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Euromech Colloquium 454, Marseille

High Resolution DNS of Turbulence and its Application to LES Modeling

A review

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Collaboration with

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Outline of Talk

I) High Resolution DNS

a) DNS with up to 4096^3 grid points

- an overview

b) Regeneration of small eddies

- a support for the idea of LES

II) LES Modeling (spectral approach)

a) Deterministic

- an application of DNS data alaysis

b) Stochastic

- an attempt for predictability analysis

I) High Resolution DNS

a) DNS with up to 4096^3 grid points

Computational Facilities & Performance

★ 1 (512^3) & ★ 2 (1024^3)

- Fujitsu VPP500/42, VPP5000/56 (Nagoya UCC)

  0.5TFLOPS(peak), Memory 0.9TB

★ 3 (2048^3) & ★ 4 (4096^3)

- Earth Simulator

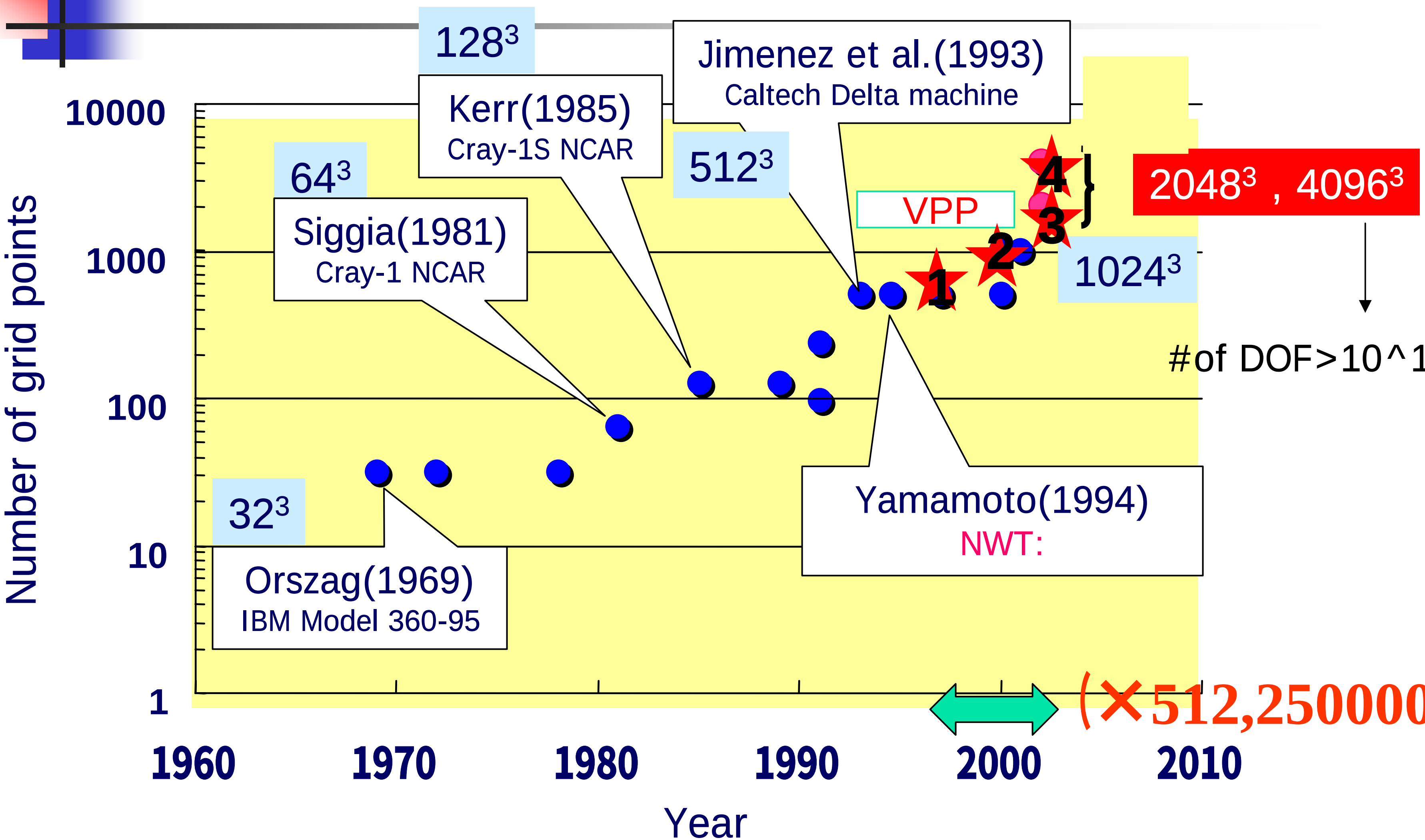
  40TFlops (peak), **16.4TFlops**(sustained),

  Memory:10TB

Yokokawa, Itakura, Uno, Ishihara & Kaneda (SC2002) ; http://www.sc-conference.org/sc2002/

History of representative DNS

Incompressible Homogeneous Isotropic Turbulence under periodic BC



Two series of DNS data

Alias free spectral method

Possible on
the Earth Simulator

VPP

Double Precision

	256^3	512^3	1024^3	2048^3	4096^3	N
➤Series 1($k_{max}\eta = 1$)	167	257	471	732	1131	R_λ
➤Series 2($k_{max}\eta = 2$)	94	173	268	429	675	R_λ

Kaneda et al. (2003)

Analysis of the DNS data by ES

underway

- DNS's up to R_λ=1200 suggest

  Normalized dissipation $\varepsilon \to$ **const, as R** $\to \infty$
- **Energy Spectrum**
- **Scaling & Statistics of 4th order velocity moments**

  mean squares of $\nabla^2 p$, $\omega \cdot \omega$, SS = $\varepsilon/(2\nu)$

- **High order structure functions,**

  pdf, joint-pdf, intermittency
- **Anisotropic scaling, effects of anisotropy,**
- **Inertial range structure,**
- **Dissipation range spectrum, ...**
- **Analysis at coarse grained level, alignment of** ω **vs. S,** Π**, etc.**

- **Direct & Qualitative Examination of Theories**

results of data analysis -1

- Energy spectrum
  - Inertial subrange

    $E(k) \propto k^{-5/3-\alpha}$ for $k\eta < 0.04$ and $R_\lambda > 500$

    $\alpha \approx 0.1$

  - Near dissipation range

    $E(k) \propto C(k\eta)^{\alpha} \exp[-\beta(k\eta)]$

    α, β, C tend to constants as $R_\lambda \to \infty$

results of data analysis -2

1. Moments of dissipation and enstrophy
   - Ratio ~ const. for $R_\lambda > 600$

2. Spectra of dissipation and enstrophy
   - $\bar{\Omega}(k) \approx \bar{D}(k) \approx C R_\lambda^{0.25} (k\eta)^{-2/3}$

3. Spectrum of pressure
   - $P(k) \sim k^{-7/3}$ for $R_\lambda > 600$

results of data analysis -3

1. Skewness and Flatness
   - Transition at $R_\lambda \sim 700$?… Not observed
   - $S \propto F^a$, $a \sim 1/3$
2. 4 rotational invariants
   - I_1, I_2, I_3, I_4 … the same R_λ-scaling for $R_\lambda > 400$
3. Scaling of fluid-particle acceleration
   - $\sim$ An empirical formula for $R_\lambda > 400$ (Hill 2002) (but for $k_{max}\eta = 1$)

Visualization

DNS data

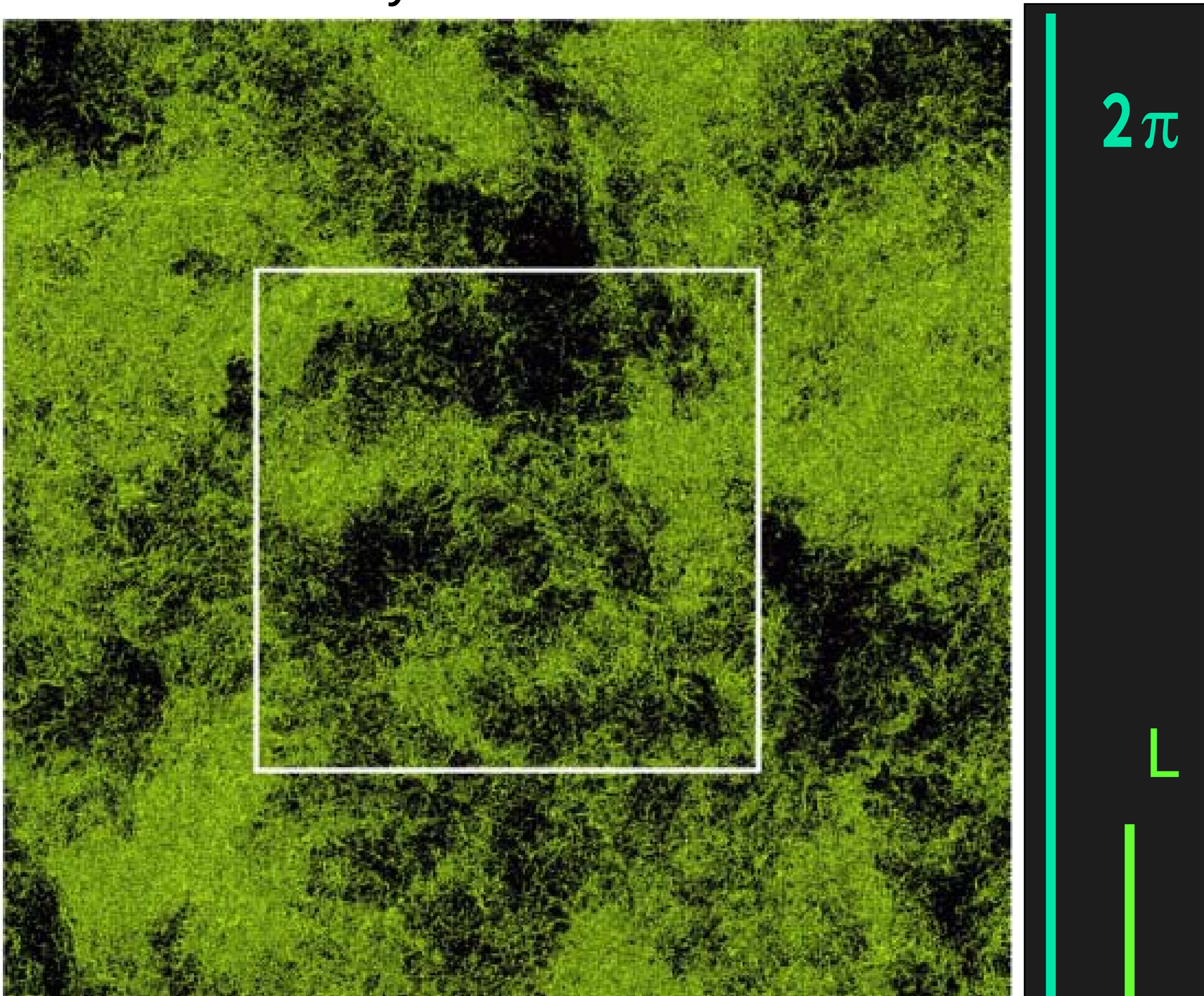
Table 2: DNS parameters and turbulence characteristics at $t = t_f$. Δt is the time increment, $\langle \epsilon \rangle$ the mean rate of energy dissipation per unit mass, and λ the Taylor micro-length scale. (Values except for $N = 4096$ are quoted from Ref. 3).

