Coherent enstrophy dissipation in the inviscid limit of 2D turbulence

Romain Nguyen van yen¹
Marie Farge¹
Kai Schneider²

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¹ Laboratoire de Météorologie Dynamique-CNRS, ENS Paris, France
² Laboratoire de Mécanique, Modélisation et Procédés Propres-CNRS, CMI-Université d'Aix-Marseille, France
Introduction
How to decompose turbulent fluctuations?

‘In 1938 Tollmien and Prandtl suggested that turbulent fluctuations might consist of two components, a diffusive and a non-diffusive. Their ideas that fluctuations include both random and non random elements are correct, but as yet there is no known procedure for separating them.’


\[
\text{turbulent fluctuations} = \text{non random} + \text{random} = \text{coherent structures} + \text{incoherent noise}
\]

\[
\text{turbulent dynamics} = \text{chaotic non diffusive} + \text{stochastic diffusive} = \text{inviscid nonlinear dynamics} + \text{turbulent dissipation}
\]

Coherent Vorticity Simulation (CVS)


Farge, Schneider, Kevlahan, Phys. Fluids, 11 (8), 1999

Definition of coherent enstrophy
1D Wavelet bases

- **Orthogonal wavelet bases** on the real line are obtained by dilating and translating a single, well chosen oscillating function.

- They have good locality properties both **in scale and space**.

The « coiflet 12 » wavelets and their corresponding energy spectra.

The construction can be generalized to any dimension using the **multiresolution formalism**.

2D Wavelet bases
Scalewise and directionwise extraction

As a first guess, we make the hypothesis that the incoherent part is an additive Gaussian noise.

Gaussian contributions will correspond to the smallest wavelet coefficients at their respective scale.

Hence we can separate them by thresholding:

\[ |\tilde{\omega}_{\mu,j,i}| \leq T_{\mu,j} \quad \rightarrow \text{incoherent} \]
\[ |\tilde{\omega}_{\mu,j,i}| > T_{\mu,j} \quad \rightarrow \text{coherent} \]

\[ \mu \text{ is the direction} \]
\[ j \text{ is the scale} \]
\[ i \text{ is the position} \]

The scalewise and directionwise thresholds are determined from the field itself using a fixed-point iterative procedure (\(*)\).

\( \ast \) Azzalini et al., ACHA 18 (2004)
Results
Coherent Vorticity Extraction

$t = 40$

N = 512
Re = $4 \times 10^3$

N = 1024
Re = $1.5 \times 10^4$

N = 2048
Re = $6 \times 10^4$

N = 4096
Re = $2.5 \times 10^5$

N = 8192
Re = $10^6$

Total

97% N
0.4% Z

98% N
2% Z

98% N
4% Z

98% N
8% Z

98% N
12% Z

Incoherent

3% N
99.6% Z

2% N
98% Z

2% N
96% Z

2% N
92% Z

2% N
88% Z

Coherent

3% N
99.6% Z

2% N
98% Z

2% N
96% Z

2% N
92% Z

2% N
88% Z
t = 40

Coherent Vorticity Extraction
• the incoherent part has a $k^{-1}$ inertial range spectrum
the incoherent part has a $k^{-1}$ inertial range spectrum
the coherent part dominates in the dissipative range (!!)
Scalewise statistics: Extraction Results

- the incoherent part has a $k^{-1}$ inertial range spectrum
- the coherent part dominates in the dissipative range (!!!)

- the incoherent part is close to marginally Gaussian
Dissipation of coherent enstrophy

Enstrophy at $t = 50$ normalized by initial enstrophy

Initial enstrophy

Enstrophy that has been dissipated between $t = 0$ and $t = 50$

Total enstrophy does not dissipate in the inviscid limit (*)

(*) Dmitruk & Montgomery 2005, Tran & Dritschel 2006
Dissipation of coherent enstrophy

Enstrophy at $t = 50$ normalized by initial enstrophy

- Coherent
- Total

Initial enstrophy

Coherent enstrophy that has been dissipated between $t = 0$ and $t = 50$

Total enstrophy does not dissipate in the inviscid limit

...but coherent enstrophy dissipates in the inviscid limit
Dissipation of coherent enstrophy

Enstrophy at \( t = 50 \) normalized by initial enstrophy

- Total enstrophy does not dissipate in the inviscid limit
- Coherent enstrophy dissipates in the inviscid limit due to the production of incoherent enstrophy
Conclusion

- **Coherent enstrophy** was defined using scalewise statistics of the vorticity field that could be obtained thanks to a wavelet transform.

- The Navier-Stokes equations at increasingly high Reynolds numbers were solved using a classical pseudo-spectral method.

- The analysis of the numerical solutions shows that **coherent enstrophy is dissipated in the inviscid limit**, even though total enstrophy is conserved.

- The remainder, **incoherent enstrophy**, gets spread between wavelet coefficients that behave like a **correlated Gaussian process** with a spectral slope -1, like the total vorticity field.

- We conjecture that only the coherent coefficients have to be solved for deterministically using **Coherent Vortex Simulation**, while the incoherent ones could be modelled by a random process.

More references:

http://wavelets.ens.fr

Numerical tools (incl. parallel wavelet transform):

http://justpmf.com/romain/kicksey_winsey