Tomographic reconstruction of Tore Supra edge plasma radiation by wavelet-vaguelette decomposition

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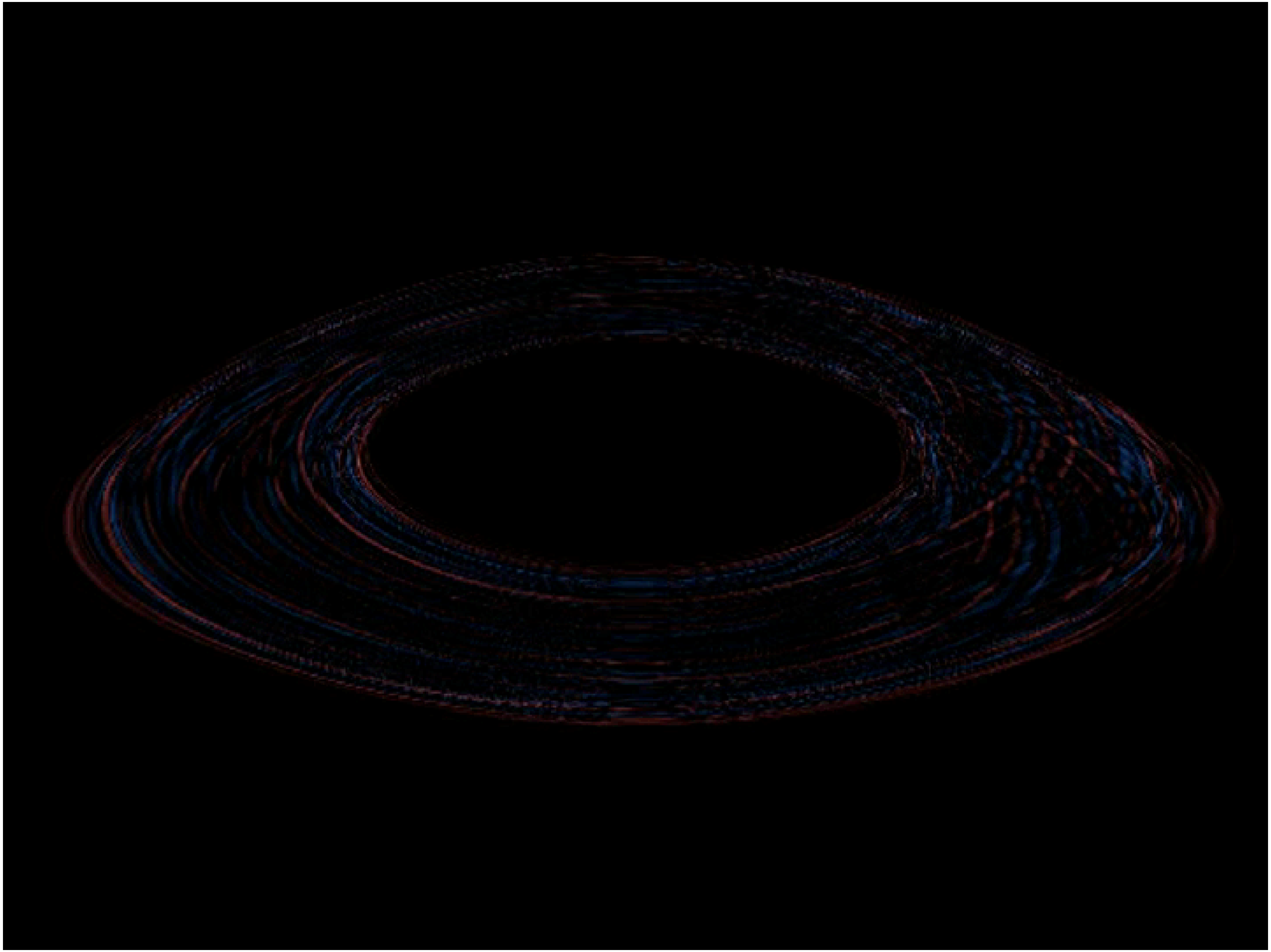
Introductory movies

- acquired by F. Brochard, G. Bonhomme, N. Fedorczak and the Tore Supra team,
- sampling rate 40kHz
- detached plasma regime, with cold radiative shell
- structures can be seen, but what **quantitative information** can we extract?



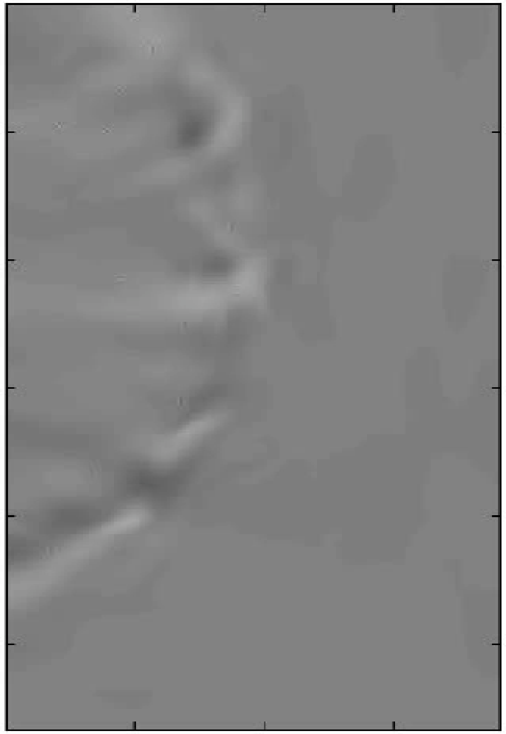
Outline

- Naive denoising
- Helical Abel transform
- Wavelet-vaguelette decomposition
- Tutorial example
- Validation
- Application to Tore Supra movies

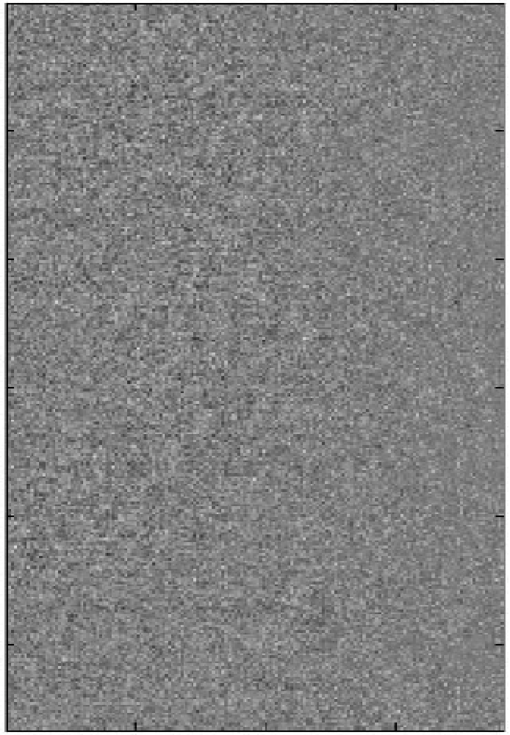
Naive denoising

- We can apply **wavelet denoising techniques** to the camera image to obtain movies that look better:

