High Resolution DNS of Turbulence and its Application to LES Modeling

A review

Y. Kaneda

Nagoya University
Collaboration with

Ishihara T.  (Nagoya Univ.)
Yoshida K.  (Nagoya Univ.)
Yokokawa M.  (Grid Computing Center)
Itakura K.  (Earth Simulator Center)
Uno A.  (Earth Simulator Center)
I) **High Resolution DNS**
   a) DNS with up to $4096^3$ grid points
      - an overview
   b) Regeneration of small eddies
      - a support for the idea of LES

II) **LES Modeling (spectral approach)**
   a) Deterministic
      - an application of DNS data analysis
   b) Stochastic
      - an attempt for predictability analysis
I) High Resolution DNS

a) DNS with up to $4096^3$ grid points
Computational Facilities & Performance

★ 1 (512³) & ★ 2 (1024³)

- Fujitsu VPP500/42, VPP5000/56 (Nagoya UCC)
  0.5TFLOPS (peak), Memory 0.9TB

★ 3 (2048³) & ★ 4 (4096³)

- Earth Simulator
  40TFlops (peak), 16.4TFlops (sustained),
  Memory: 10TB

Yokokawa, Itakura, Uno, Ishihara & Kaneda (SC2002);
http://www.sc-conference.org/sc2002/
History of representative DNS

Incompressible Homogeneous Isotropic Turbulence under periodic BC

- Orszag (1969): IBM Model 360-95
- Kerr (1985): Cray-1S NCAR
- Siggia (1981): Cray-1 NCAR
- Jimenez et al. (1993): Caltech Delta machine
- Yamamoto (1994): NWT

Number of grid points

- 32^3
- 64^3
- 128^3
- 512^3
- 1024^3
- 2048^3, 4096^3

Year

- # of DOF > 10^{11}

(■ 512,250000)
Two series of DNS data

Series 1 \((k_{\text{max}} \eta = 1)\)

Series 2 \((k_{\text{max}} \eta = 2)\)

Possible on the Earth Simulator

Kaneda et al. (2003)
Analysis of the DNS data by ES

- DNS’s up to $R_\lambda = 1200$ suggest
  - Normalized dissipation $\varepsilon \rightarrow \text{const, as } R \rightarrow \infty$
- Energy Spectrum
- Scaling & Statistics of 4th order velocity moments
  - Mean squares of $\bar{u}^2$, $\omega \omega$, $SS = \varepsilon/(2\nu)$
- High order structure functions, pdf, joint-pdf, intermittency
- Anisotropic scaling, effects of anisotropy,
- Inertial range structure,
- Dissipation range spectrum, ...
- Analysis at coarse grained level, alignment of $\omega$ vs. $S$, $\Pi$, etc.

Direct & Qualitative Examination of Theories
results of data analysis -1

- **Energy spectrum**
  - **Inertial subrange**
    \[ E(k) \propto k^{-5/3-\alpha} \quad \text{for } k\eta < 0.04 \text{ and } R_\lambda > 500 \]
    \[ \alpha \approx 0.1 \]
  - **Near dissipation range**
    \[ E(k) \propto C(k\eta)^\alpha \exp[-\beta(k\eta)] \]
    \[ \alpha, \beta, C \text{ tend to constants as } R_\lambda \to \infty \]
1. **Moments of dissipation and enstrophy**
   - Ratio $\sim$ const. for $R_\lambda > 600$

2. **Spectra of dissipation and enstrophy**
   - \( \bar{\Omega}(k) \approx \bar{D}(k) \approx CR_\lambda^{0.25} (k\eta)^{-2/3} \)

3. **Spectrum of pressure**
   - \( P(k) \sim k^{-7/3} \) for $R_\lambda > 600$
1. Skewness and Flatness
   - Transition at $R_\lambda \sim 700$ ?… Not observed
   - $S \propto F^a$, $a \sim 1/3$

2. 4 rotational invariants
   - $I_1, I_2, I_3, I_4$ … the same $R_\lambda$-scaling for $R_\lambda > 400$

3. Scaling of fluid-particle acceleration
   - $\sim$ An empirical formula for $R_\lambda > 400$ (Hill 2002)
     (but for $k_{\text{max}} \propto = 1$)
Visualization
Table 2: DNS parameters and turbulence characteristics at $t = t_f$. $\Delta t$ is the time increment, $\langle \epsilon \rangle$ the mean rate of energy dissipation per unit mass, and $\lambda$ the Taylor micro-length scale. (Values except for $N = 4096$ are quoted from Ref. 3).

