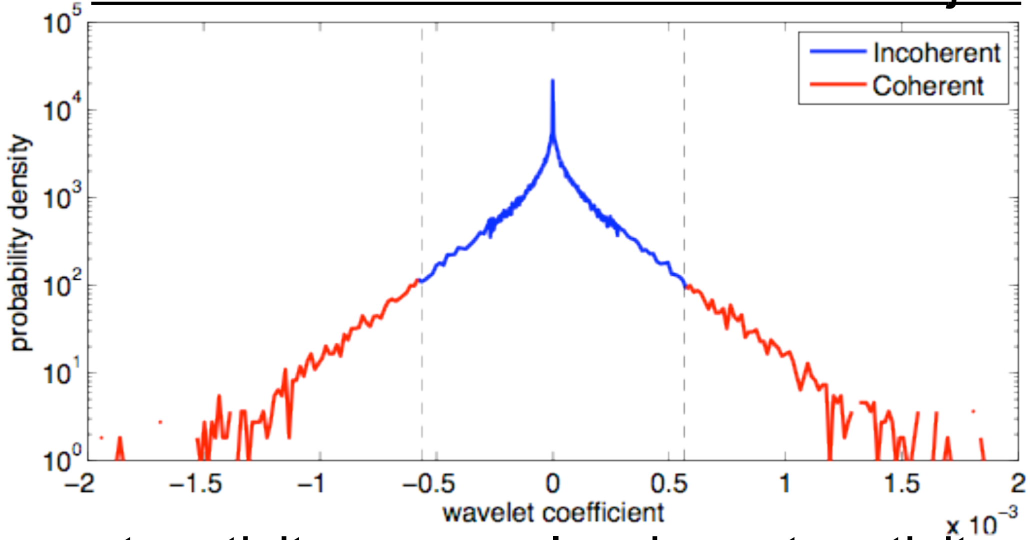
The threshold is defined at each scale by:

$$\Theta = q\sigma$$

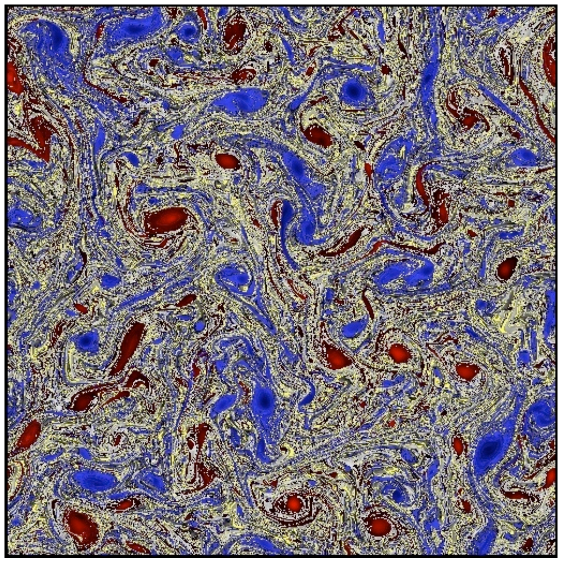
constant parameter → $q$

standard deviation → $\sigma$

PDF of wavelet coefficients at scale j=8




Total vorticity



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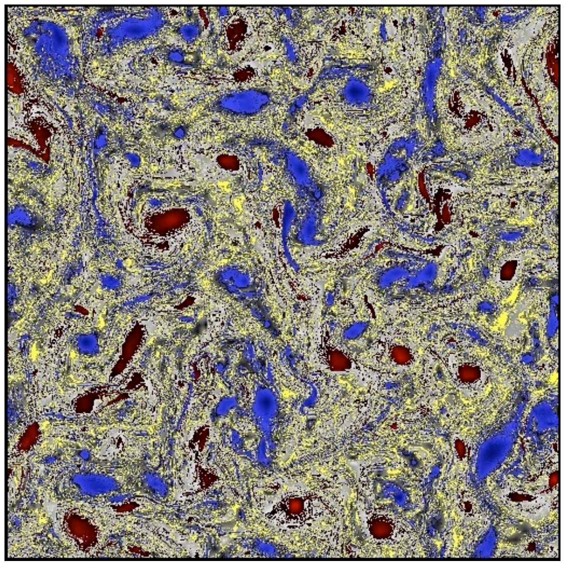
Coherent vorticity