raw movie



denoised movie



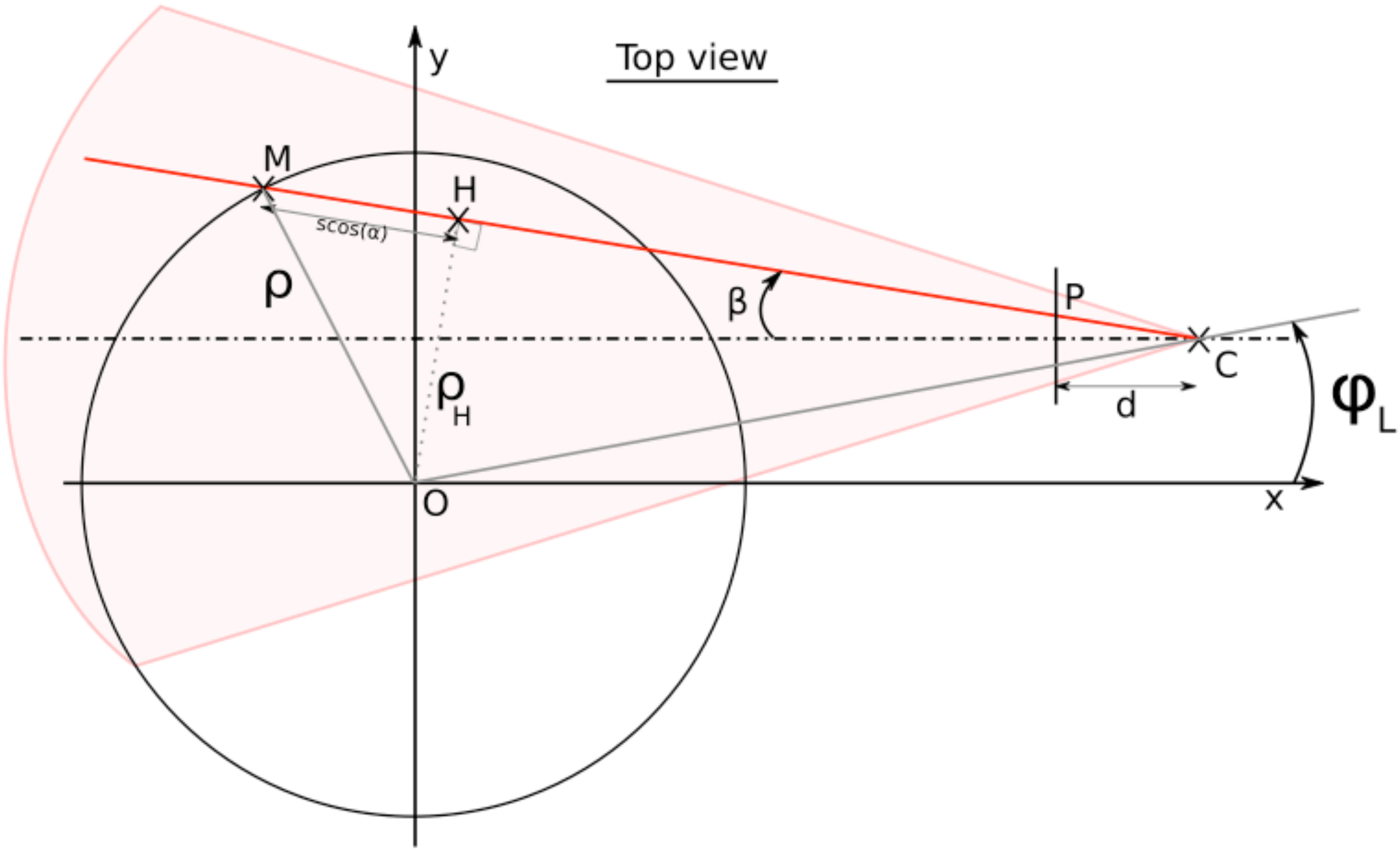
noise

- Some useful information can be extracted from these movies (see Brochard et al., EPS Meeting 2009), but not much, due to **flattening effects**.

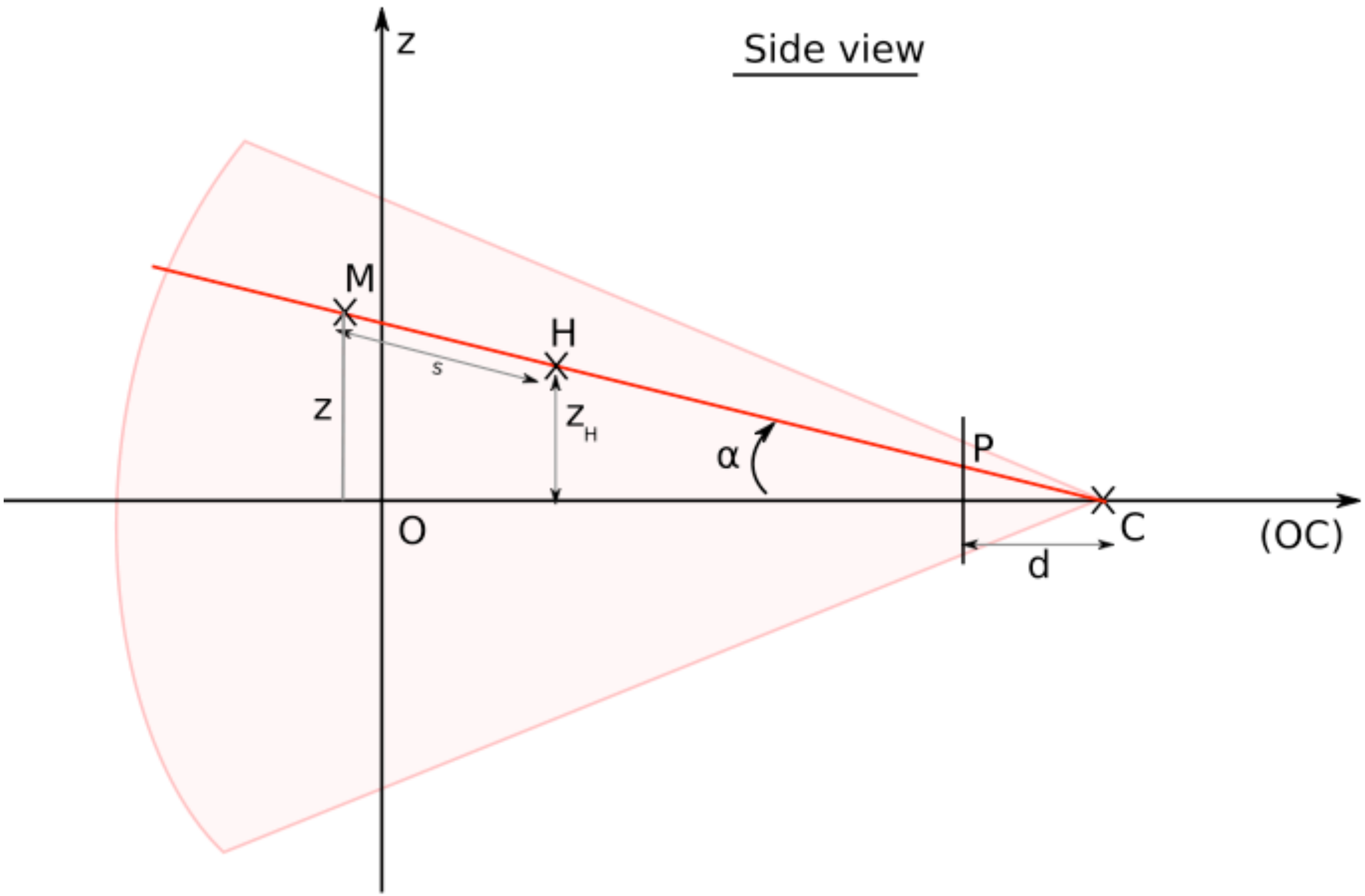
Alternative : tomographic inversion

- We only have one fixed camera, so we cannot do classical tomography (Radon transform), and not even stereography,
- However it may be possible to **invert** the flattening operation under the assumption that the emissivity is **almost constant along field lines**,
- Similar to axisymmetry hypothesis underlying Abel transform (used in observation of galaxies and probing of mechanical devices)
- But more complicated due to toroidal geometry, magnetic helicity & shear, etc.