Series	N	R_λ	k_{max}	$\Delta t(\times 10^{-3})$	t_f	$\nu(\times 10^{-4})$	$\langle \epsilon \rangle$	L	λ	$\eta(\times 10^{-3})$
1	256	167	121	1.0	10	7.0	0.0849	1.13	0.203	7.97
	512	257	241	1.0	10	2.8	0.0902	1.02	0.125	3.95
	1024	471	483	0.625	10	1.1	0.0683	1.28	0.090	2.10
	2048	732	965	0.4	10	0.44	0.0707	1.23	0.056	1.05
	4096	1131	1930	0.25	4.52	0.173	0.0752	1.09	0.034	0.51
2	256	94	121	1.0	10	20	0.0936	1.10	0.326	17.1
	512	173	241	1.0	10	7.0	0.0795	1.21	0.210	8.10
	1024	268	483	0.625	10	2.8	0.0829	1.12	0.130	4.03
	2048	429	965	0.4	10	1.1	0.0824	1.01	0.082	2.00
	4096	675	1930	0.25	3.8	0.44	0.0831	1.05	0.052	1.01

×2000

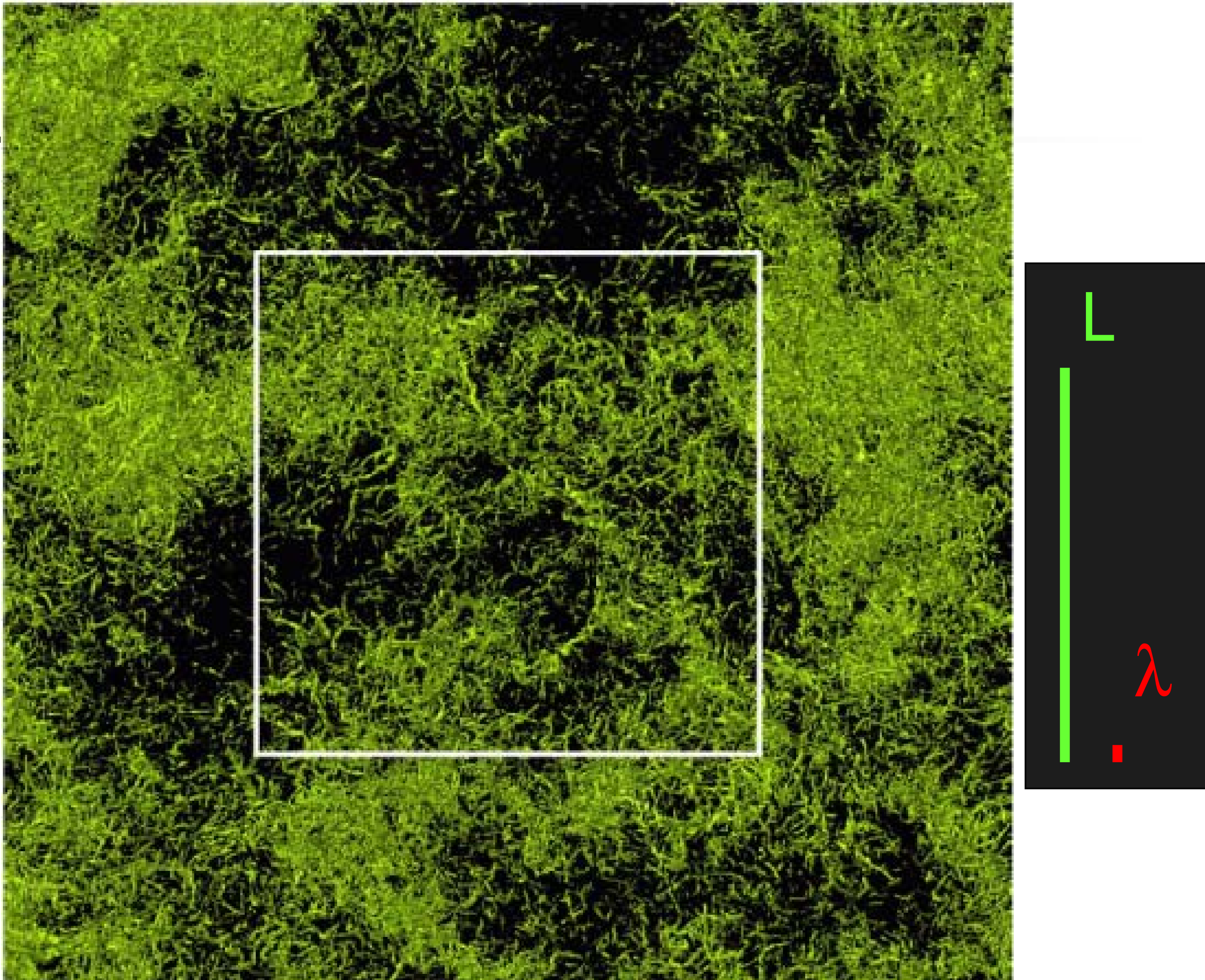
×1000

Image of Flow Field (Vorticity)
by DNS with N^3=2048^3



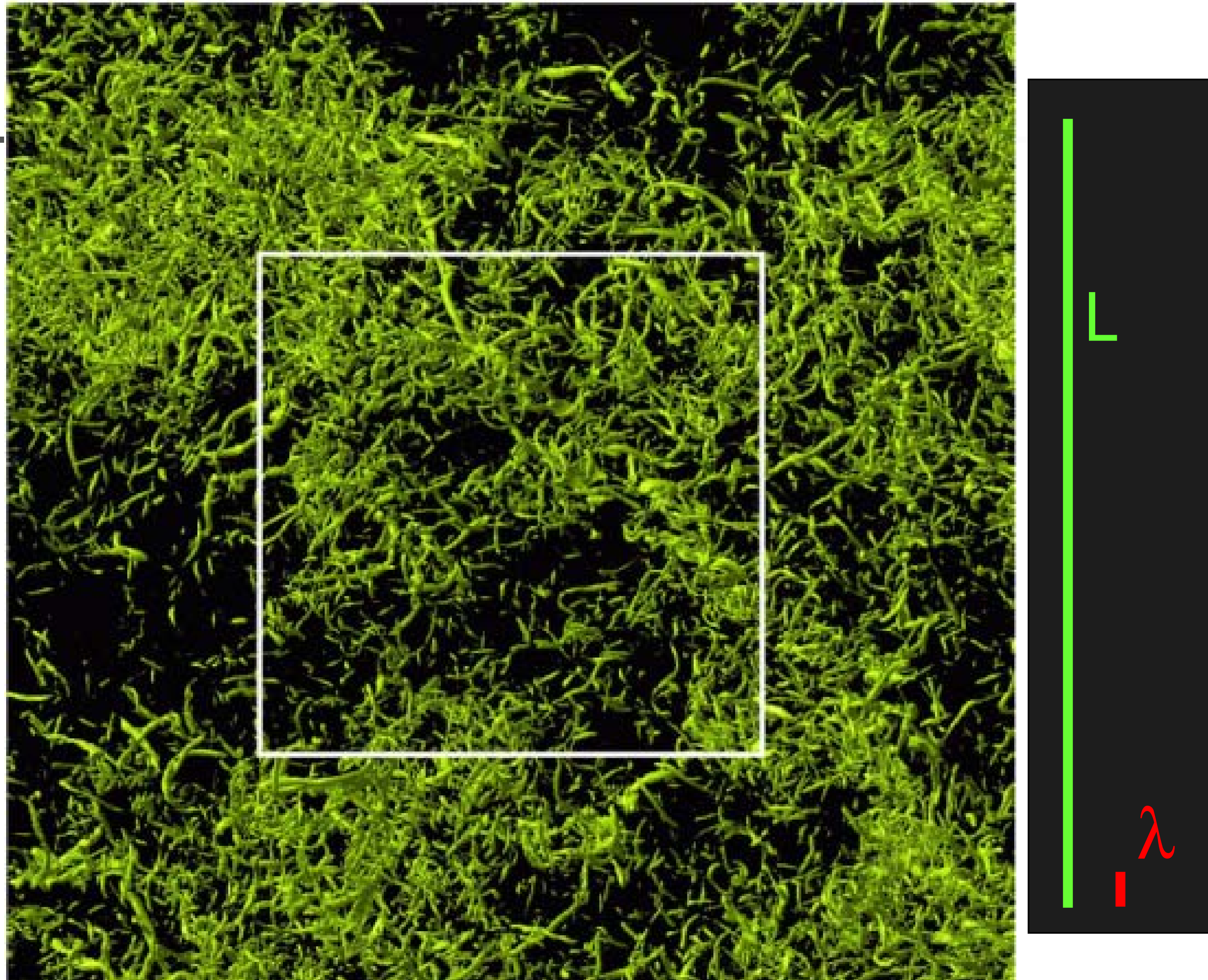
SC2002)