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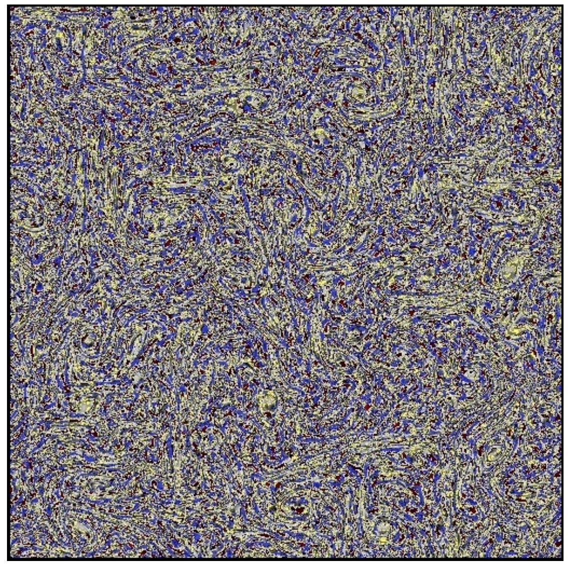
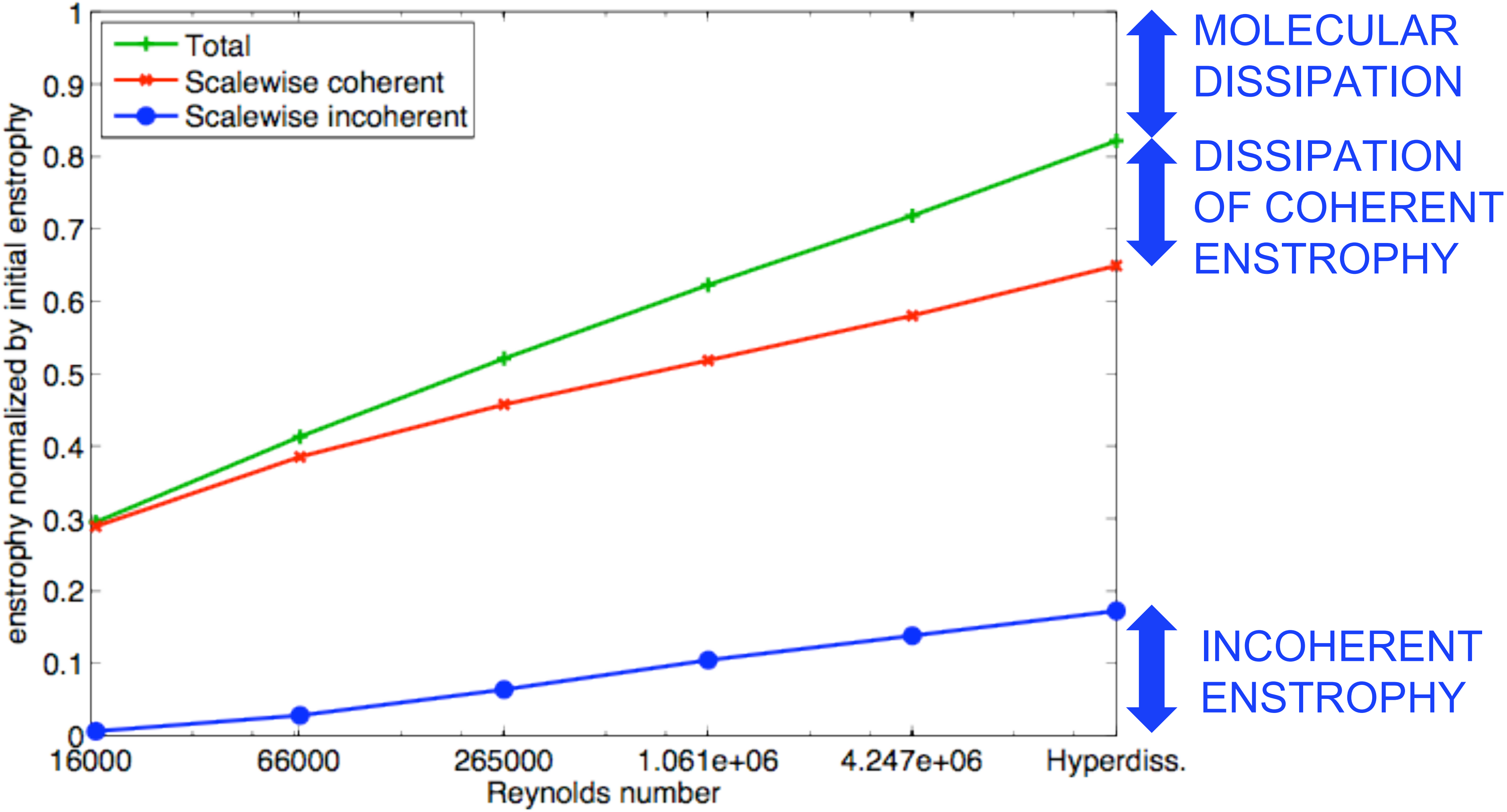
Incoherent vorticity

