Coherent enstrophy dissipation in the inviscid limit of 2D turbulence

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Introduction

How to decompose turbulent fluctuations?

'In 1938 Tollmien and Prandtl suggested that turbulent fluctuations might consist of two components, a diffusive and a non-diffusive. Their ideas that fluctuations include both random and non random elements are correct, but as yet there is no known procedure for separating them.'

Hugh Dryden, Adv. Appl. Mech., 1, 1948

turbulent fluctuations
= **non random** + **random**
= **coherent structures** + **incoherent noise**

turbulent dynamics
= chaotic **non diffusive** + stochastic **diffusive**
= **inviscid nonlinear** dynamics + **turbulent dissipation**

Coherent Vorticity Simulation (CVS)

*Farge et al., Fluid Dyn. Res., **10**, 229, 1992*

*Farge, Schneider, Kevlahan, Phys. Fluids, **11** (8), 1999*

*Farge, Pellegrino, Schneider Phys. Rev, Lett. **87** (5), 2001*

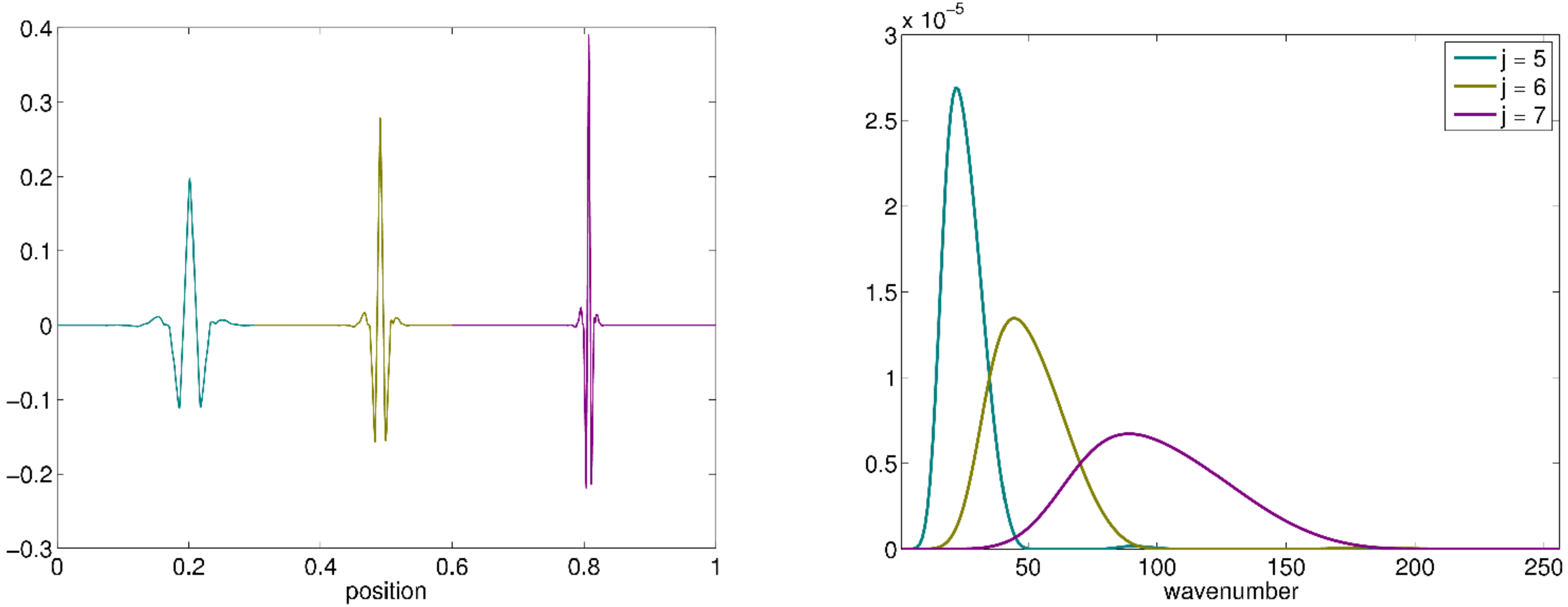
Definition of coherent enstrophy

1D Wavelet bases

- **Orthogonal wavelet bases** on the real line are obtained by dilating and translating a single, well chosen oscillating function.

- They have good locality properties both **in scale and space**.