<table>
<thead>
<tr>
<th>Series</th>
<th>$N$</th>
<th>$R_\Lambda$</th>
<th>$k_{\text{max}}$</th>
<th>$\Delta t \times 10^{-3}$</th>
<th>$t_f$</th>
<th>$\nu \times 10^{-4}$</th>
<th>$\langle \epsilon \rangle$</th>
<th>$L$</th>
<th>$\lambda$</th>
<th>$\eta \times 10^{-3}$</th>
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<td>1.05</td>
<td>0.052</td>
<td>1.01</td>
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Compensated Spectra of $\Omega$ and $D$

Fig. 5. $\Omega(k)$ (thick lines) and $D(k)$ (thin lines) spectra compensated by $R_\lambda^{-0.25}(k\eta)^{2/3}/(\nu^{-5} \langle \varepsilon \rangle^7)^{1/4}$.
Energy spectrum & energy transfer

(at statistically steady state)

\[ \Pi(k) = \int_{k}^{\infty} T(k) dk \]

\( R_\lambda = 1450 \)

G. Saddoughi and S. V. Veeravalli

Experimental values (from Sreenivasan 1995)

\[ C_K = 1.62 \pm 0.17 \]

ALHDIA (Kraichnan 1966)

\[ C_K = 1.77 \]

LRA (Kaneda 1986)

\[ C_K = 1.72 \]
$$\Pi = \varepsilon \quad \text{(width, flat, stationarity)}$$

$$\Pi(k) = \int_{k}^{\infty} T(k)dk$$

Some difference from DNS with lower resolution: -2

$$N=2048, \quad k_{\text{max}} \eta \sim 1 \quad R_\lambda \sim 732$$

(Phys Fluids 12(2003), L21-L24)
FIG. 5: Compensated energy spectra from DNSs with (A) $512^3$, $1024^3$, and (B) $2048^3$, $4096^3$ grid points. Scales on the right and left are for (A) and (B), respectively.

(Phys Fluids 12(2003),L21-L24)
Summary of I-a

- Structure at small scales vs. large eddies vs. clusters
  like leaves/twigs/branches/trees vs. forest (cf. CS2002)

  Q: Is the vortex so that important?
  for the understanding large scale dynamics

- 2048, 4096 DNS give wide inertial range
  → enables quantitative examination
  of theories of inertial range
  example: II-a
I) High Resolution DNS

b) Regeneration of small eddies
   - a support for the idea of LES
Velocity fields with different initial states in higher $\kappa$

**Case 1**

Kinetic energy contour

**Case 2**
The method of numerical experiments

Initial condition

Prepare two different 3D isotopic turbulence

\[ \hat{u}^{(1)}(k, t), \hat{u}^{(2)}(k) \]

Copy \( \hat{u}^{(1)}(k) \) to \( u^{(2)}(k) \) for \( |k| < k_{init} \)
Time marching

\[ \hat{u}^{(1)}(k, t) \]  
\[ \hat{u}^{(2)}(k, t) \]

Measure error growth quantitatively

2 parameters

The max wave number of assimilated range: \( k_c \)

The time interval of assimilation: \( T \)
Regeneration of Small Eddies

By K.Yoshida
$128^3, kc=32$ vorticity by Yamaguchi, Yoshida
Summary of 1-b

- Importance of Large eddies
  small eddies are subordinate
  butterfly effect vs. lizard-tail effect

→

A support for the soundness of the idea of LES
II) LES Modeling (spectral approach)

a) Deterministic LES
   - an application of DNS data analysis
Spectral LES

Navier-Stokes Eq.

\[
\left( \frac{\partial}{\partial t} + \nu k^2 \right) u_i(k) = M_{imn}(k) \sum_{k=p+q} u_m(p) u_n(q) + f_i(k, t),
\]

LES Model Eq.

\[
\left( \frac{\partial}{\partial t} + [\nu + \nu_e(k|k_c)]k^2 \right) \tilde{u}_i(k) = M_{imn}(k) \sum_{k=p+q} \tilde{u}_m(p) \tilde{u}_n(q) + f_i(k, t),
\]

Cutoff wavenumber \( k_c \)

Spectral eddy viscosity

How to determine?
Requirement for the model

- Energy Spectrum

\[ E(k) = \frac{1}{2} \sum_{k-1/2 < |k'| < k+1/2} \langle u(k') \cdot u(-k') \rangle, \]

Require the model to simulate \( E(k) \)

\[ \dot{E}(k) = E(k), \quad \frac{\partial}{\partial t} \dot{E}(k) = \frac{\partial}{\partial t} E(k), \quad \text{for } k < k_c. \]

\[ \nu_e(k \mid k_c) = -\frac{T(k) - \mathcal{T}(k \mid k_c)}{2k^2 E(k)} \]

\[ \left( \frac{\partial}{\partial t} + 2\nu k^2 \right) E(k) = T(k) \]

\[ \left( \frac{\partial}{\partial t} + 2[\nu + \nu_e(k \mid k_c)k^2] \right) \overline{E}(k) = \overline{T_c}(k \mid k_c) \]
2P closures