Problem geometry



Problem geometry



Camera model

- The flux received by a pixel is proportional to the integral of the emissivity along the line of sight :

$$I(x,y)=\int_{0}^{+\infty} S(\psi,\theta,\varphi)\mathrm{d}s$$

- ψ,θ,φ are functions of (x,y) and s, parametrizing the line of sight which passes through (x,y) in field line coordinates,

- ψ,θ,φ can be obtained from an analytical model for simple cases, or using a magnetic reconstruction code (this second option will not be considered here)

Simplifying hypotheses

- We assume that field lines follow the equation:

$$\varphi - q\theta = 0$$

- Main hypothesis : S is piecewise **constant along field lines**, and jumps occur outside the camera image.

- In practice we have $k_{//} << k_{\perp}$, so that S varies **slowly** along field lines.

- In the following we only consider **circular cross section**, so that we may take:

$$\psi \equiv r$$

Generalized Abel transform

- Under these hypotheses, we get an operator K

$$S(r,\theta) \xrightarrow{K} I(x,y)$$

$$L^2\left([r_1,r_2]\times[\theta_1,\theta_2]\right) \xrightarrow{K} L^2\left([x_1,x_2]\times[y_1,y_2]\right)$$

- We assume that the intervals are well chosen so that **K is one to one** (but we do not attempt to prove it here)

- Our goal is to invert K in a stable way in the presence of noise.

Discretization

- The pixels on the camera image live on a cartesian grid

$$\left\{(x_i, y_j)\right\}, i = 1..N_x, j = 1..N_y$$

- The domain of interest in the (r, θ) plane is discretized as well:

$$\left\{(r_k, \theta_l)\right\}, k = 1..N_r, l = 1..N_\theta$$

- Finally, the integral is discretized using the method of rectangles.

Discretization

- We can now represent K as a sparse matrix, by computing

$$K_{ijkl} = K(\delta_{r_k}\delta_{\theta_l})(x_i, y_j)$$

- Typically a few percent of the coefficients are nonzero.

Wavelet-vaguelette decomposition

- Singular value decomposition (**SVD)** allows optimal representation of operators but yield **global modes**, so that turbulent signals typically do not have a sparse representation,
- Wavelet-vaguelette decomposition (**WVD**) is a suboptimal representation, but preserves locality and thus offers **better sparsity** (Donoho 1992),
- The basic idea is to expend the **unknown source** over a wavelet basis, which induces a representation of the signal:

$$S = \sum_{\lambda \in \Lambda} \tilde{S}_\lambda \psi_\lambda \quad \Longrightarrow \quad I = KS = \sum_{\lambda \in \Lambda} \tilde{S}_\lambda K \psi_\lambda$$

Wavelet-vaguelette decomposition

- Define the **vaguelettes** $(u_\lambda)_{\lambda \in \Lambda}$ and $(v_\mu)_{\mu \in \Lambda}$ by:

$$K\psi_\mu = \kappa_\mu v_\mu$$

$$K^* u_\lambda = \kappa_\lambda \psi_\lambda$$

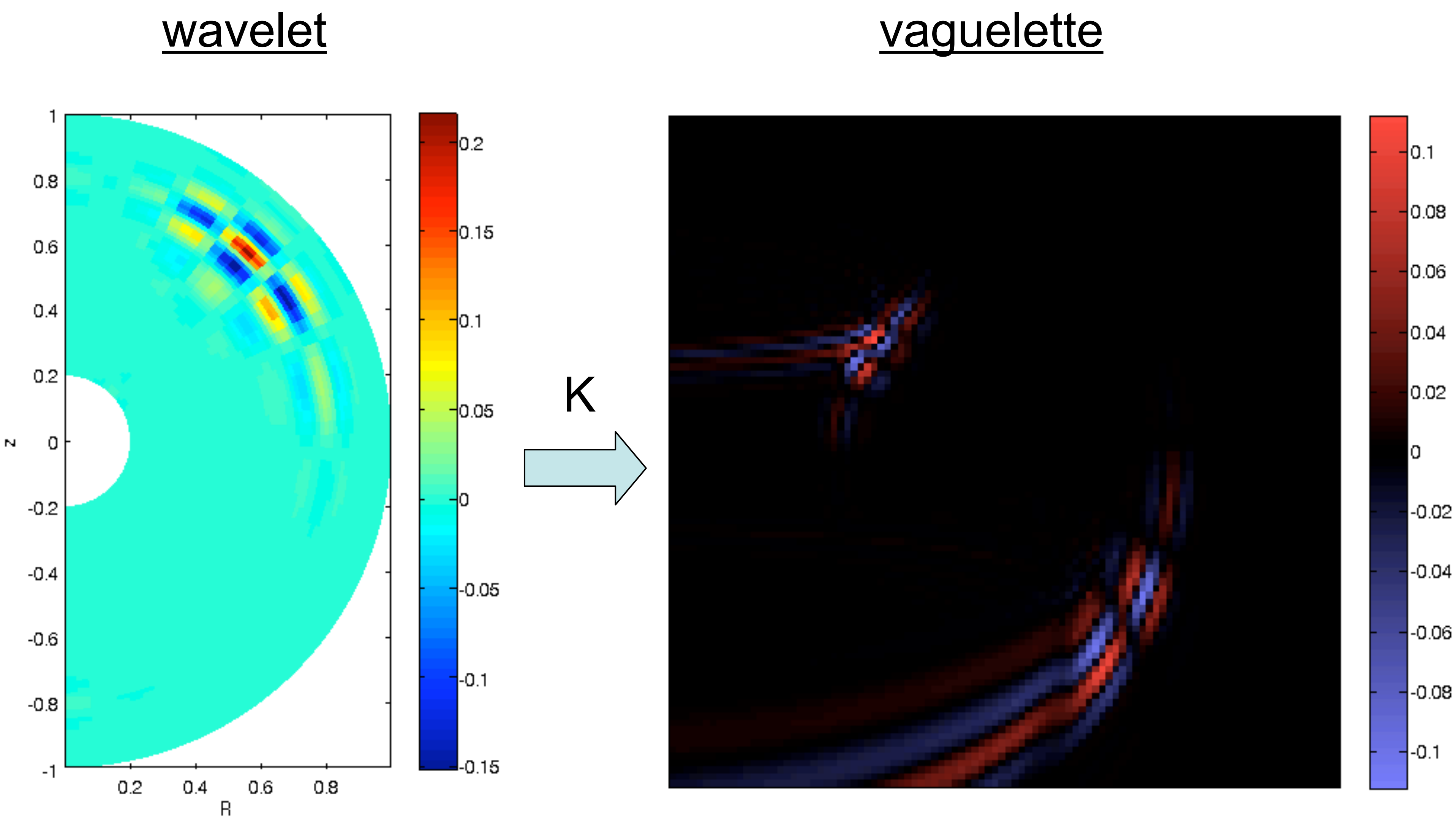
where the κ_μ are constants chosen so that $\left\| v_\mu \right\|_2 = 1$
then we have the biorthogonality relation

$$\left\langle u_\lambda \middle| v_\mu \right\rangle = \delta_{\lambda\mu}$$

and therefore the reconstruction formula :

$$\boxed{S = \sum_{\lambda \in \Lambda} \left\langle I \middle| u_\lambda \right\rangle \kappa_\lambda^{-1} \psi_\lambda}$$

What do vaguelettes look like?