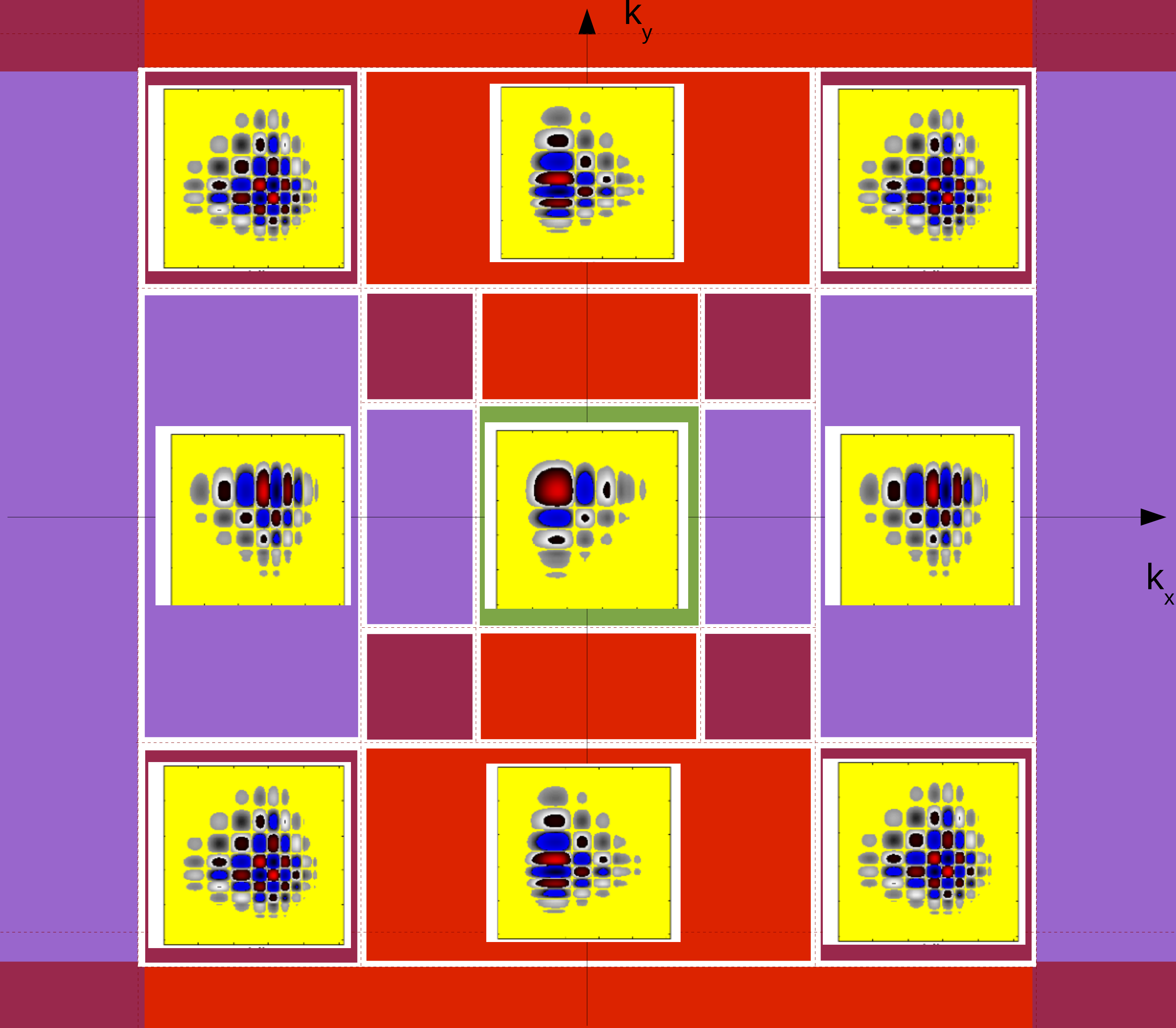
The « coiflet 12 » wavelets and their corresponding energy spectra.



the construction can be generalized to any dimension using the **multiresolution formalism**.

S. Mallat, *A wavelet tour of signal processing*, Academic Press (1999)

2D Wavelet bases



Scalewise and directionwise extraction

As a first guess, we make the hypothesis that the incoherent part is an **additive Gaussian noise**.

Gaussian contributions will correspond to the **smallest wavelet coefficients** at their respective scale.

Hence we can separate them by **thresholding**:

$$|\tilde{\omega}_{\mu,j,\mathbf{i}}| \leq T_{\mu,j} \quad \rightarrow \text{incoherent}$$

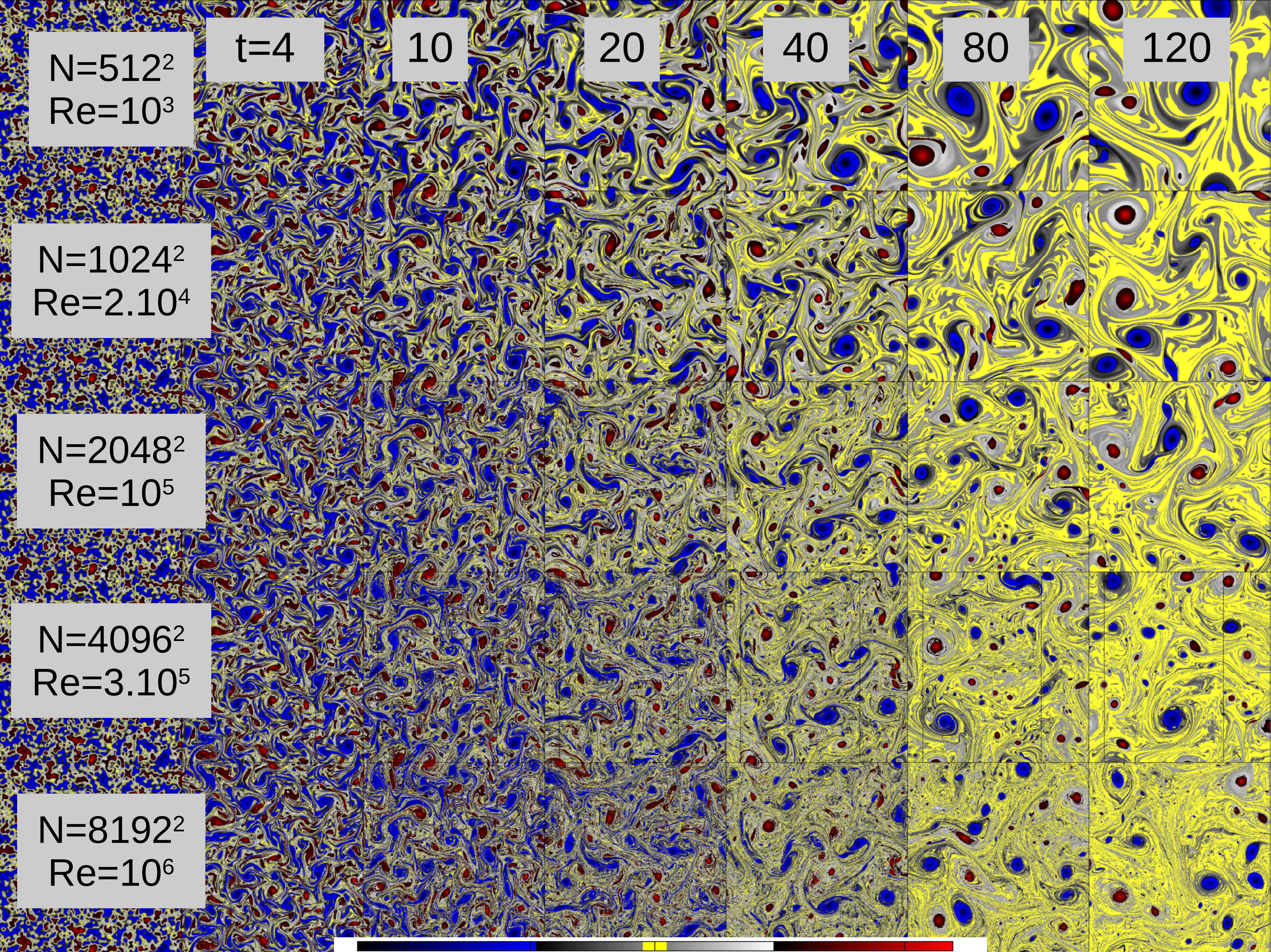
$$|\tilde{\omega}_{\mu,j,\mathbf{i}}| > T_{\mu,j} \quad \rightarrow \text{coherent}$$