Closed equations for 2-point statistics

2-point closures

**LRA** (Lagrangian Renormalized Approximation)
- Simplest among Lagrangian closures
- Free from any ad-hoc parameter
- Fully consistent with Galilean invariance/Kolmogorov spectrum
Example of performance of the LRA for 2nd order moments:

Equilibrium Energy Spectrum by the LRA & Experiments

Gotoh, Nagaki, and Kaneda

FIG. 1. Comparison of the one-dimensional energy spectrum determined by (Phys Fluids 12(2000), 155-168) the LRA (MLRA) with the experimental data (Refs. 25 and 26).
$T(k)$ in 2P closures

\[ T(k) = \int \int_{\triangle} dpdq \ k^3pqb_{kpq}\theta_{kpq}q^{-2}E(q)[p^{-2}E(p) - k^{-2}E(k)], \]

\[ \tilde{T}(k|k_c) = \int \int_{\triangle_1} dpdq \ k^3pqb_{kpq}\tilde{\theta}_{kpq}q^{-2}\tilde{E}(q)[p^{-2}\tilde{E}(p) - k^{-2}\tilde{E}(k)], \]

\[ \theta_{kpq} = \int_{-\infty}^{t} ds \ G(k, t, s)G(p, t, s)G(q, t, s), \]

$G(k,t,s)$: Lagrangian response function
Assume $k_c$ is in the inertial subrange.

- Substitute similarity solution of $E(k)$ and $G(k)$ of LRA into the equations for $T(k)$ (Universality in small scales).

$$E(k) = K_o \epsilon^{2/3} k^{-5/3}, \quad K_o = 1.72$$

- Simplification, $\tilde{G}(k) = G(k)$. 
Spectral eddy viscosity

\[ \nu_e(k|k_c, t) = [\tilde{\varepsilon}(t)]^{1/3} k_c^{-4/3} \nu_e^*(\frac{k}{k_c}), \]

\[ \tilde{\varepsilon}(t + \Delta t) = \int_{k < k_c} dk \ 2\nu_e(k|k_c, t) k^2 u(k) \cdot u(-k), \]
LES of 3D turbulence

$t = 0$

$\text{# of deg. of freedom } \rightarrow 1/32000$

against DNS with $1024^3$

$t = T$
LES model of 2D turbulence with inverse cascade range

Negative eddy viscosity
Application: finite difference schemes

- LES based on Gaussian Filter (GLES)
  - Gaussian Filter -- easily applied to FD schemes

Spectral LES

GLES

Gaussian Filter

Resolved wavenumber range in SP

Resolved wavenumber range in FD
LES applied to FD schemes (1)

**DNS**
- SH spectral
- 2048×1024 (T682)

**LES**
- Double Fourier
- 512×256

\[ E(k) \propto k^{-5/3} \]
LES applied to FD schemes (2)

**DNS** 2048×1024 (T682)
SH spectral

**LES CCD** 512×256

$E(k) \propto k^{-5/3}$
Application to stratified turbulence

Assume $k_c$ is in the inertial subrange.

In SGS,

$u(x)$ -- quasi isotropic turbulence,
Density fluctuation field -- almost passive scalar.

Eddy viscosity and Eddy diffusivity

\[ \nu_e(k \mid k_c) = \varepsilon^{1/3} k_c^{-4/3} \nu_e \left( \frac{k}{k_c} \right) \]

\[ \kappa_e(k \mid k_c) = \varepsilon^{1/3} k_c^{-4/3} \kappa_e \left( \frac{k}{k_c} \right) \]

Eddy Prandtl number

\[ \Pr_e(k \mid k_c) = \frac{\nu_e(k \mid k_c)}{\kappa_e(k \mid k_c)} \]
LES of stratified turbulence

\[ N = 3\pi \]
\[ k_b = 231 \]

Computed resolution \( 512^3 \)

Visualized resolution \( 64^3 \)

Isosurface of
\[ \rho = +2\sigma_\rho \] (red and blue)
\[ \rho = -2\sigma_\rho \]
\[ \omega = 2\sigma_\omega \] (green)
Vertical shear spectrum

\[ S(r) = \sum_{i=1}^{3} \left( \frac{\partial u_i}{\partial x_3}(x) \frac{\partial u_i}{\partial x_3}(x + re_3) \right) \rightarrow S(k) \]

\[ \log_{10} \left[ \frac{S(k)}{S_b} \right] \]