Denoising

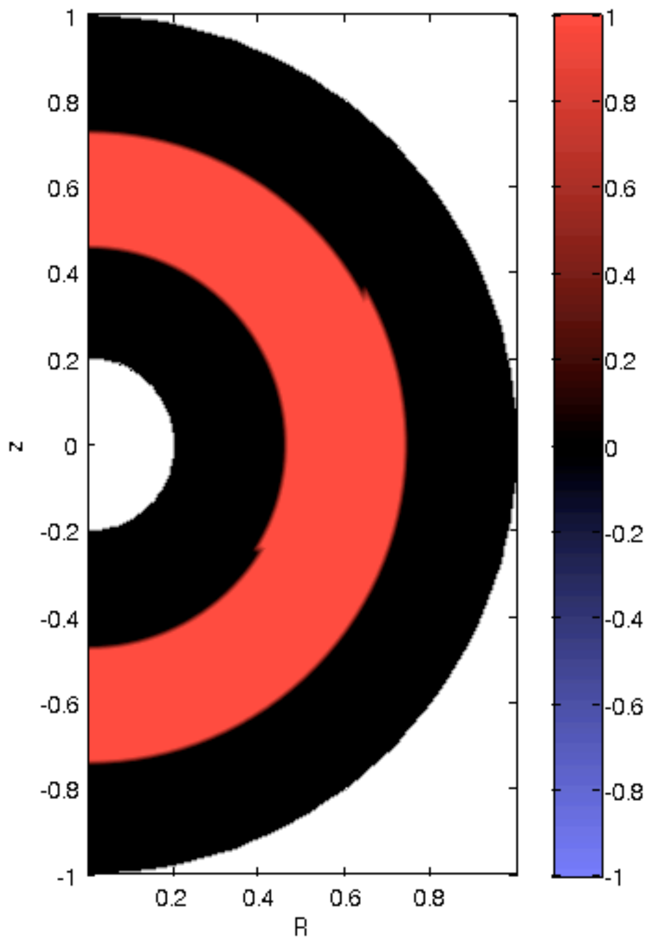
- Denoising is achieved by thresholding coefficients

$$\boxed{S_{WVD} = \sum_{\lambda \in \Lambda} F_\theta(\langle I | u_\lambda \rangle) \kappa_\lambda^{-1} \psi_\lambda}$$

where F is a thresholding function and the threshold is determined by an iterative algorithm (see Azzalini et al., *ACHA* **18**, 2005)

Example : toric shell

- Take the following emissivity map:



Sharp fronts at r = 0.43 and r = 0.76, good test case for wavelet methods.

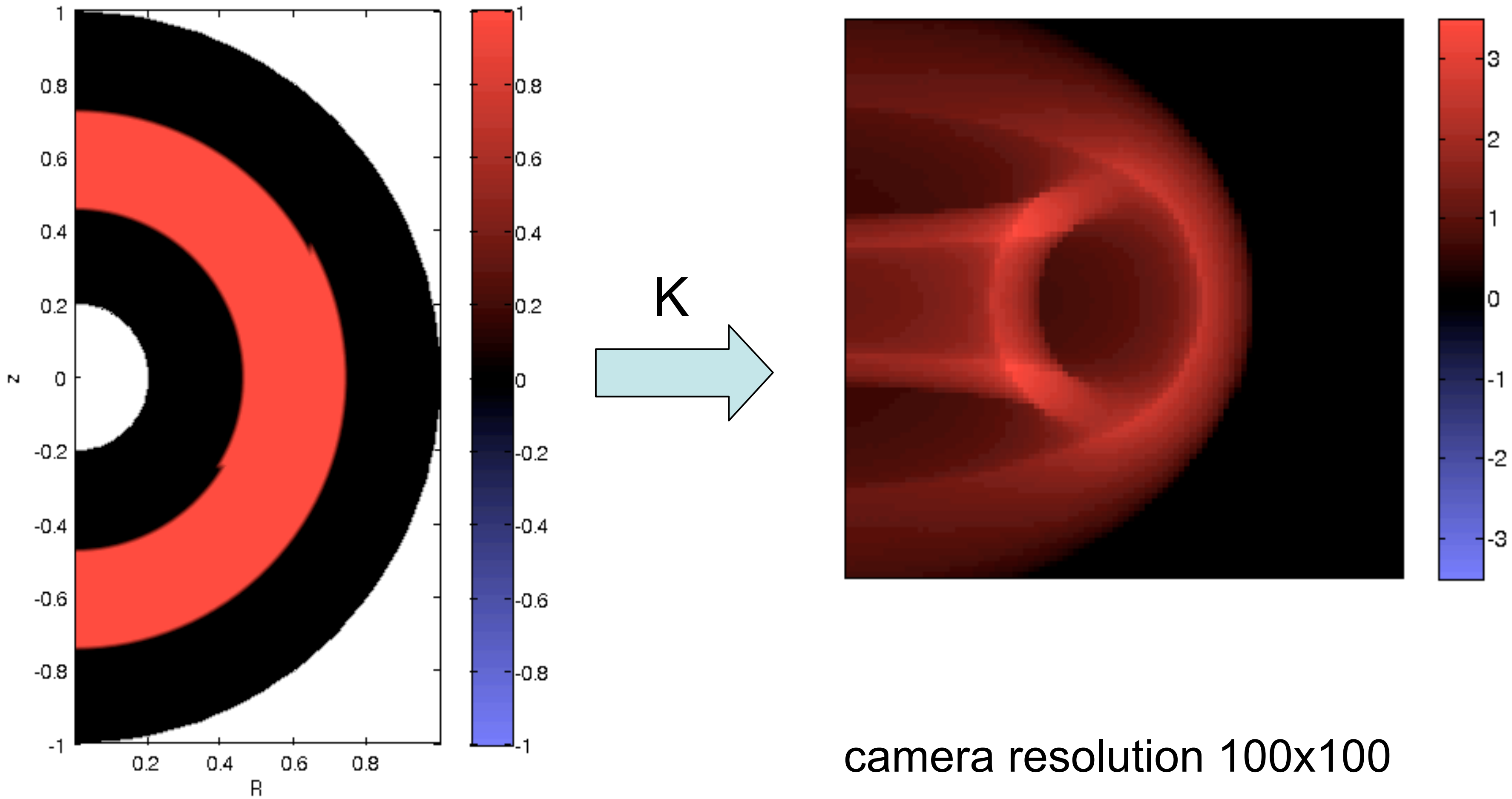
•Periodic in θ direction but not in r.

•64x64 grid in (r, θ).

•No Shafranov shift, infinite q.