Close up view-1



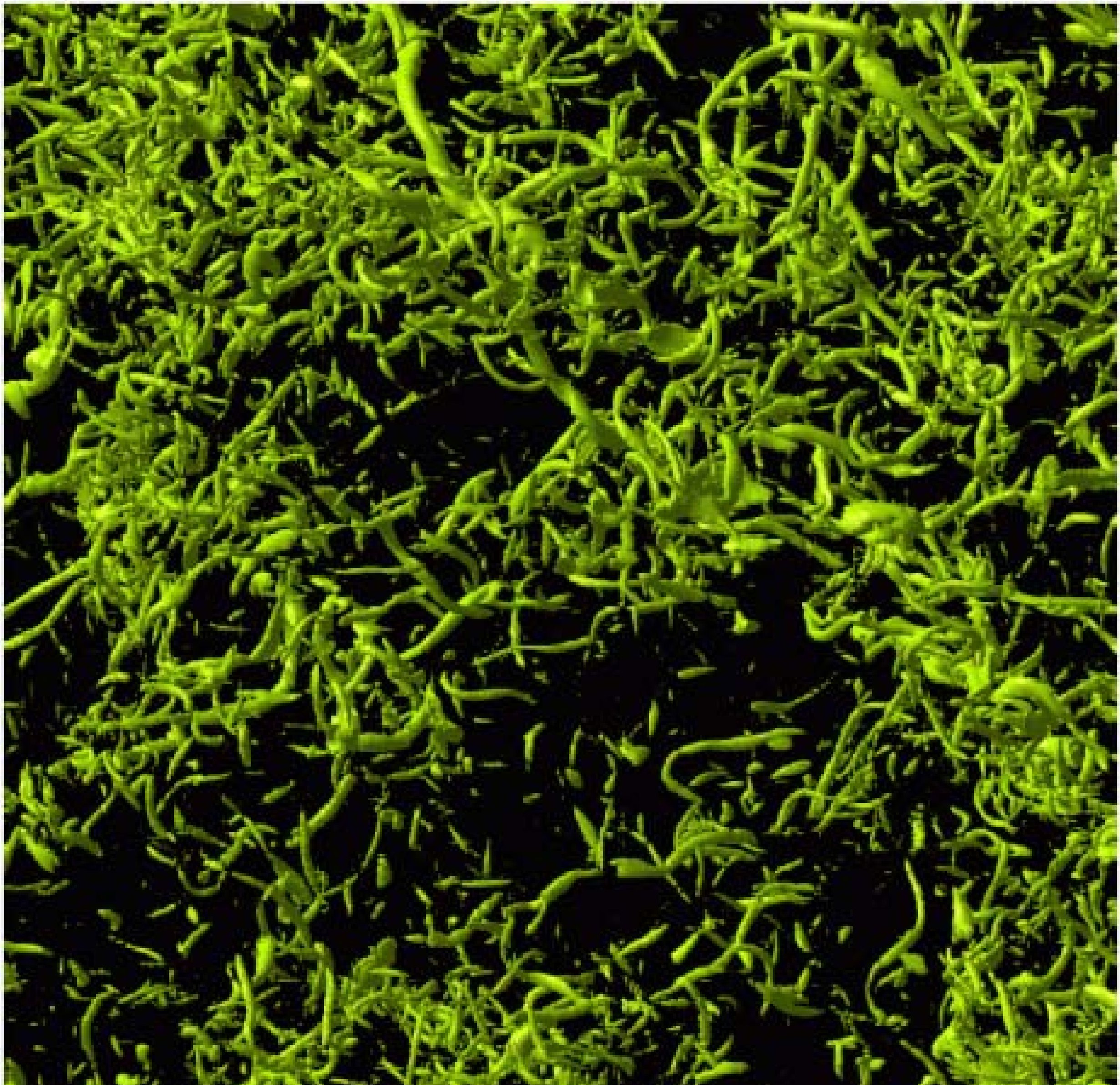
SC2002)

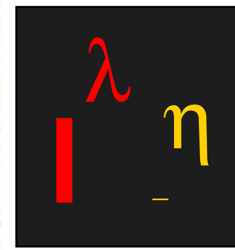
Close up view-2



SC2002)

Close up view-3





SC2002)

Compensated Spectra of Ω and D

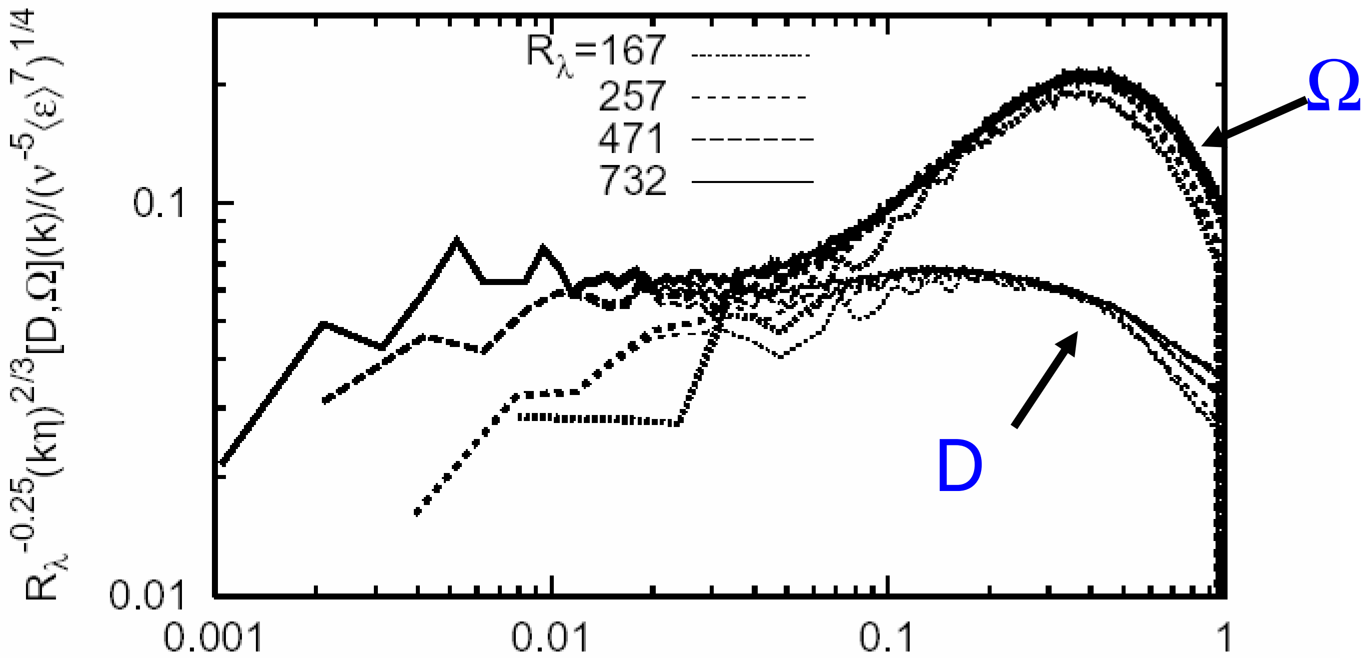
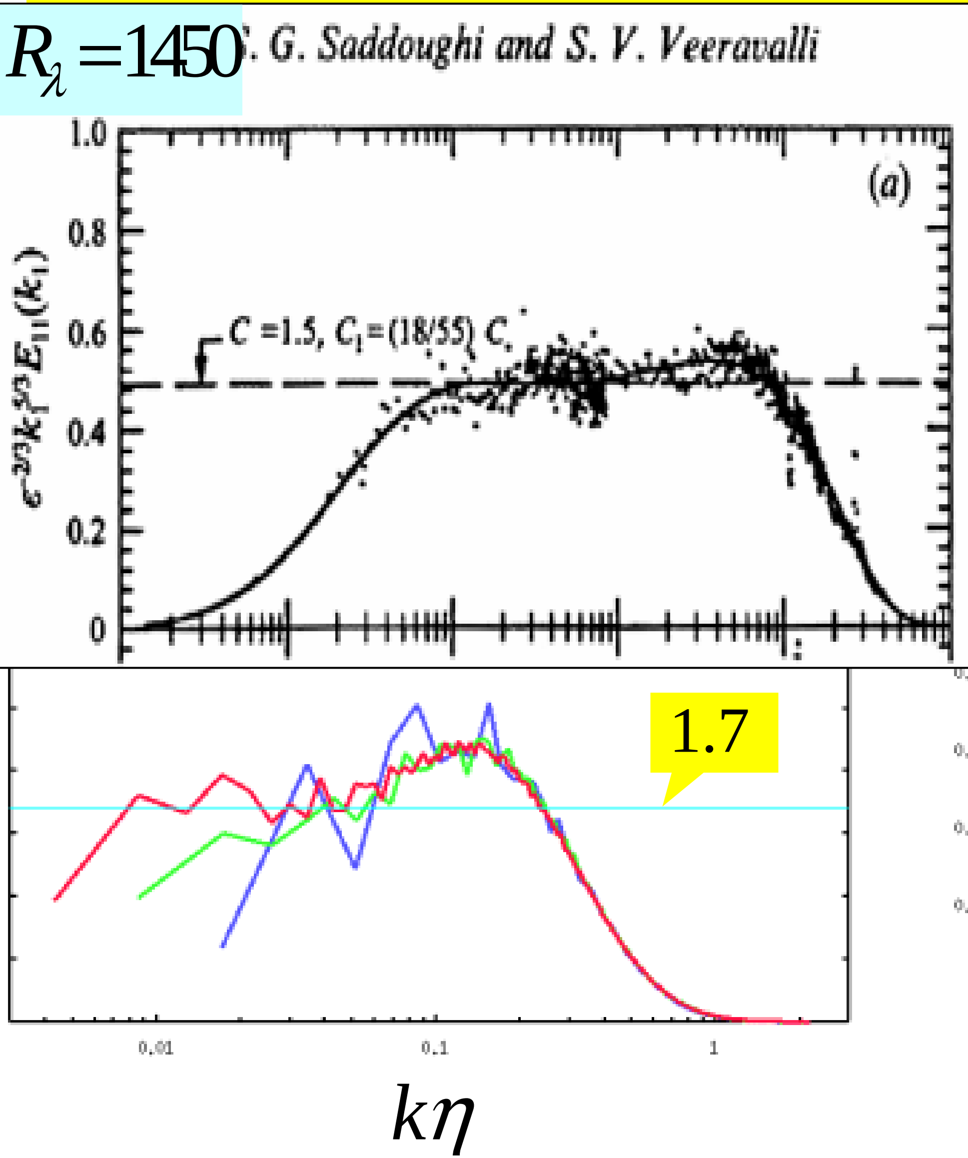


Fig. 5. $\Omega(k)$ (thick lines) and $D(k)$ (thin lines) spectra compensated by $R_\lambda^{-0.25}(k\eta)^{2/3}/(\nu^{-5}\langle\epsilon\rangle^7)^{1/4}$.