μ is the direction
j is the scale
i is the position

The scalewise and directionwise thresholds are determined from the field itself using a fixed-point iterative procedure (*).

(*) Azzalini et al., *ACHA* **18** (2004)

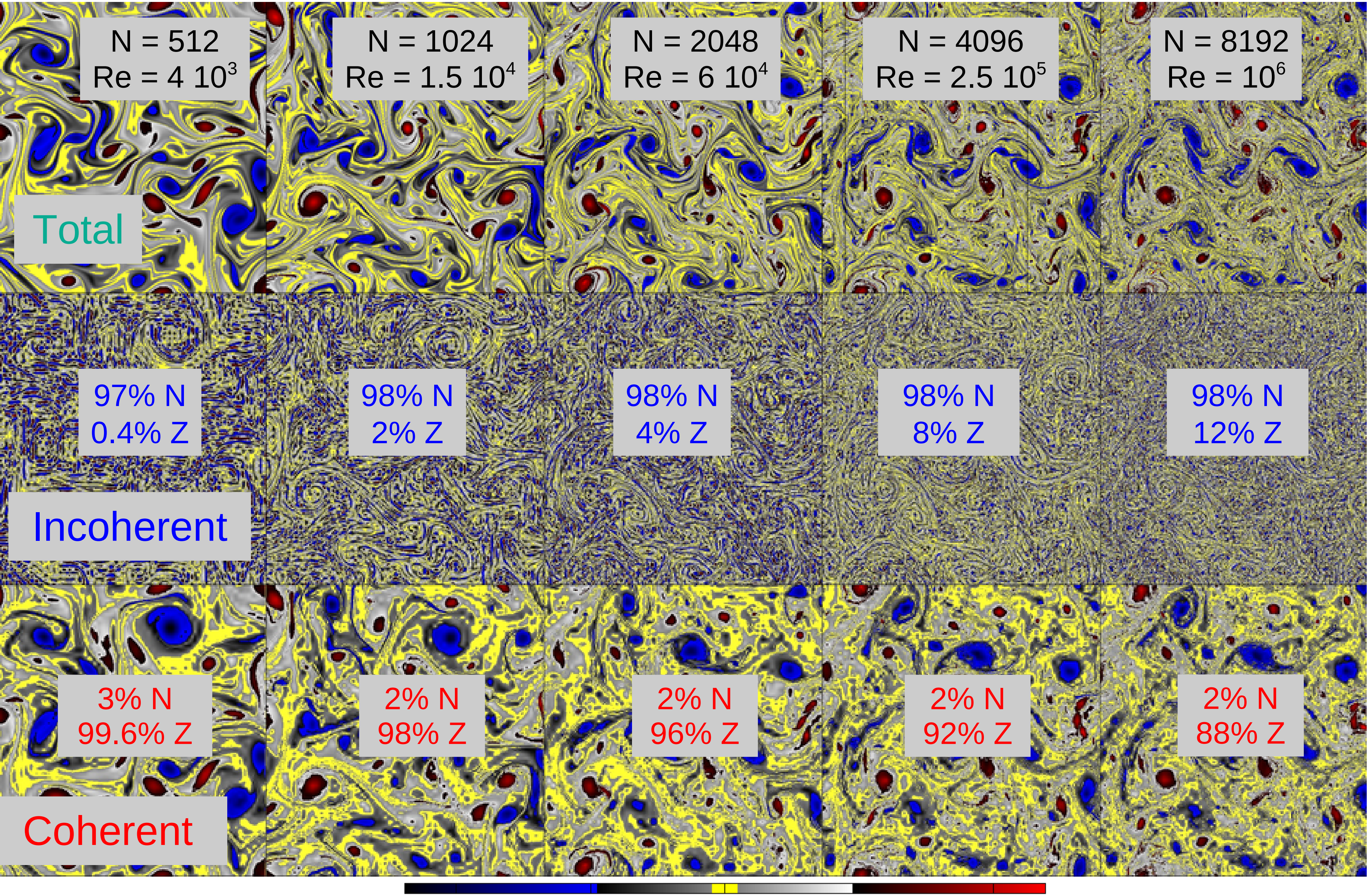
Results



N=512²
Re=10³
t=4
10
20
40
80
120
N=1024²
Re=2.10⁴
N=2048²
Re=10⁵
N=4096²
Re=3.10⁵
N=8192²
Re=10⁶