Example : toric shell, forward transform

- First apply the forward transform

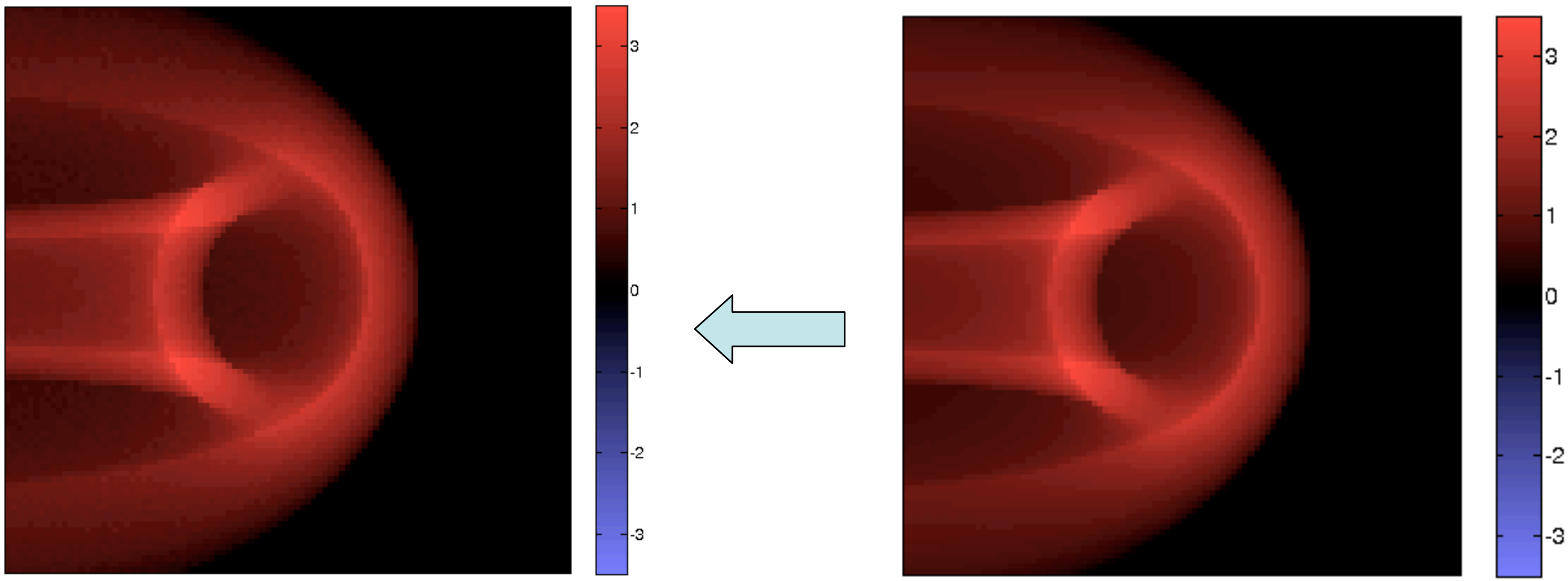


camera resolution 100x100

appearance of critical curves due to flattening effect

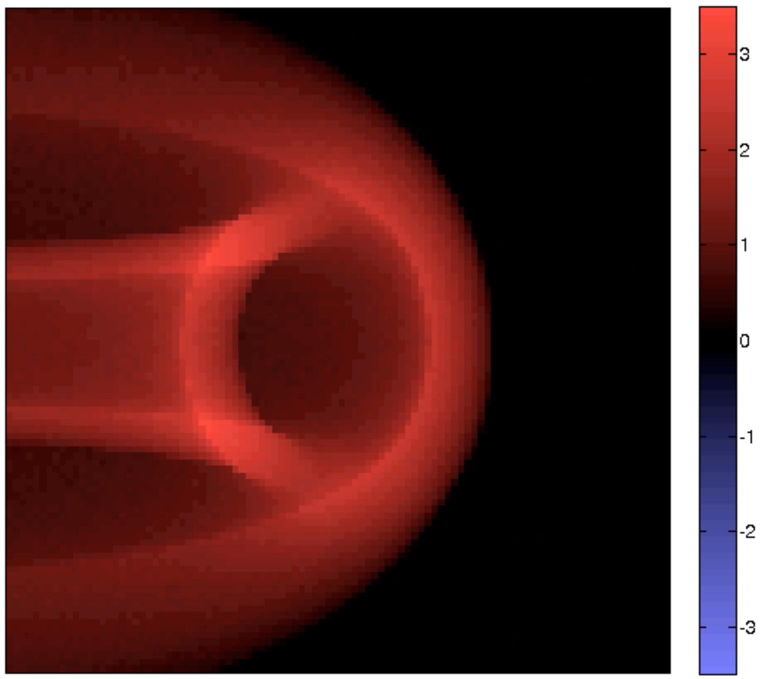
Example : toric shell + noise

- Then add some noise

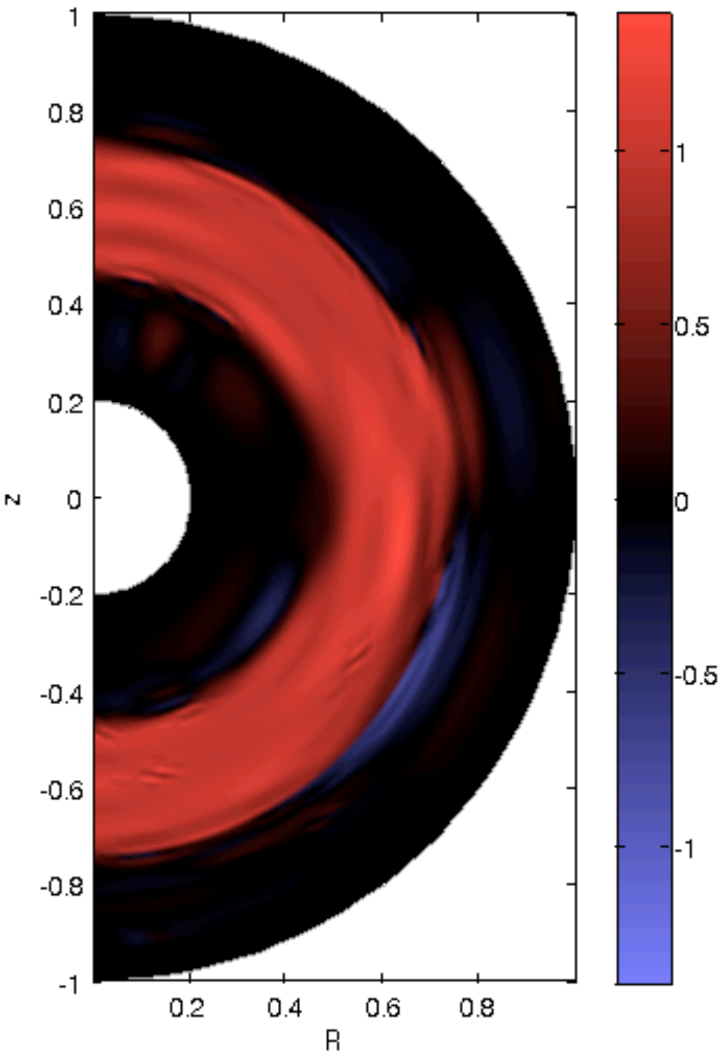


Example : toric shell + noise

- Finally apply WVD

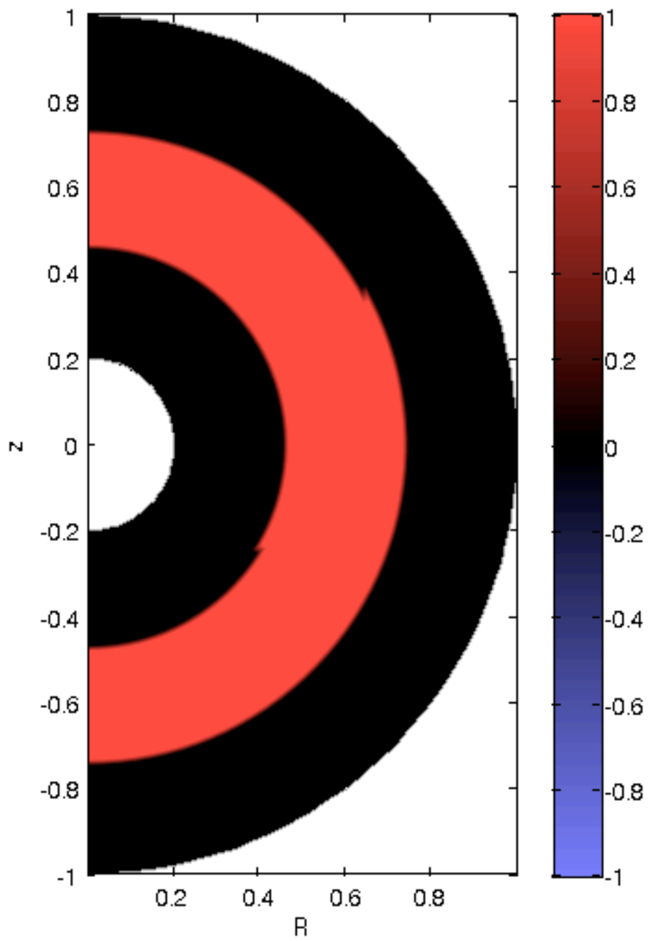