J.Phys.Soc Jpn (2003),983-98

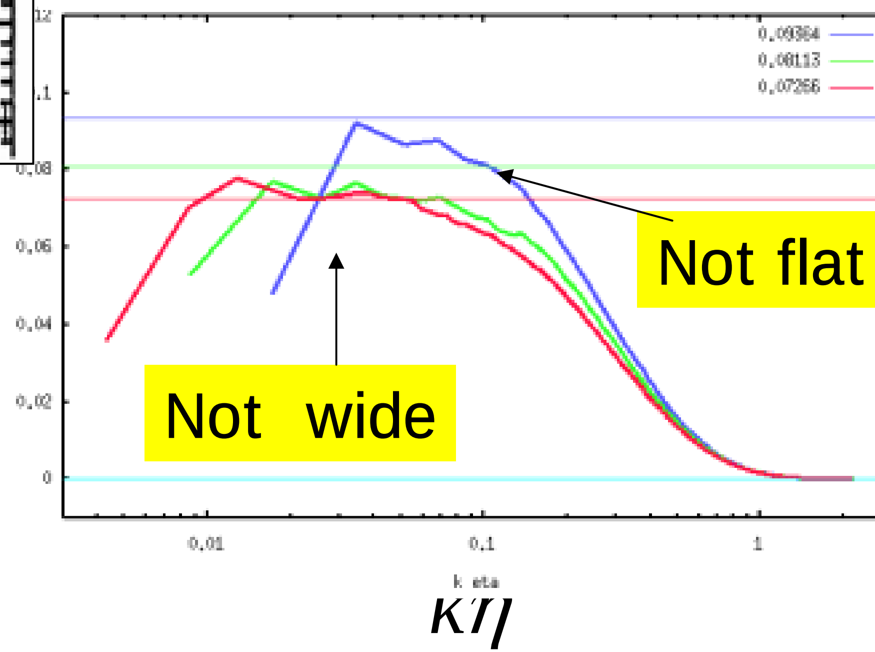
ower resolution: -1



energy transfer

(at statistically steady state)

$$\Pi(k) = \int_k^\infty T(k)dk$$



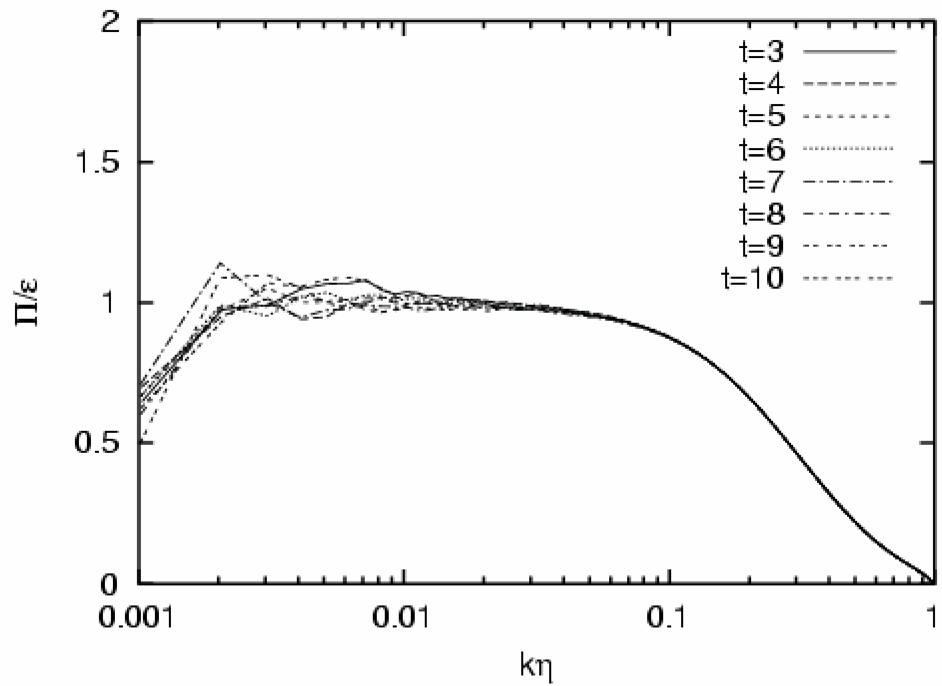
$C_K = 1.62 \pm 0.17$ Experimental values (from Sreenivasan 1995)

$C_K = 1.77$ ALHDIA (Kraichnan 1966) $C_K = 1.72$ LRA (Kaneda 1986)

Some difference from DNS with lower resolution: -2

$\Pi \underset{?}{=} \varepsilon$ (width, flat, stationarity)

$$\Pi(k) = \int_k^\infty T(k)dk$$



N=2048, $k_{max}\eta \sim 1$ $R_\lambda \sim 732$

(Phys Fluids 12(2003) L21-L24)

Energy Spectrum

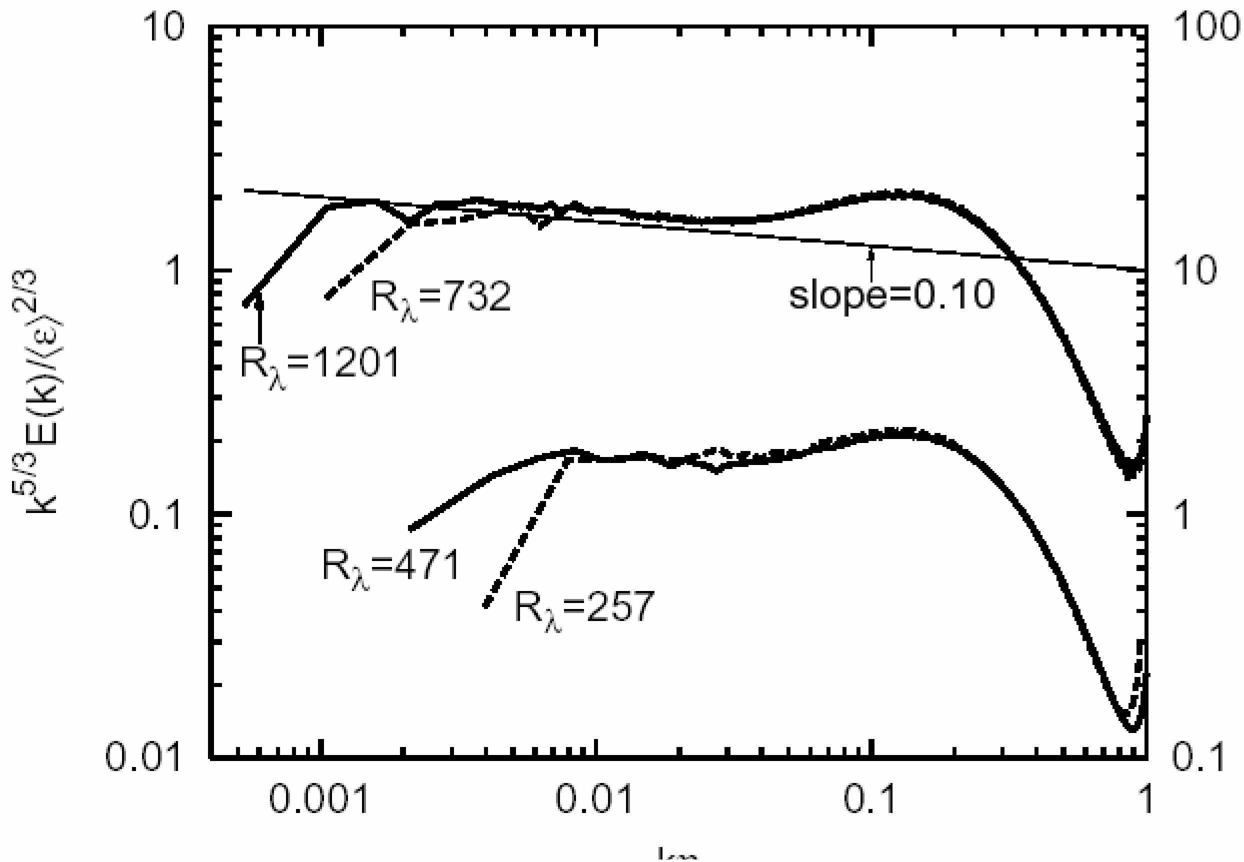


FIG. 5: Compensated energy spectra from DNSs with (A) 512^3, 1024^3, and (B) 2048^3, 4096^3 grid points. Scales on the right and left are for (A) and (B), respectively.