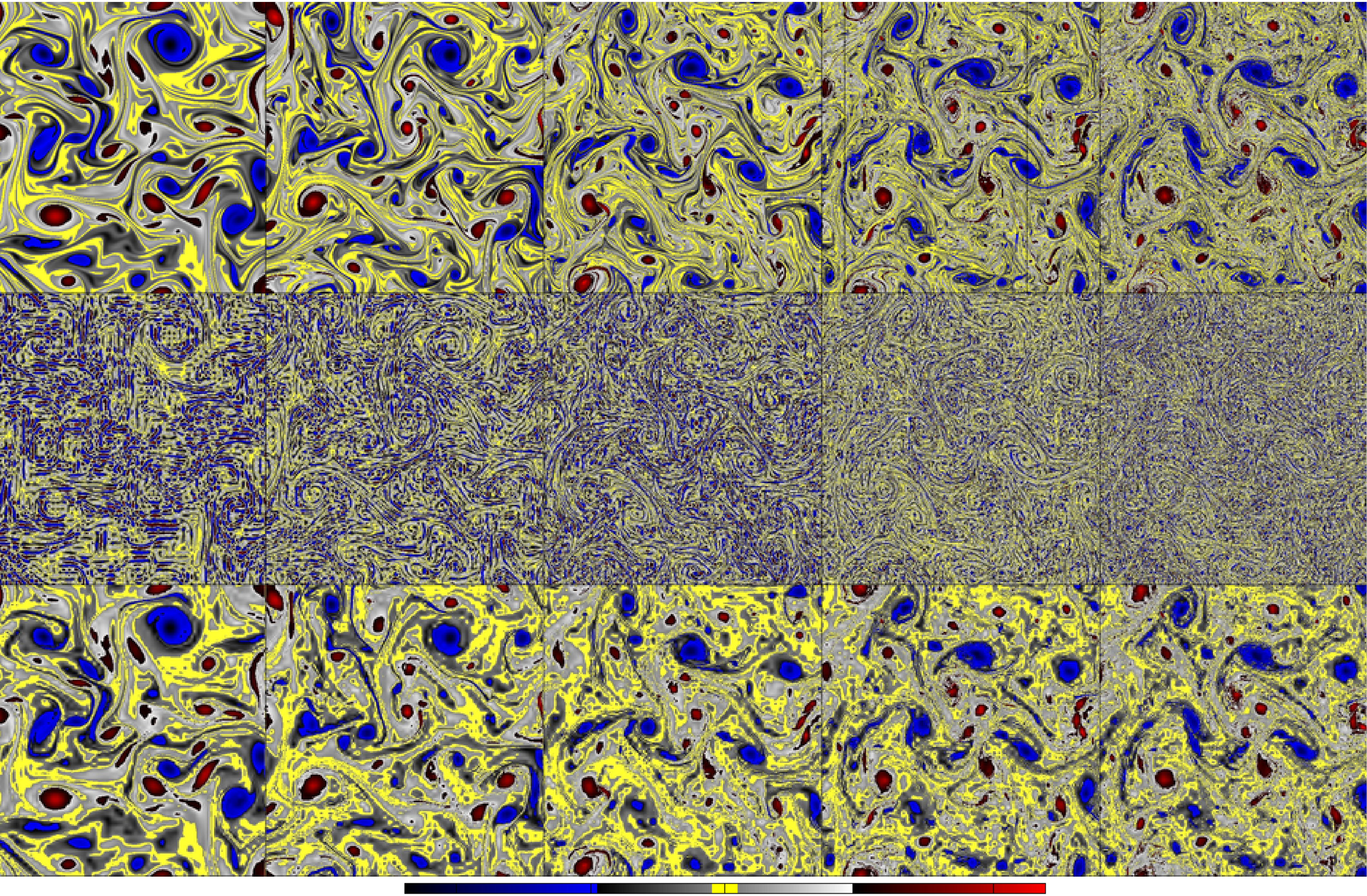
t = 40

Coherent Vorticity Extraction



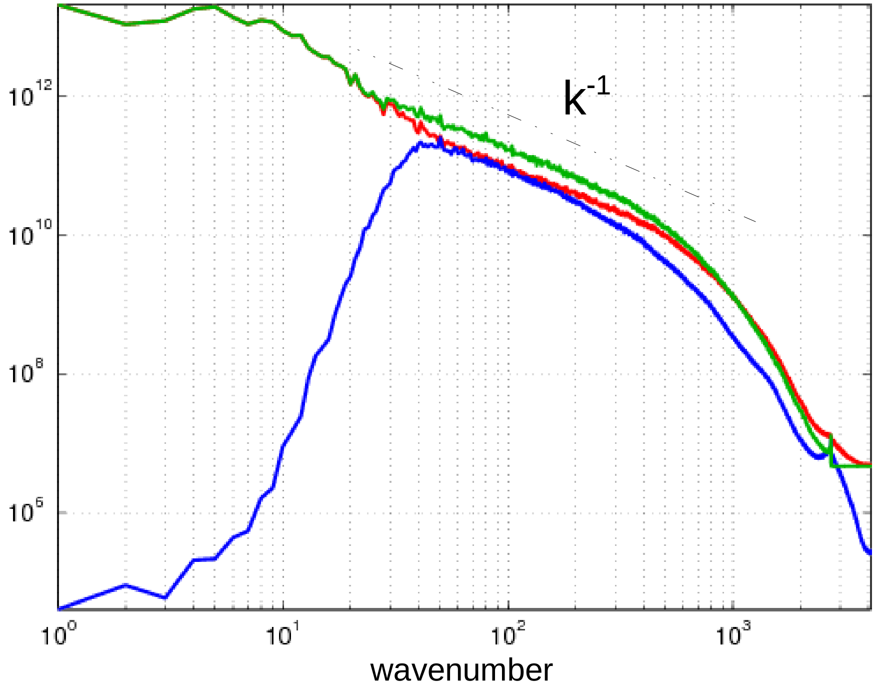
t = 40

Coherent Vorticity Extraction

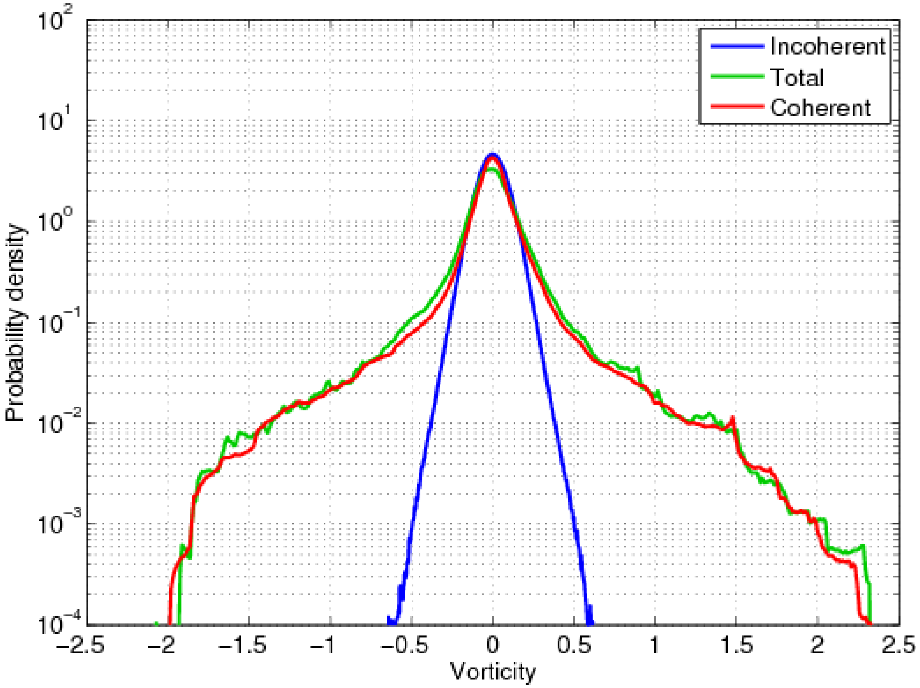