K^{-1}

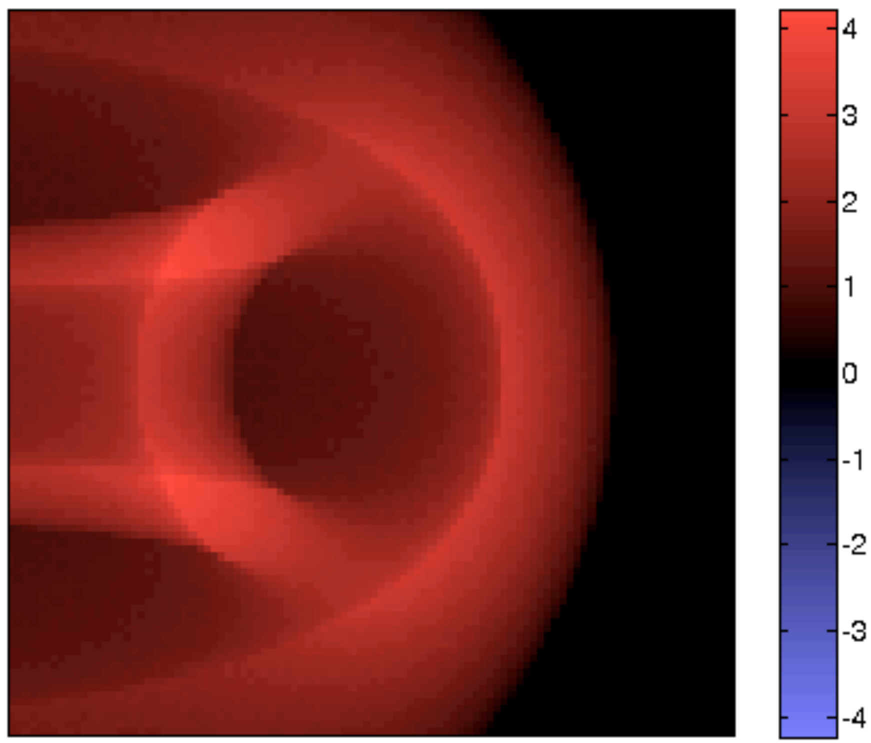


Example : toric shell, inverse transform

- First apply the forward transform



K-1

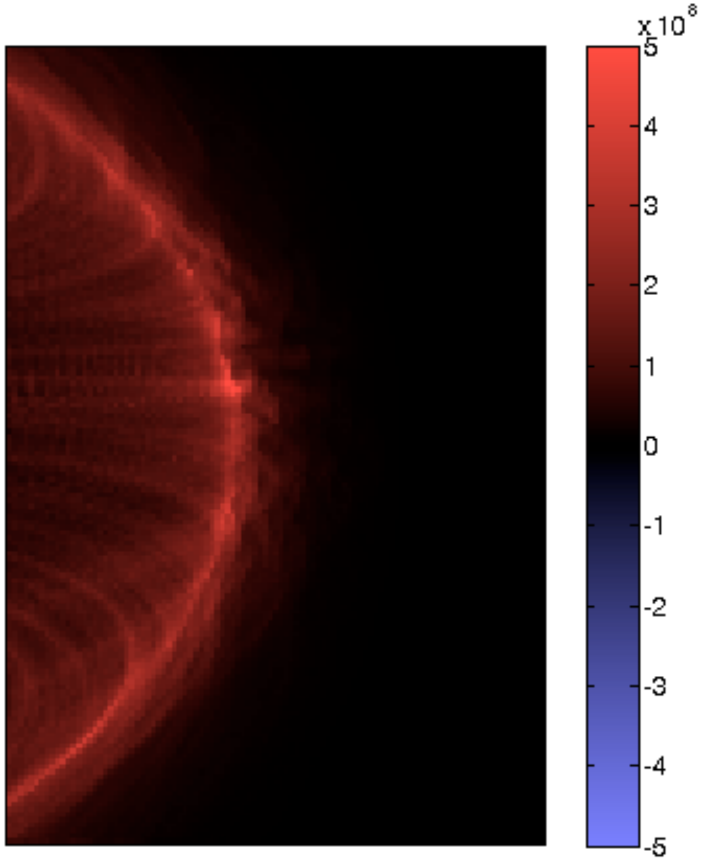


camera resolution 100x100

appearance of critical curves due to flattening effect

Validation

<u>independently simulated camera image</u>



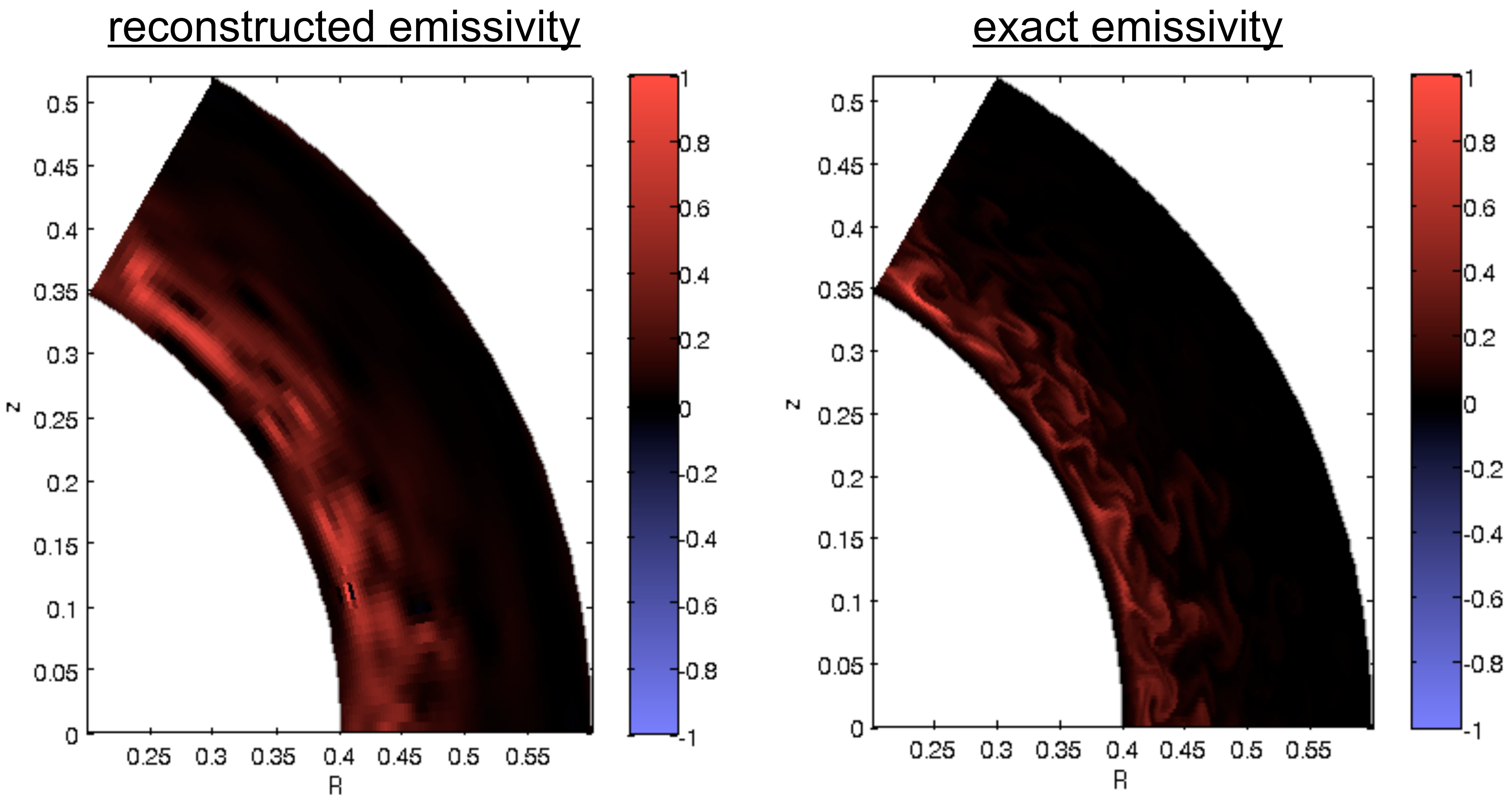
- density data from the Tokam code (Y. Sarazin, P. Gendrih),

