Wavelets meet Burgulence : CVS-filtered Burgers equation

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http://wavelets.ens.fr

Coherent vortex simulation (CVS)…

… is a multiscale method for computing turbulent flows.

Traditional spectral methods	CVS
Expand the solution over a family of modes selected *a priori*. Numerical accuracy requires regularity of the solution, usually enforced by high viscosity.	Expand the solution over an **adaptive** set of wavelets, selected via **nonlinear** thresholding. Discard the small wavelet coefficients.

A simple question: does CVS also need viscosity, or does nonlinear thresholding take care of dissipation ?

Roadmap:

- Take the 1D Burgers equation as a toy model,
- Solve it with a traditional Fourier pseudo-spectral method,
- Apply nonlinear wavelet thresholding at every timestep,
- Compare with the known analytical solution of the inviscid equation.

A toy model : the 1D Burgers equation

- The inviscid limit is mathematically well understood,
- Shocks are an extreme case of inhomogeneous regularity,
- There is a known analytical solution.

Viscous Burgers equation

$$\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial u^2}{\partial x}=\nu\frac{\partial^2 u}{\partial x^2}$$

Inviscid Burgers equation

$$\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial u^2}{\partial x}=0$$

- Periodic boundary conditions on [-1,1].
- The initial condition is one realization of a Gaussian white noise, viscously integrated for 0.3 time units to develop shocks.
- The equations are discretized on a uniform grid with N points.
- Our scheme is carefully adjusted to have negligible intrinsic dissipation.