Scalewise statistics: Extraction Results

enstrophy spectra



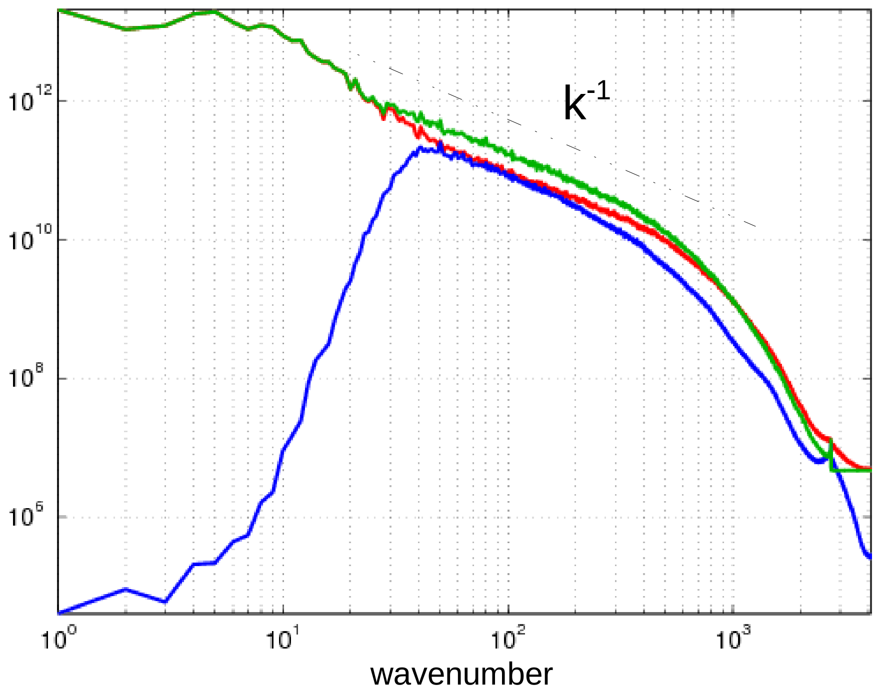
vorticity PDF



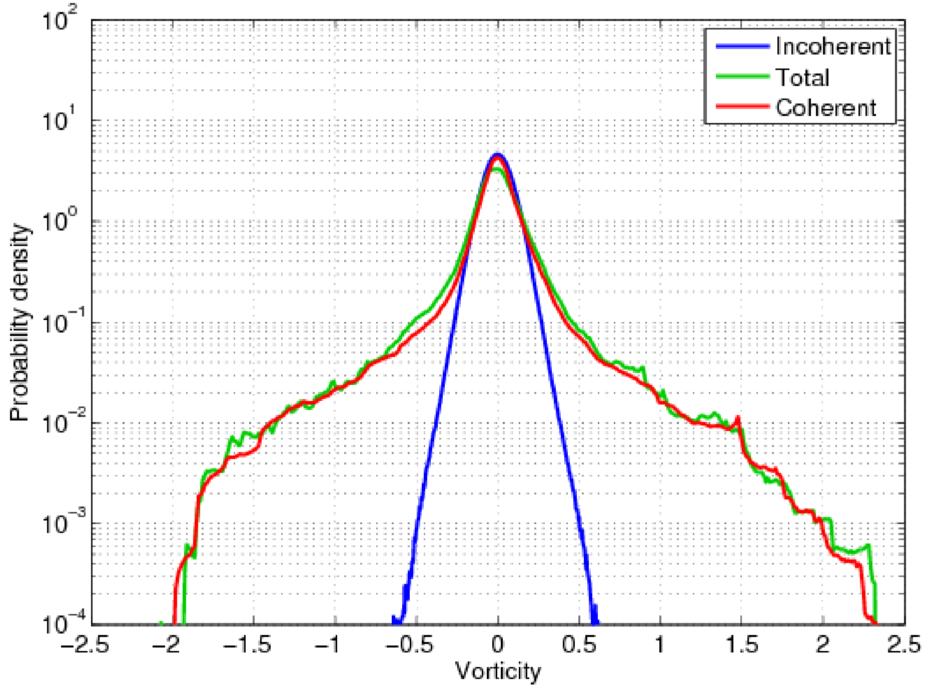
- the incoherent part has a k^{-1} inertial range spectrum