- camera simulated by N. Fedorczak by accumulating projections of successive poloidal cross sections

- the method is independent from our own discretization of K

- fixed q=3, no Shafranov shift

Validation result



Not too bad given the low resolution of the image

Validation result

independently simulated camera image

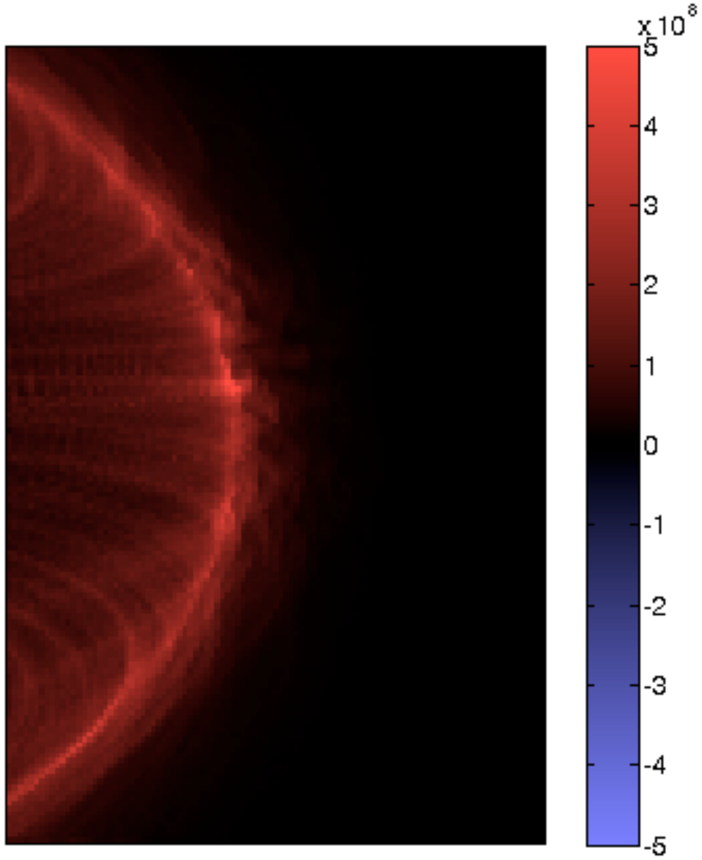
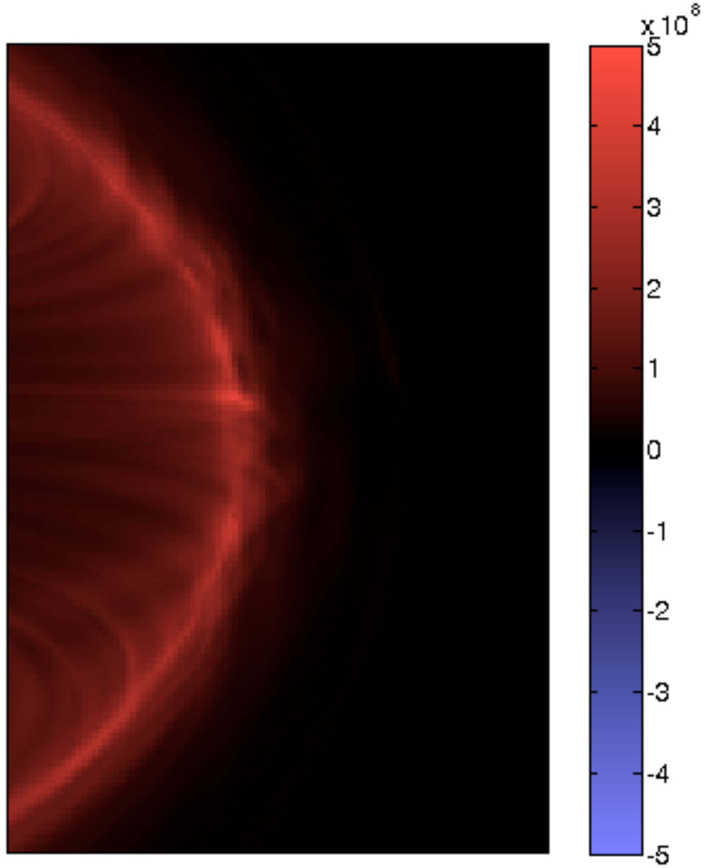
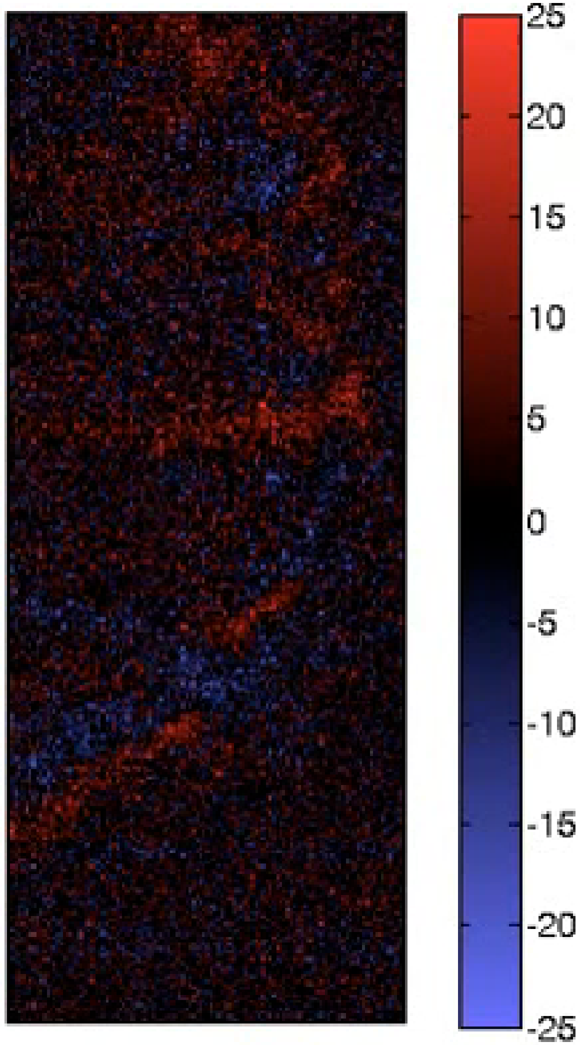