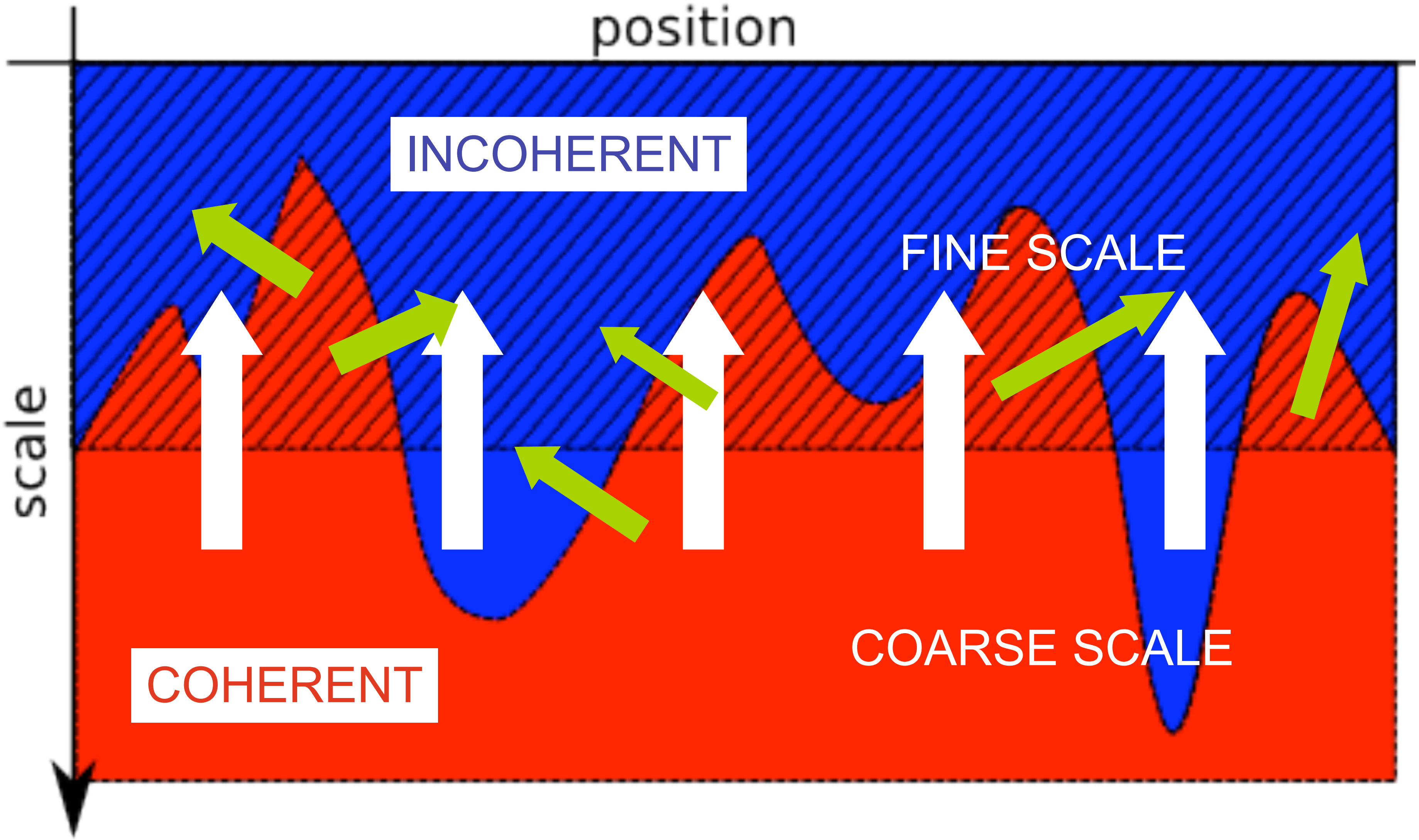


# Dissipation of coherent enstrophy

$$\frac{Z(t_1)}{Z(0)}$$



# Flow dissipation seen in wavelet space



# Summary and conclusion

- In the simplified framework of 2D incompressible fluids, we have shown that two dissipative mechanisms could exist completely **independently** of each other:
  - a purely macroscopic tendency to randomization of the flow,
  - a residual microscopic effect occuring in very localized structures.
- Impossible to distinguish them using Fourier analysis.
- But **wavelets** can be used to study them.

# Perspectives

- The interaction of the two dissipative mechanisms could be studied in the framework of a forced wall-bounded 2D flow.
- Is there **collaboration** or **competition** between them?
- What predictions can we make from the knowledge of the coherent flow only?
- What is the statistical uncertainty associated with these predictions?
- Can this approach be extended to 3D flows?

# References


- RNVY, M. Farge, K. Schneider, D. Kolomenskiy, N. Kingsbury, *Physica D* **237**, p. 2151

- RNVY, D. del Castillo-Negrete, K. Schneider, M. Farge, G. Chen, *J. Comp. Phys.* **229** p. 2821

- RNVY, M. Farge, K. Schneider, *ESAIM:Proc* **29** p. 89

- RNVY, M. Farge, K. Schneider, "Energy dissipating structures produced by walls in two-dimensional flows at vanishing viscosity", *submitted to Phys. Rev. Lett.*

- RNVY, M. Farge, K. Schneider, "Scale-wise coherent vorticity extraction for conditional statistical modelling of 2D homogeneous turbulence", *submitted to Physica D.*

- G. Khujadze, RNVY, K. Schneider, M. Oberlack, M. Farge, *accepted in CTR Proceedings 2010, Stanford-NASA Ames.*

# Proust on dissipation…

*Le temps efface tout comme effacent les vagues*

*Les travaux des enfants sur le sable aplani*

*Nous oublierons ces mots si précis et si vagues*

*Derrière qui chacun nous sentions l'infini.*

Marcel Proust