Scalewise statistics: Extraction Results

enstrophy spectra



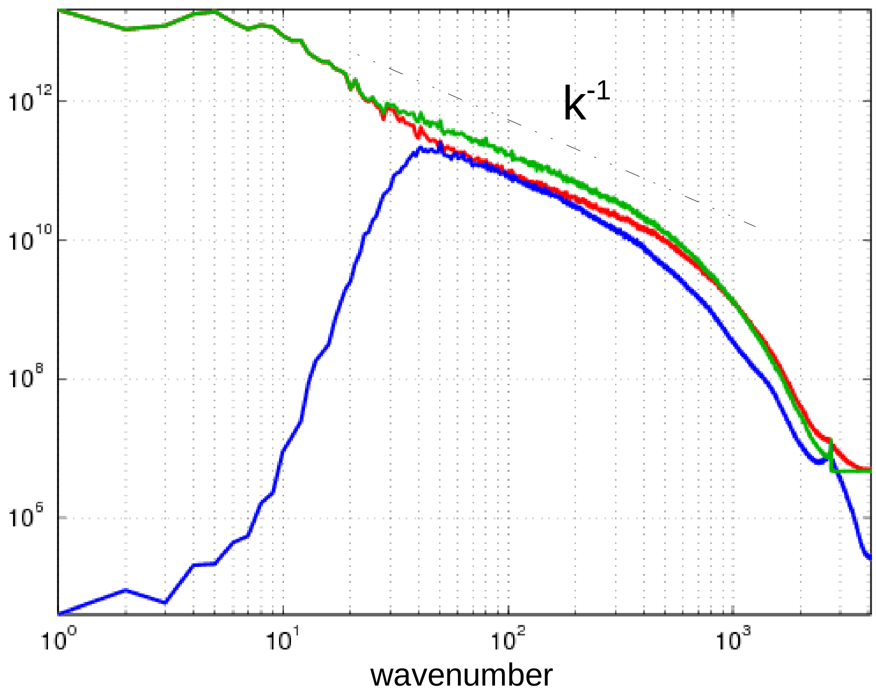
vorticity PDF



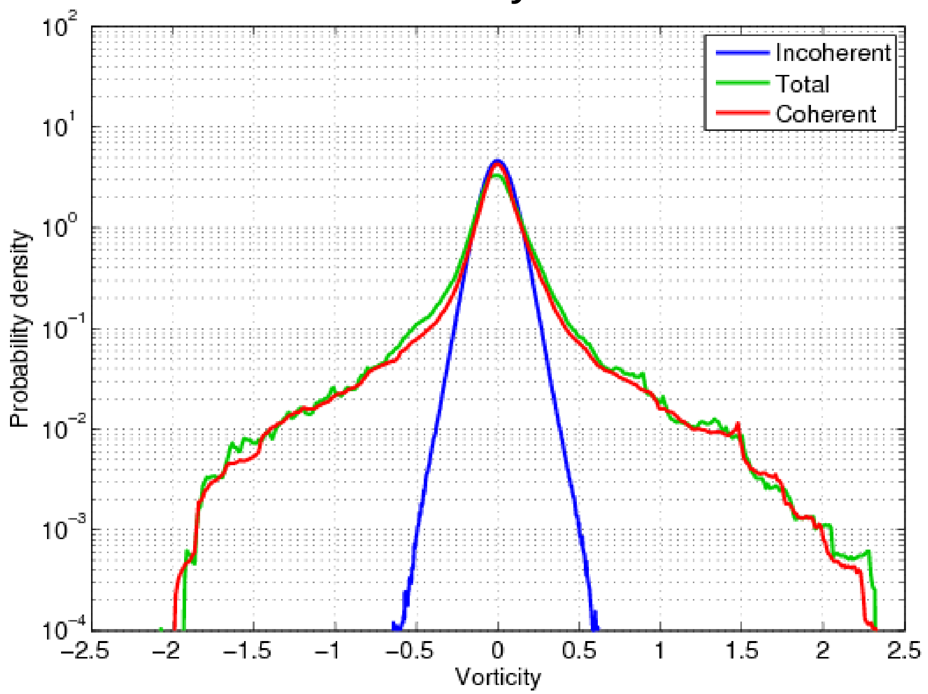
- the incoherent part has a k^{-1} inertial range spectrum
- the coherent part dominates in the dissipative range (!!)