(Phys Fluids 12(2003),L21-L24)

Summary of I-a

- Structure at small scales vs. large
  eddies vs. clusters

  like leaves/twigs/branches/trees vs. forest (cf. CS2002)

  Q: Is the vortex so that important ?
  for the understanding large scale dynamics

- 2048, 4096 DNS give wide inertial range
  → enables quantitative examination
  of theories of inertial range
  example : II-a

I)   High Resolution DNS

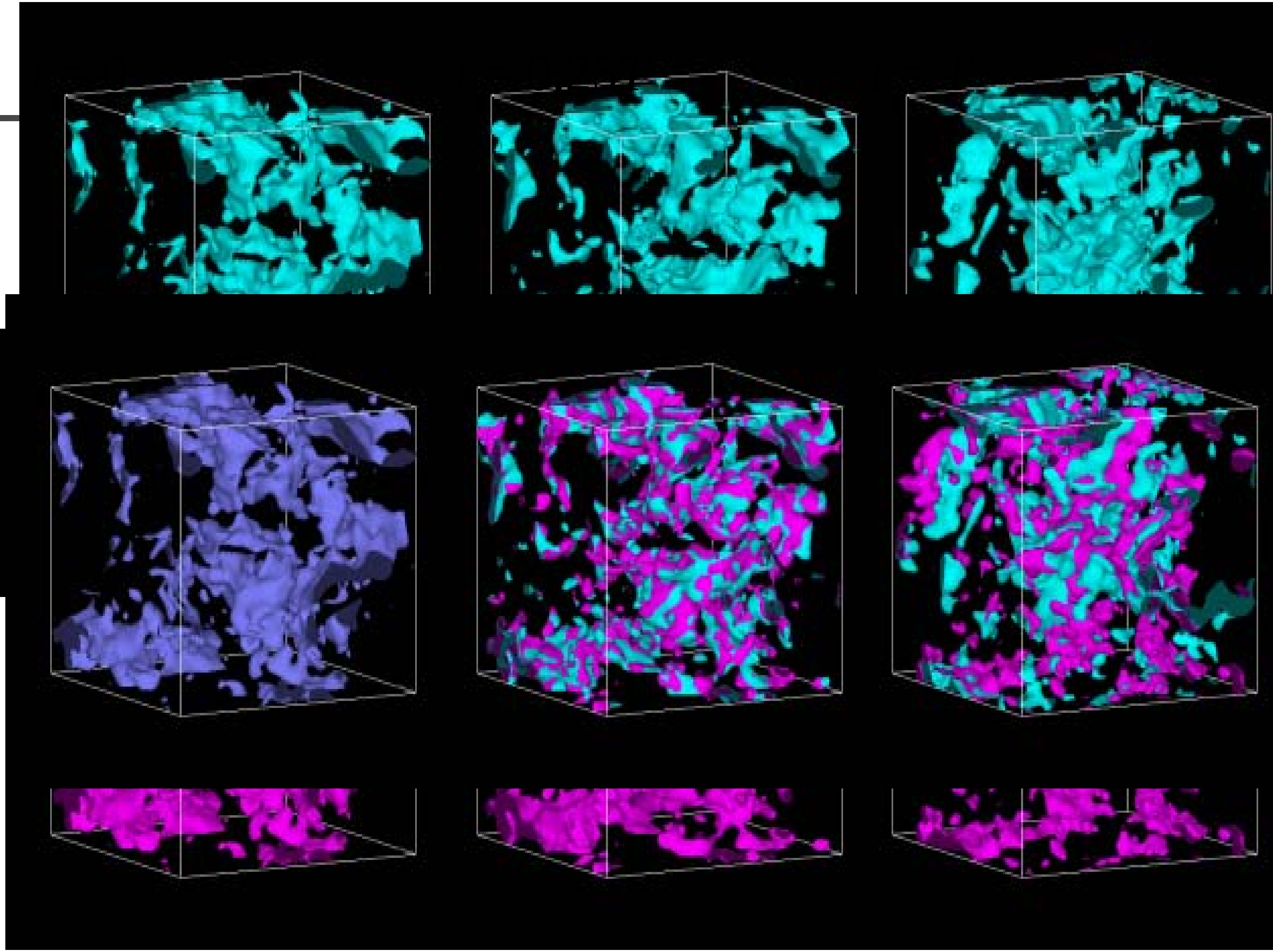
b) Regeneration of small eddies

- a support for the idea of LES

Velocity fields with different initial states in higher *k*

Case 1

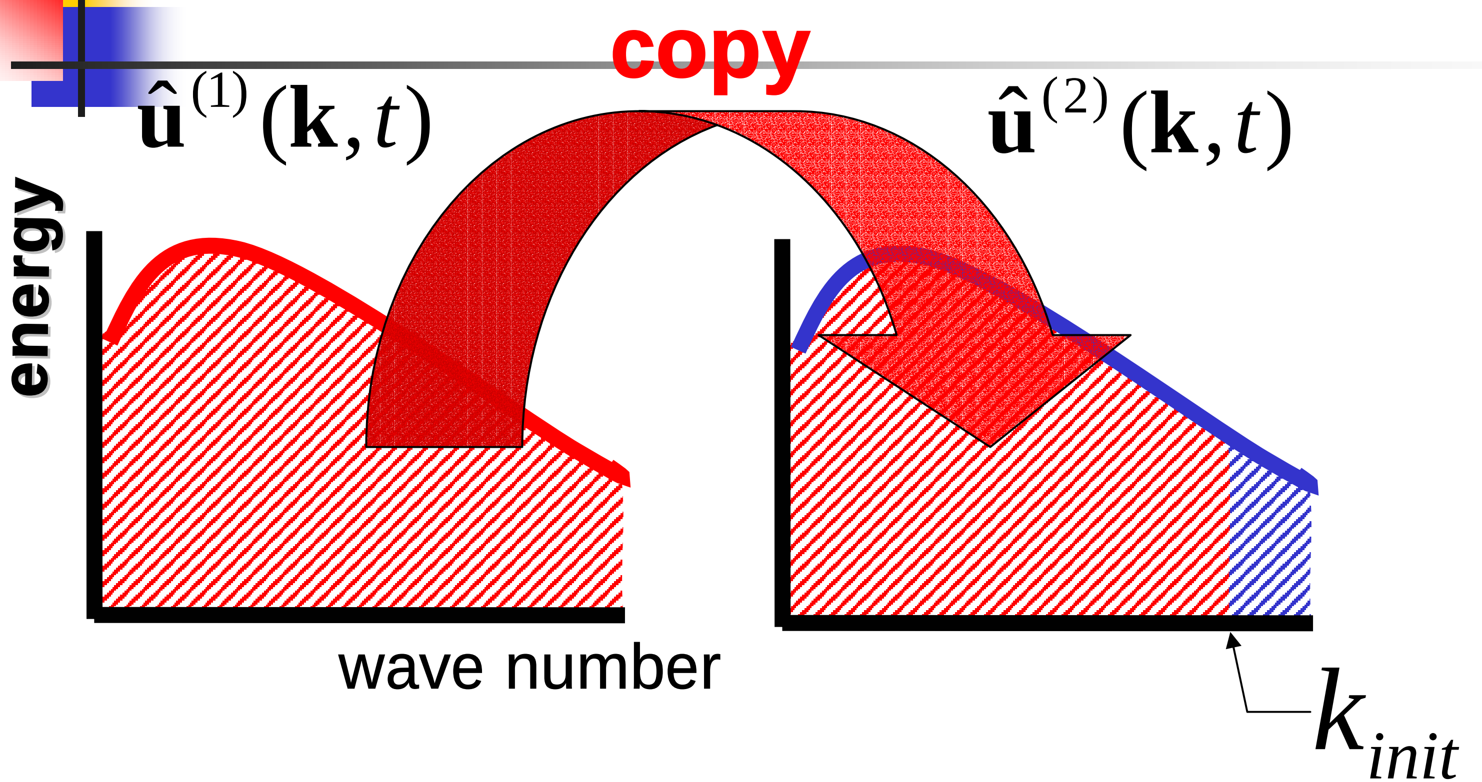
Kinetic energy contour



Case 2

The method of numerical experiments

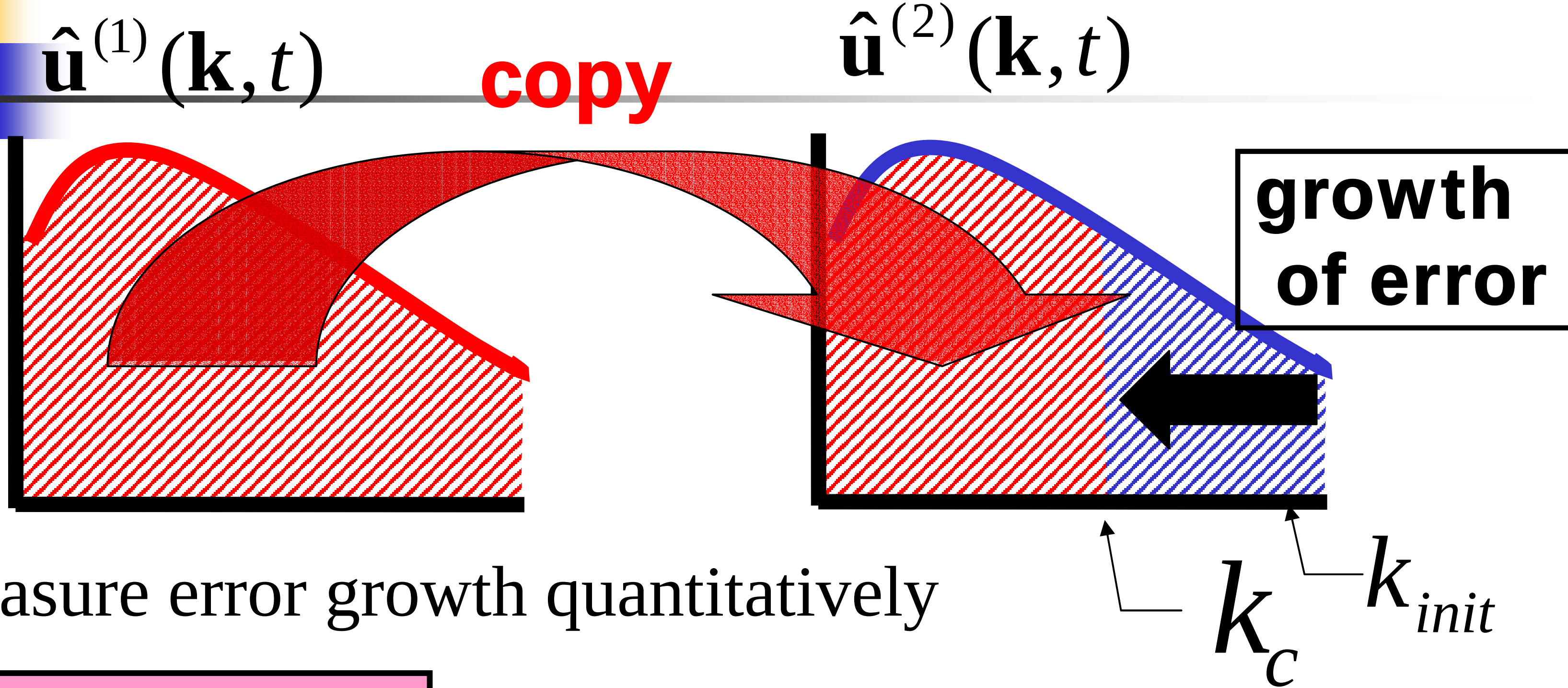
Initial condition



Prepare two different 3D isotopic urbulence $\hat{\mathbf{u}}^{(1)}(\mathbf{k}), \hat{\mathbf{u}}^{(2)}(\mathbf{k})$