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with $\nu > 0$ = **CASE 1**

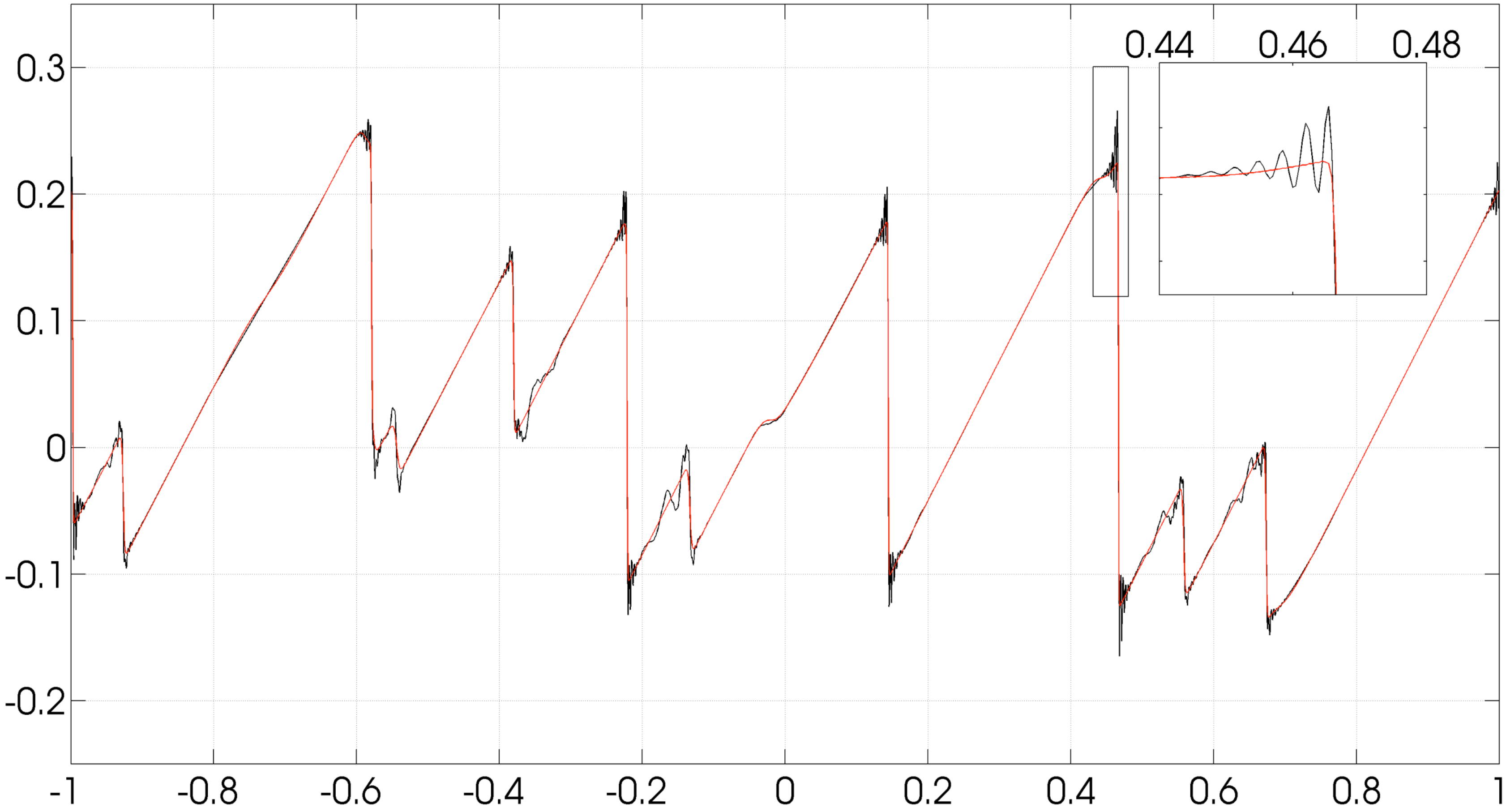
Inviscid Burgers equation

$$\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial u^2}{\partial x}=0$$

+ CVS filter = **CASE 2**

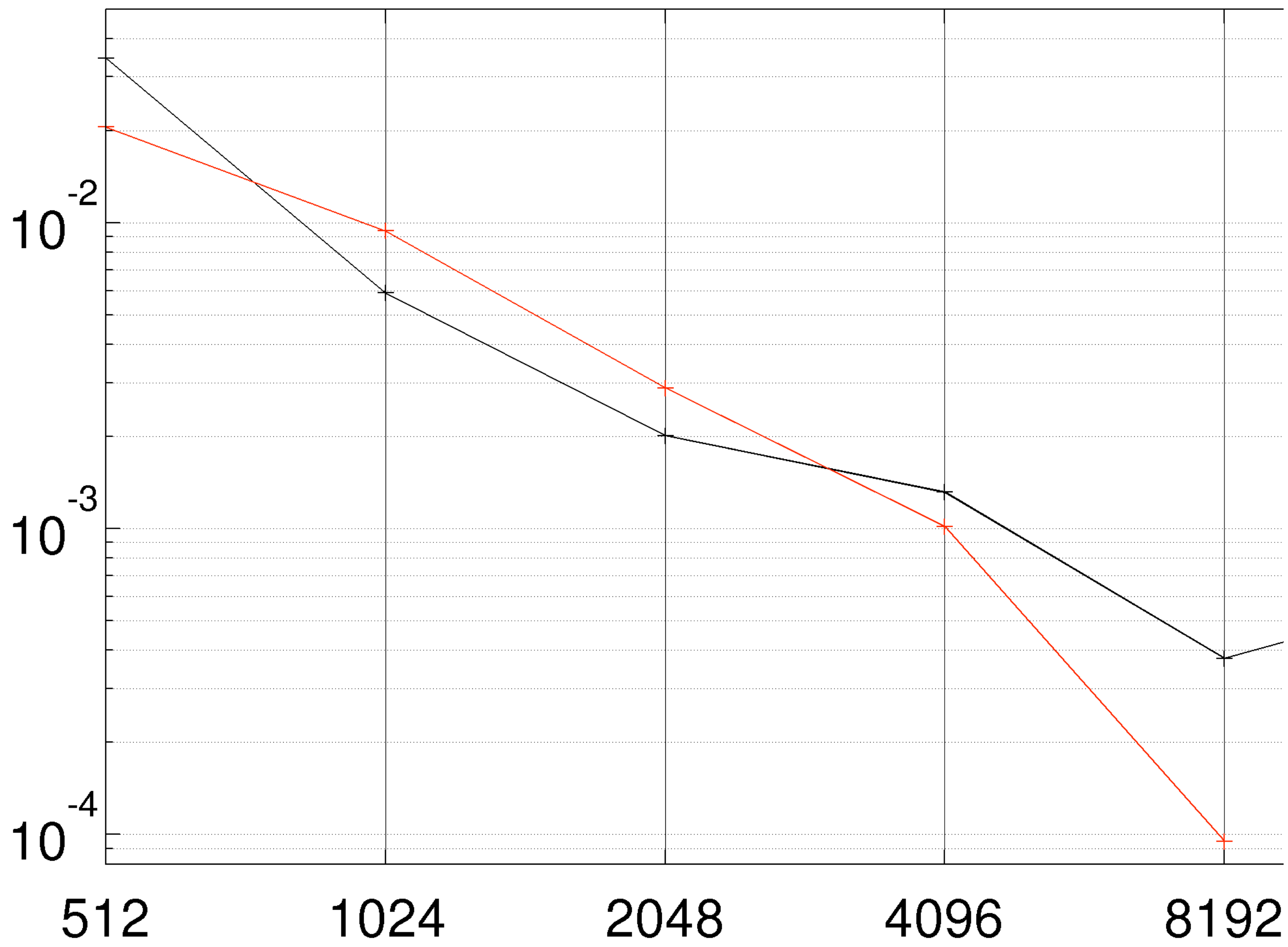
Results

For a given resolution, the CVS-filtered equation is equivalent to the viscous equation with a small ad-hoc viscosity.



Results

When resolution increases, viscous and filtered solutions converge to the entropy solution at the same rate.



The filter is on the poster.

Thank you !

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