Scalewise statistics: Extraction Results

enstrophy spectra



vorticity PDF

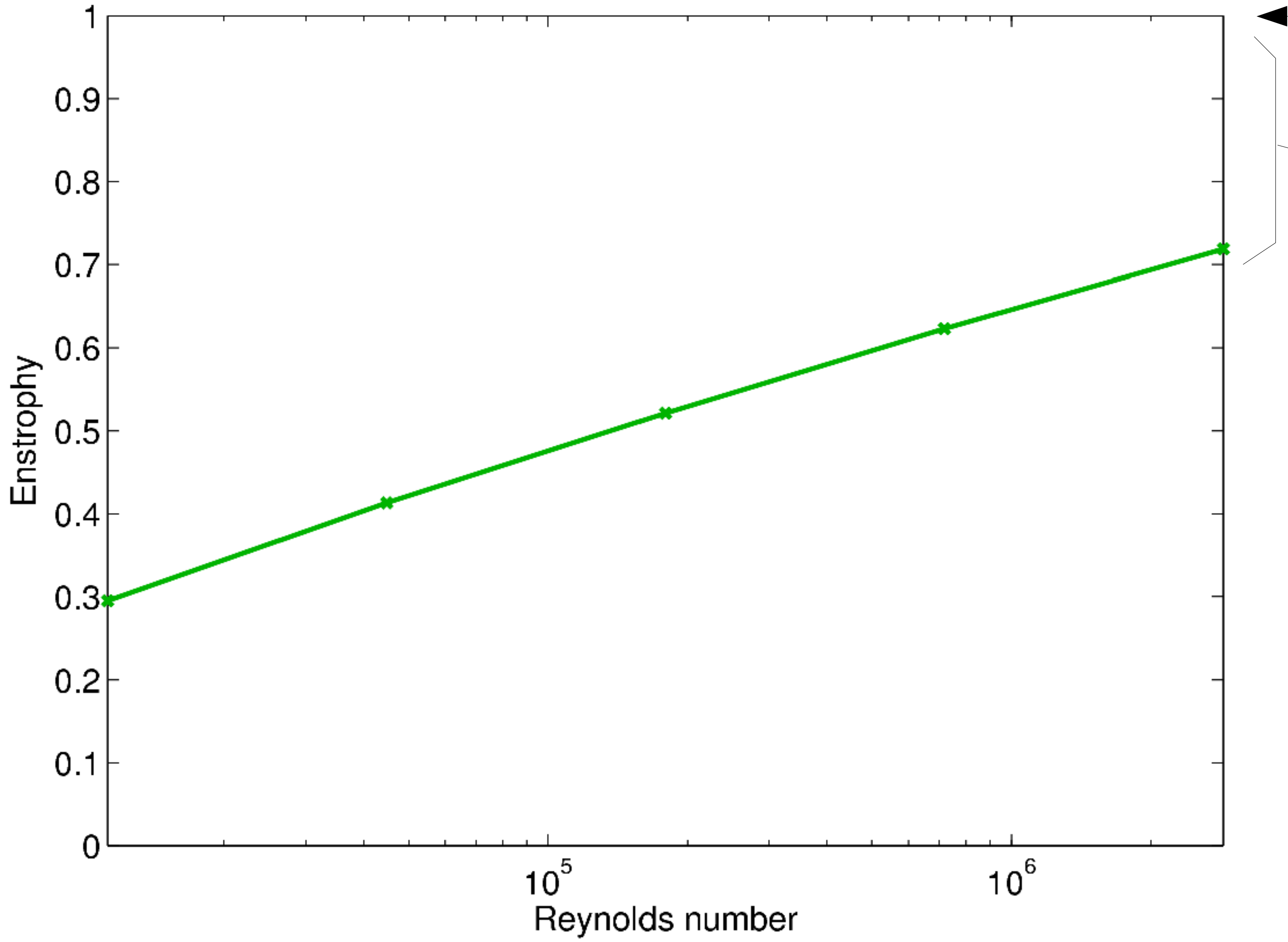


- the incoherent part has a k^{-1} inertial range spectrum
- the coherent part dominates in the dissipative range (!!)

- the incoherent part is close to marginally Gaussian

Dissipation of coherent enstrophy

enstrophy at t = 50 normalized by initial enstrophy



initial enstrophy

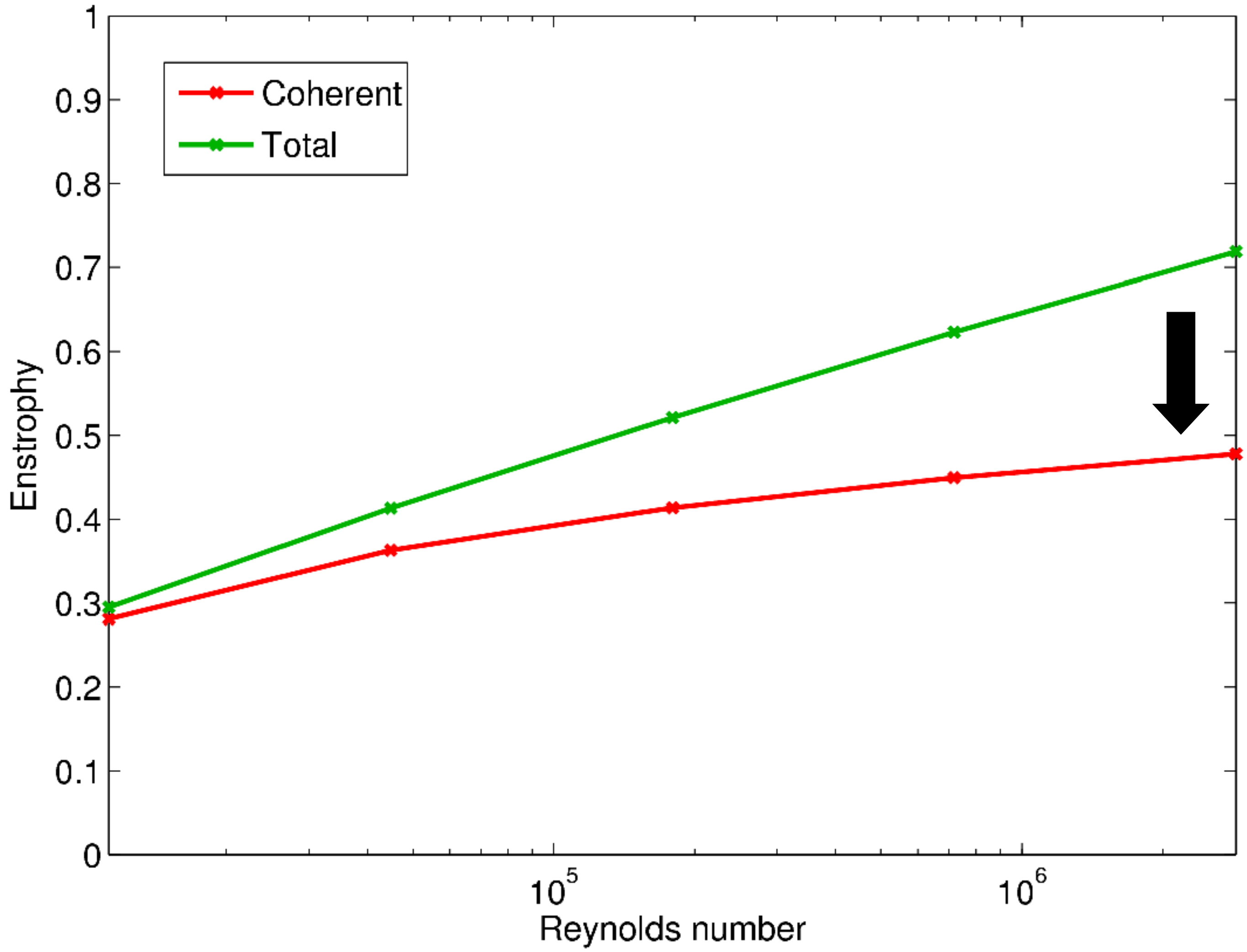
enstrophy that has been dissipated between t = 0 and t = 50

total enstrophy **does not dissipate** in the inviscid limit (*)