Copy $\mathbf{u}^{(1)}(\mathbf{k})$ to $\mathbf{u}^{(2)}(\mathbf{k})$ for $|\mathbf{k}| < k_{init}$

Time marching



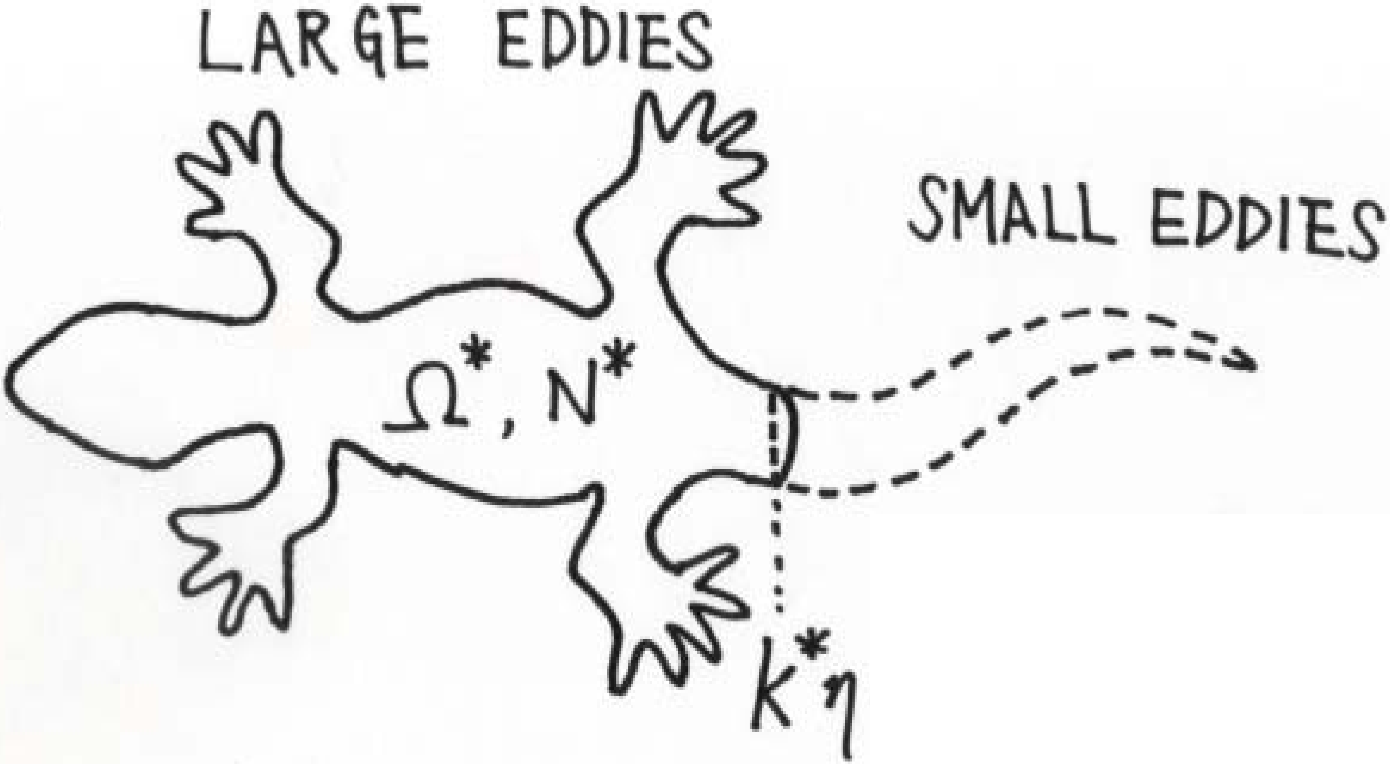
measure error growth quantitatively

2 parameters

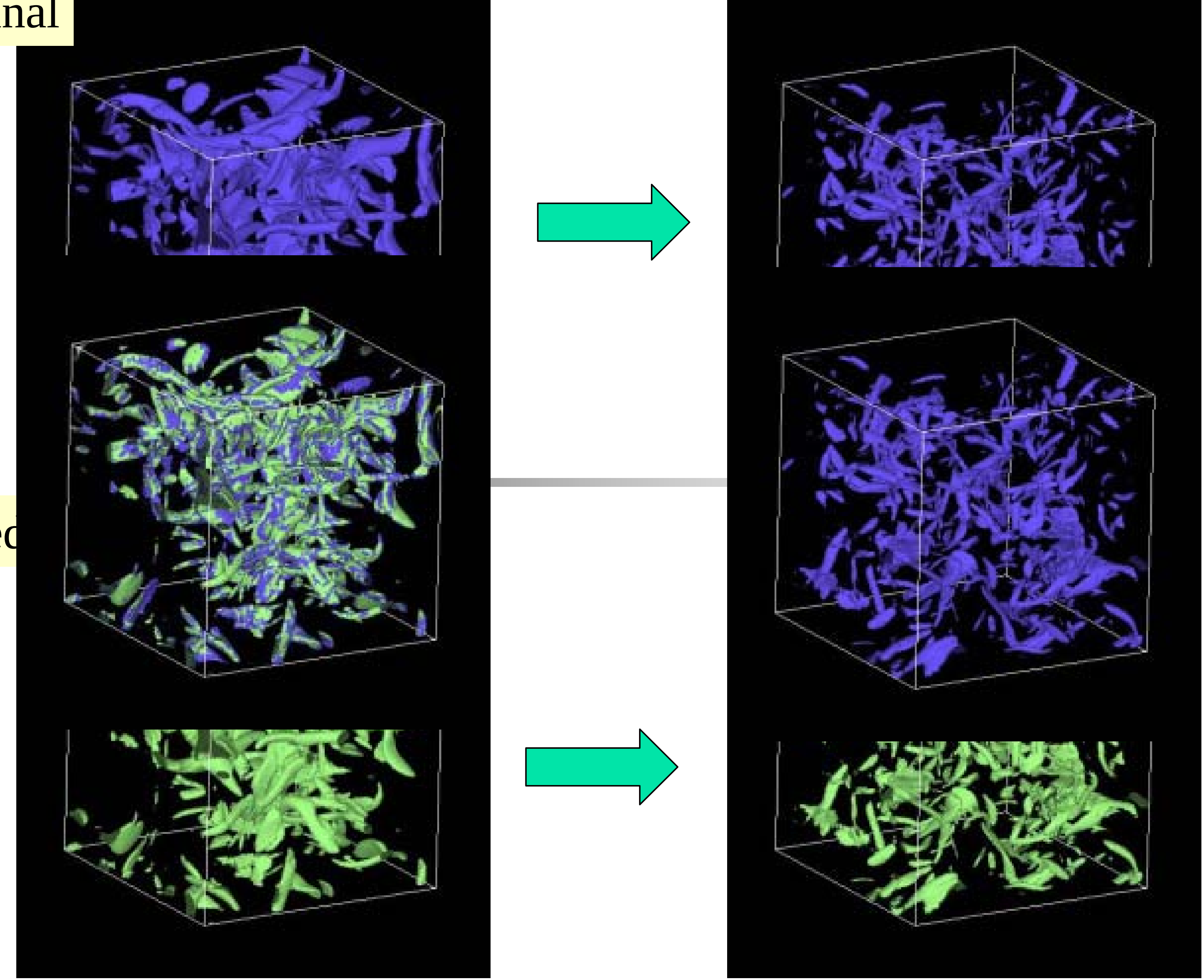
The max wave number of assimilated range : k_c

The time interval of assimilation : T

Regeneration of Small Eddies



By K.Yoshida



128^3 kc=32 vorticity

by Yamaguchi Yoshida

Summary of I-b

- Importance of Large eddies

  small eddies are subordinate

  butterfly effect vs. lizard-tail effect

→

A support for the soundness of

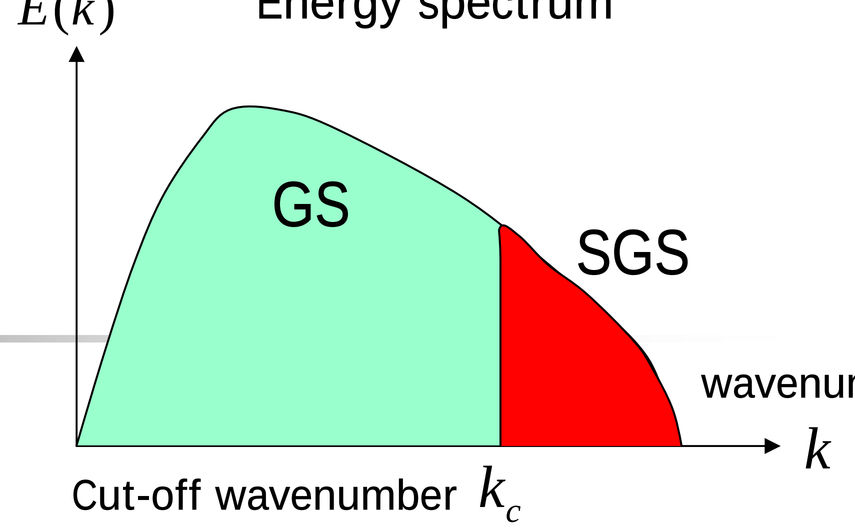
the idea of LES

II) LES Modeling (spectral approach)

a) Deterministic LES

- an application of DNS data analysis

Spectral LES



Navier-Stokes Eq.

$$\left(\frac{\partial}{\partial t} + \nu k^2\right) u_i(\boldsymbol{k}) = M_{imn}(\boldsymbol{k}) \sum_{\boldsymbol{k}=\boldsymbol{p}+\boldsymbol{q}} u_m(\boldsymbol{p}) u_n(\boldsymbol{q}) + f_i(\boldsymbol{k}, t),$$

LES Model Eq.

$$\left(\frac{\partial}{\partial t} + [\nu + \nu_e(k|k_c)]k^2\right) \tilde{u}_i(\boldsymbol{k}) = M_{imn}(\boldsymbol{k}) \sum_{\boldsymbol{k}=\boldsymbol{p}+\boldsymbol{q}} \tilde{u}_m(\boldsymbol{p}) \tilde{u}_n(\boldsymbol{q}) + f_i(\boldsymbol{k}, t),$$

$(k, p, q \leq k_c)$ k_c Cutoff wavenumber

Spectral eddy viscosity

How to determine ?