\( \propto k^{-1} \)

\( \propto k^{-1/3} \)

\( \propto k^{1/3} \)

(Gargett et al., 1984)
An application of DNS-data analysis

- Examination of Model:
  - Eddy viscosity
  - Comparison with DNS and theory

Compensated energy spectrum

Series 1
II) LES Modeling (spectral approach)

b) Predictability & Stochastic LES
LES so far

Good for energy

but ....
LES & Predictability

From the viewpoint of the reduction of information:

\[ ? = < ?, \mathbf{A} > + (\text{Res.}) \]

- **Projection to a space A**
- **Residue**

- the Dim of Res. is huge
  \[ \text{Dim (Res.)} \gg \text{Dim (A)} \]
  (in fact, the correlation between model and DNS is poor)

- Difference of \( u_1, u_2 \)

**Impossibility** to identify small scale conditions/noise

\[ \rightarrow \text{inevitable uncertainty, unpredictability} \]
Error growth due to uncertainty in SGS

\( u^{(1)}, u^{(2)} \): Two velocity field with different initial conditions in large wavenumber modes \( (k > k_c) \).

Difference between two fields

\[ \delta u = u^{(1)} - u^{(2)} \]

becomes non-zero in small wavenumber modes \( (k \leq k_c) \) for \( t > 0 \).
Uncertainty due to SGS uncertainty \( \rightarrow \) Predictability

Case 1

Kinetic energy contour

Case 2

\( t = 0 \)
\( t = 0.25T \)
\( t = T \)
Probabilistic LES (PLES) model

- Estimate the prediction error due to the uncertainty in SGS.
- Introduce random external forcing.

\[
\left( \frac{\partial}{\partial t} + \left[ \nu + \mu_e(k|k_c) \right] k^2 \right) \tilde{u}_i^{(\alpha)}(k) = M_{imn}(k) \sum_{k=p+q} \tilde{u}_m^{(\alpha)}(p) \tilde{u}_n^{(\alpha)}(q) + f_i(k,t) + (f_{e}^{(\alpha)})_i(k|k_c,t), \quad \alpha = 1, 2
\]

eddy viscosity

Forcing spectrum

\[
F(k|k_c,t) = 4\pi k^2 \int_{-\infty}^{t} ds \langle f_{e}^{(\alpha)}(k|k_c,t) \cdot f_{e}^{(\alpha)}(-k|k_c,s) \rangle.
\]

cf. Kraichnan, Bertoglio, Chasnov
Requirement for the PLES model

- Error Spectrum

\[ \Delta(k) = \frac{1}{4} \sum_{k-1/2 < |k'| < k+1/2} \langle \delta u(k') \cdot \delta u(-k') \rangle. \]

Require the model to simulate \( E(k) \) and \( \Delta(k) \)

\[
\begin{align*}
\tilde{E}(k) &= E(k), \\
\frac{\partial}{\partial t} \tilde{E}(k) &= \frac{\partial}{\partial t} E(k), \\
\tilde{\Delta}(k) &= \Delta(k), \\
\frac{\partial}{\partial t} \tilde{\Delta}(k) &= \frac{\partial}{\partial t} \Delta(k),
\end{align*}
\]

for \( k < k_c \).

DNS

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + 2\nu k^2 \right) E(k) &= T(k), \\
\left( \frac{\partial}{\partial t} + 2\nu k^2 \right) \Delta(k) &= S(k),
\end{align*}
\]

SLES

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + 2[\nu + \mu_e(k|k_c)] k^2 \right) \tilde{E}(k, t) &= \tilde{T}(k) + F_e(k|k_c), \\
\left( \frac{\partial}{\partial t} + 2[\nu + \mu_e(k|k_c)] k^2 \right) \tilde{\Delta}(k, t) &= \tilde{S}(k) + F_e(k|k_c),
\end{align*}
\]
Eddy viscosity and random forcing in PLES

- After some simplifications
  - $\tilde{G}(k) = G(k)$
  - $\Box(k) = E(k)$ for $k > k_c$

\[
\mu_e(k|k_c) = \epsilon^{1/3} k_c^{4/3} \mu_e^*(k/k_c),
\]
\[
F_e(k|k_c) = 2K_0 \epsilon k_c^{5/3} \mu_f^*(k/k_c),
\]
\[
\mu_e^* - \mu_f^* = \nu_e^*
\]
介质 (k) in DNS and PLES

PLES of 2D turbulence with inverse cascade range

Eddy viscosity and random forcing

Summary

- Spectral LES
  without ad-hoc parameter-tuning

- DNS data →
  a comparative test for the theory of
  1) eddy viscosity, 2) triad interaction, localness

- Probabilistic LES
  predictability