(*) Dmitruk & Montgomery 2005, Tran & Dritschel 2006

Dissipation of coherent enstrophy

enstrophy at t = 50 normalized by initial enstrophy



initial enstrophy

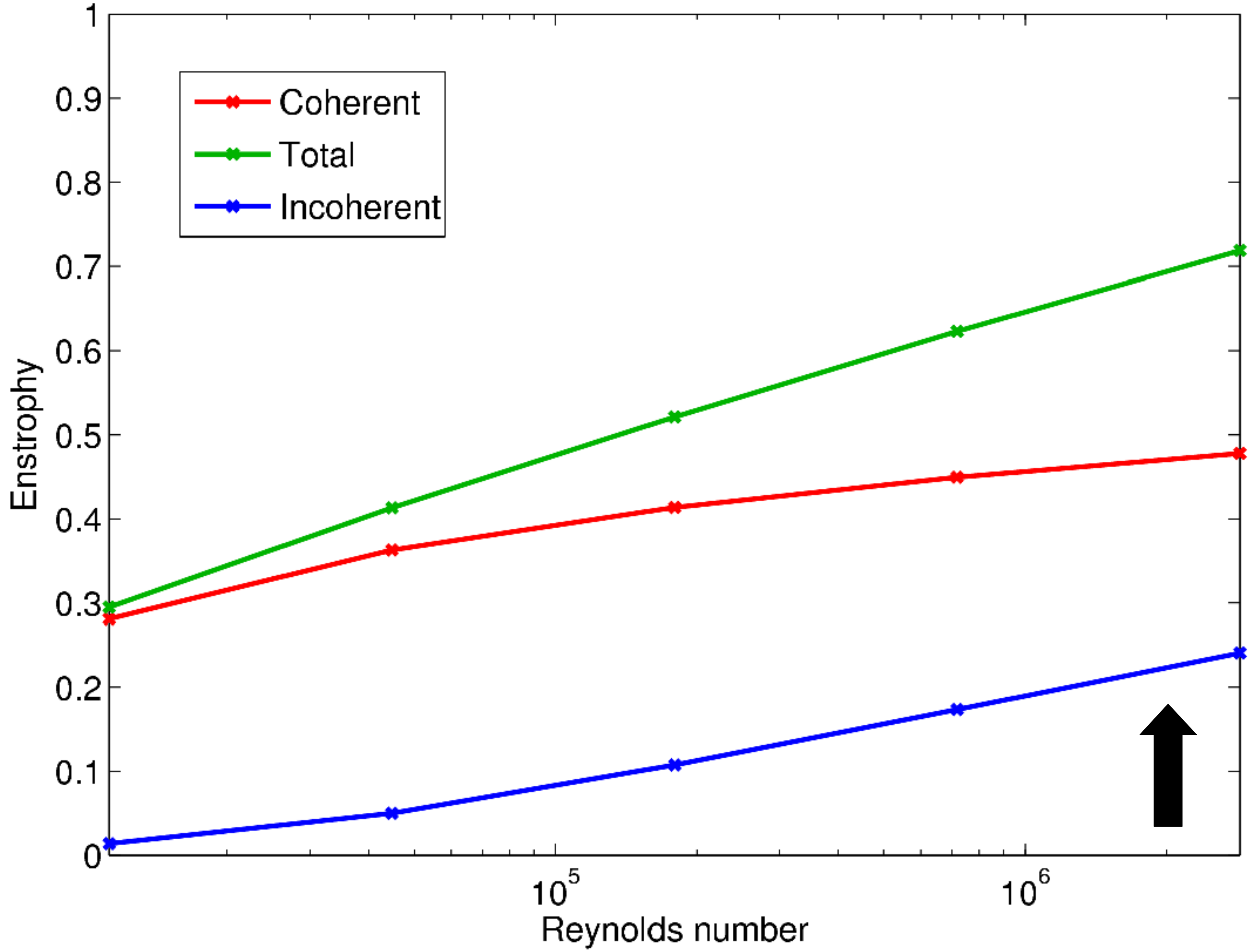
coherent
enstrophy that has
been dissipated
between t = 0
and t = 50

total enstrophy
does not dissipate
in the inviscid limit

...but
coherent enstrophy
dissipates
in the inviscid limit

Dissipation of coherent enstrophy

enstrophy at t = 50 normalized by initial enstrophy



total enstrophy
does not dissipate
in the inviscid limit

...but
coherent enstrophy
dissipates
in the inviscid limit

due to the production
of incoherent enstrophy

Conclusion

- **Coherent enstrophy** was defined using scalewise statistics of the vorticity field that could be obtained thanks to a wavelet transform.
- The Navier-Stokes equations at increasingly high Reynolds numbers were solved using a classical pseudo-spectral method.
- The analysis of the numerical solutions shows that **coherent enstrophy is dissipated in the inviscid limit**, even though total enstrophy is conserved.
- The remainder, **incoherent enstrophy**, gets spread between wavelet coefficients that behave like a **correlated Gaussian process** with a spectral slope -1, like the total vorticity field.
- We conjecture that only the coherent coefficients have to be solved for deterministically using **Coherent Vortex Simulation**, while the incoherent ones could be modelled by a random process.

More references:

http://wavelets.ens.fr

Numerical tools (incl. parallel wavelet transform):

http://justpmf.com/romain/kicksey_winsey