Requirement for the model

- Energy Spectrum $E(k) = \frac{1}{2} \sum_{k-1/2<|\boldsymbol{k}'|<k+1/2} \langle \boldsymbol{u}(\boldsymbol{k}') \cdot \boldsymbol{u}(-\boldsymbol{k}') \rangle,$

Require the model to simulate E(k)

$$\tilde{E}(k) = E(k), \quad \frac{\partial}{\partial t}\tilde{E}(k) = \frac{\partial}{\partial t}E(k), \quad \text{for } k < k_c.$$

$$\nu_e(k|k_c) = -\frac{T(k) - \tilde{T}(k|k_c)}{2k^2 E(k)}$$

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right)E(k) = T(k)$$

$$\left(\frac{\partial}{\partial t} + 2[\nu + \nu_e(k|k_c)k^2]\right)\tilde{E}(k) = \tilde{T}_c(k|k_c)$$

2P closures

- Closed equations for 2-point statistics

2-point closures

LRA (**Lagrangian Renormalized Approximation)**

- Simplest among Lagrangian closures
- Free from any ad-hoc parameter
- Fully consistent with

  Galilean invariance/Kolmogorov spectrum

Example of performance of the LRA for 2nd order moments:

Equilibrium Energy Spectrum   by the LRA & Experiments

Gotoh, Nagaki, and Kaneda

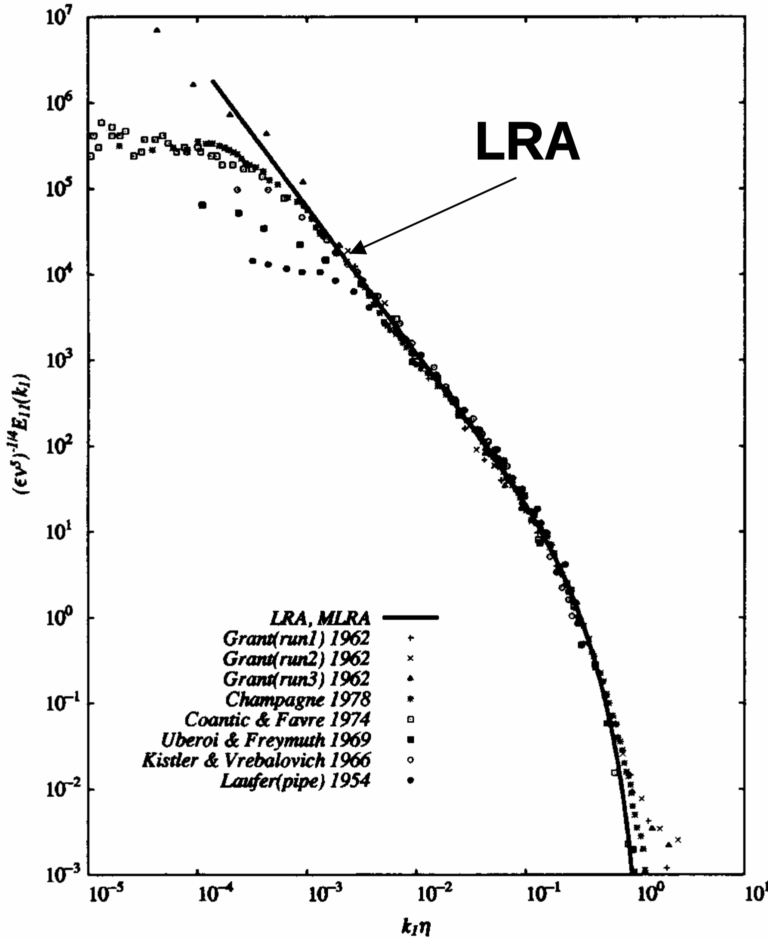


FIG. 1. Comparison of the one-dimensional energy spectrum determined by the LRA (MLRA) with the experimental data (Refs. 25 and 26).

(Phys Fluids 12(2000),155-168)

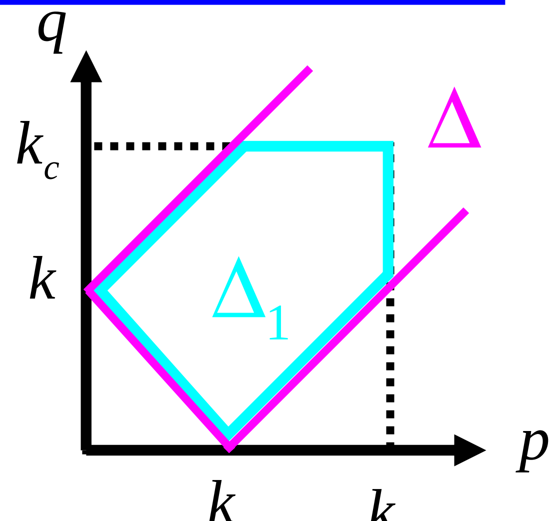
T(k) in 2P closures

$$T(k) = \int\int_{\triangle} dpdq\ k^3 pq b_{kpq}\theta_{kpq} q^{-2} E(q)[p^{-2}E(p) - k^{-2}E(k)],$$

$$\tilde{T}(k|k_c) = \int\int_{\triangle_1} dpdq\ k^3 pq b_{kpq}\tilde{\theta}_{kpq} q^{-2} \tilde{E}(q)[p^{-2}\tilde{E}(p) - k^{-2}\tilde{E}(k)],$$

$$\theta_{kpq} = \int_{-\infty}^{t} ds\ G(k,t,s)G(p,t,s)G(q,t,s),$$

G(k,t,s): Lagrangian response function



- Assume k_c is in the inertial subrange.
  - Substitute similarity solution of E(k) and G(k) of LRA into the equations for T(k) (**Universality** in small scales) .

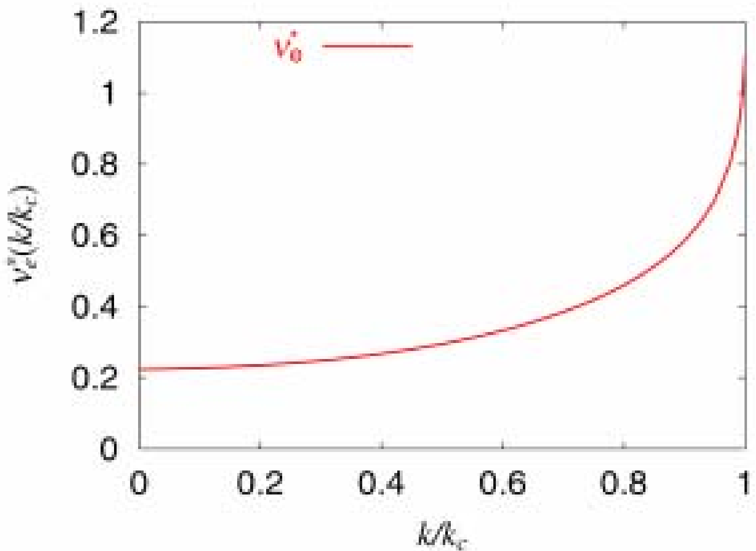
$$E(k) = K_o \epsilon^{2/3} k^{-5/3}, \qquad K_o = 1.72$$

- Simplification, $\tilde{G}(k) = G(k)$.

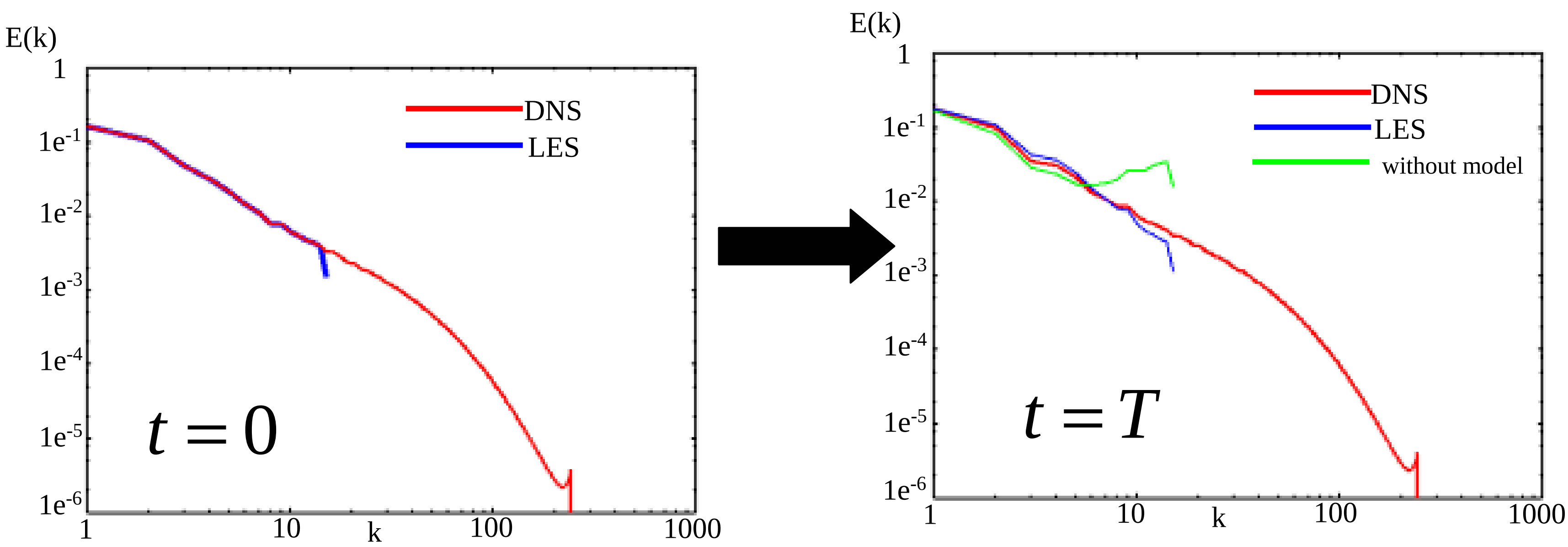
Spectral eddy viscosity

$$\nu_e(k|k_c,t) = [\tilde{\epsilon}(t)]^{1/3} k_c^{-4/3} \nu_e^* \left(\frac{k}{k_c}\right),$$

$$\tilde{\epsilon}(t+\Delta t) = \int_{k<k_c} d\boldsymbol{k}\; 2\nu_e(k|k_c,t) k^2 \boldsymbol{u}(\boldsymbol{k}) \cdot \boldsymbol{u}(-\boldsymbol{k}),$$