image regenerated from inverted emissivity

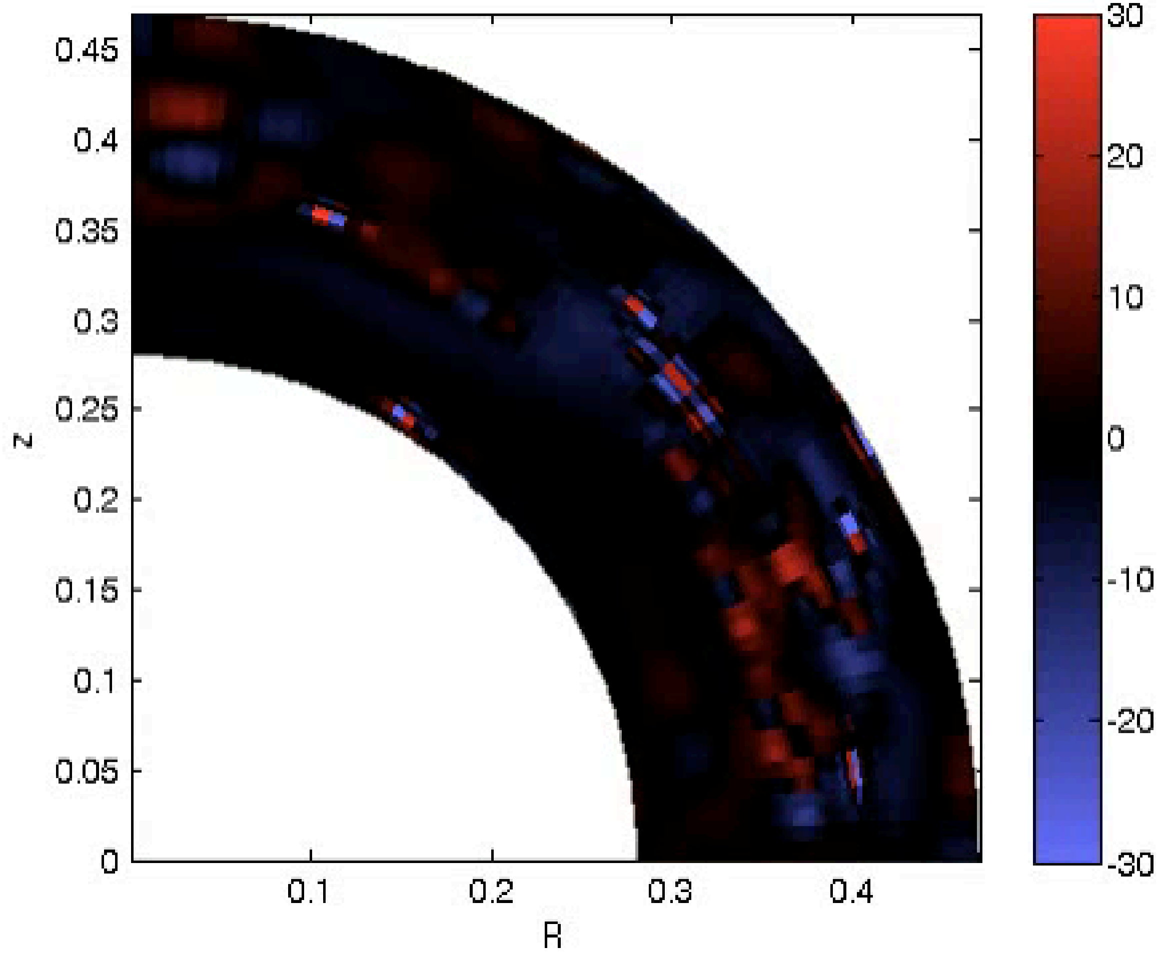


Application to experimental data

raw movie

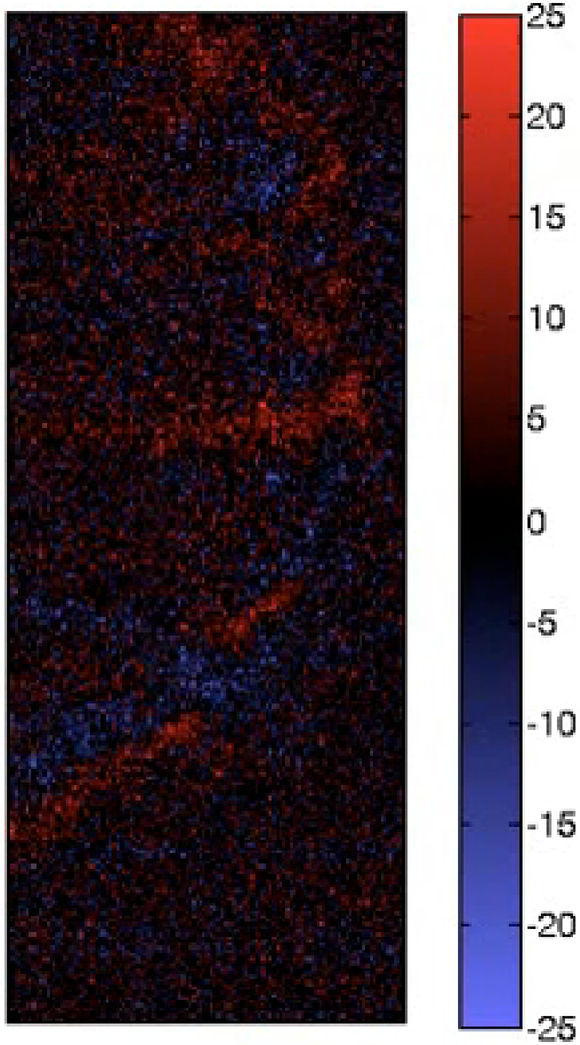


reconstructed emissivity

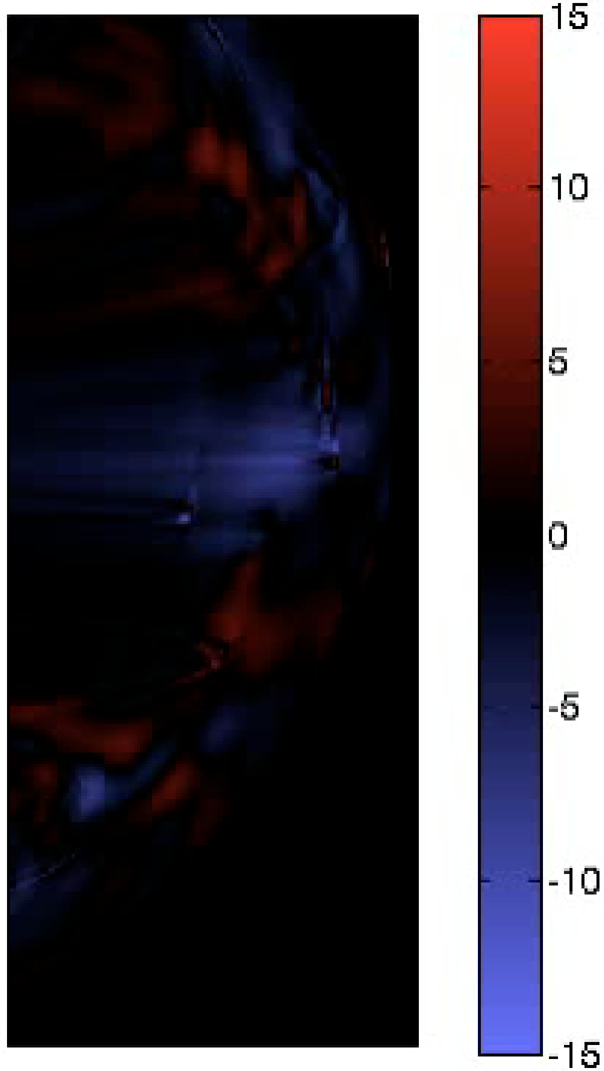


Application to experimental data

raw movie



reconstructed emissivity



Perspectives

- There is room for improvement
- It is hoped that some quantitative features of edge turbulence can be retrieved from the inverted movies.
- Further validation is in progress (against bolometry data)
- The best validation would be against an experiment for which the emissivity is known from another diagnostic.

Thank you !

- And thanks to Jamie Gunn, Frédéric Lemoine, Stella Oldenburger, Maximilien Bolot

- See also:
  - Poster number 12 about PIC denoising
  - http://wavelets.ens.fr
  - http://justpmf.com/romain

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Summary

- We have developed an efficient “numerical camera” and computed a sparse matrix representation of it,
- We have to take care of flattening effects when interpreting camera images !
- To invert the matrix, we have used a wavelet-vaguelette decomposition,
- The inversion was roughly validated using an independently simulated image,
- A first application to Tore Supra data was shown.