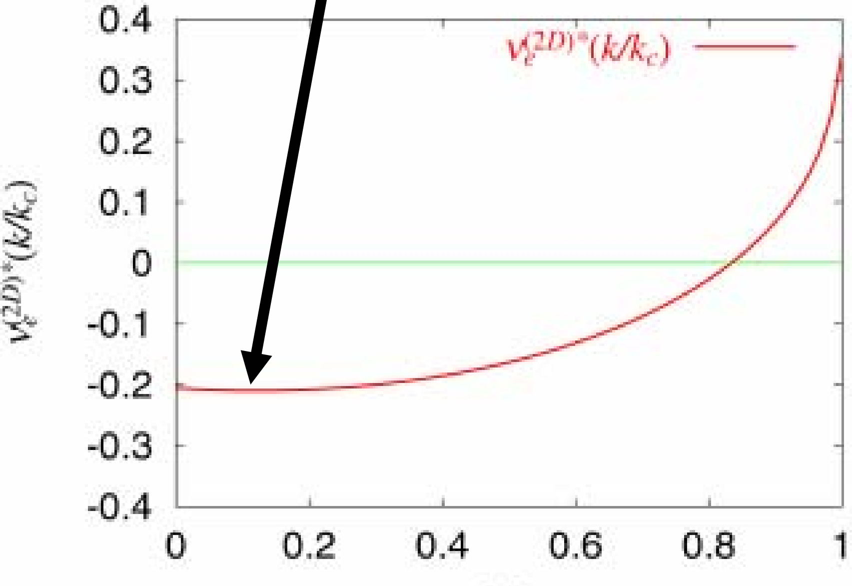
LES of 3D turbulence

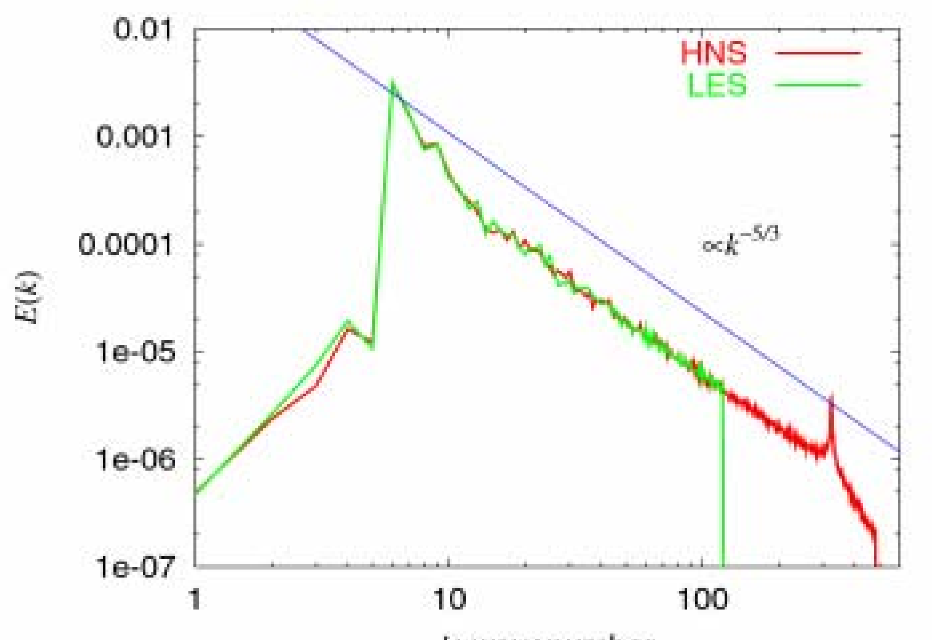


# of deg. of freedom → 1/32000 against DNS with 1024^3

LES model of 2D turbulence with inverse cascade range

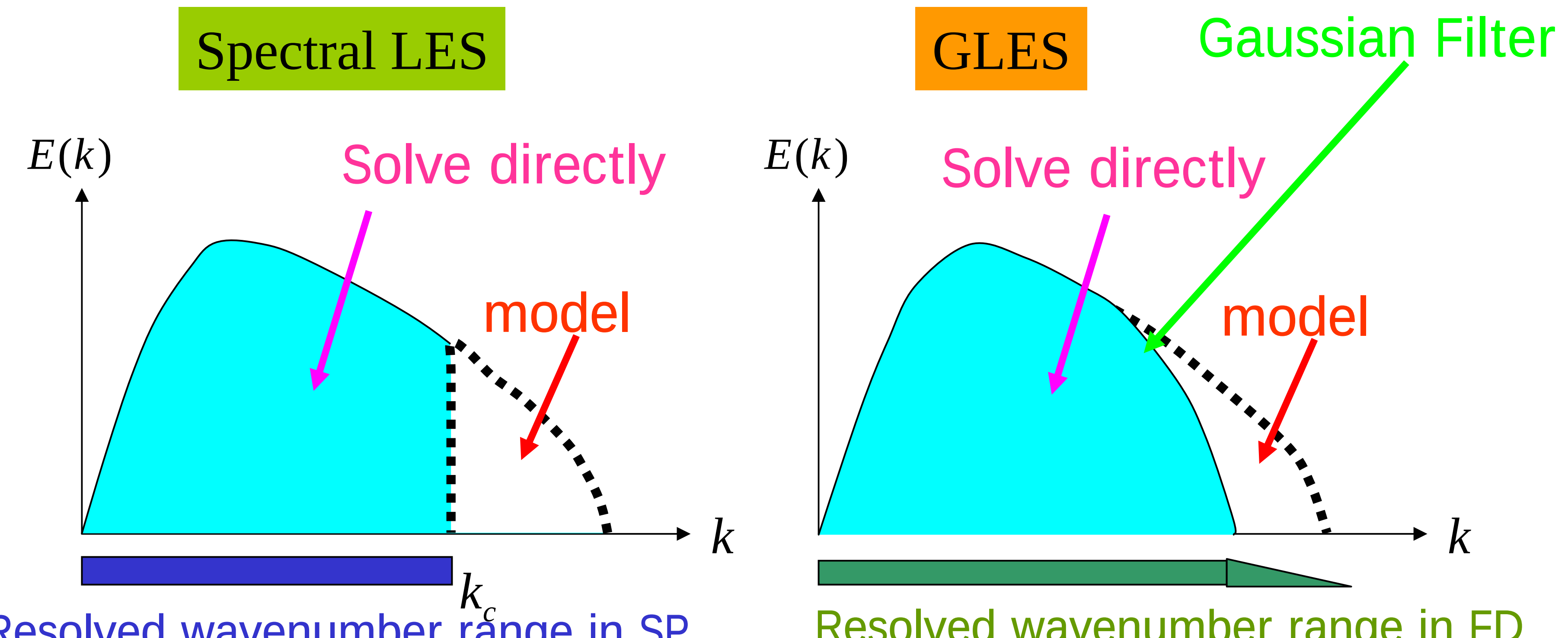
Negative eddy viscosity





Application : finite difference schemes

- LES based on Gaussian Filter（GLES)
  - Gaussian Filter -- easily applied to FD schemes



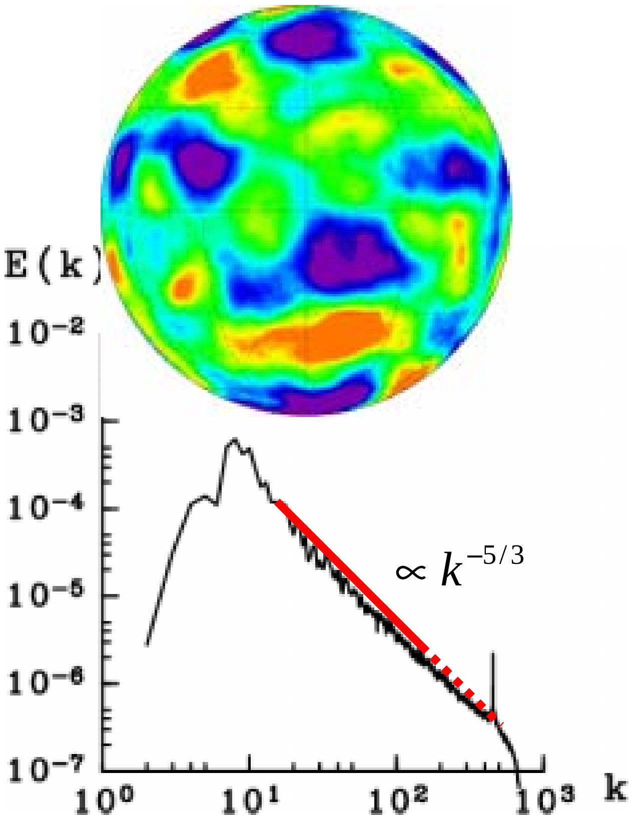
LES applied to FD schemes (1)

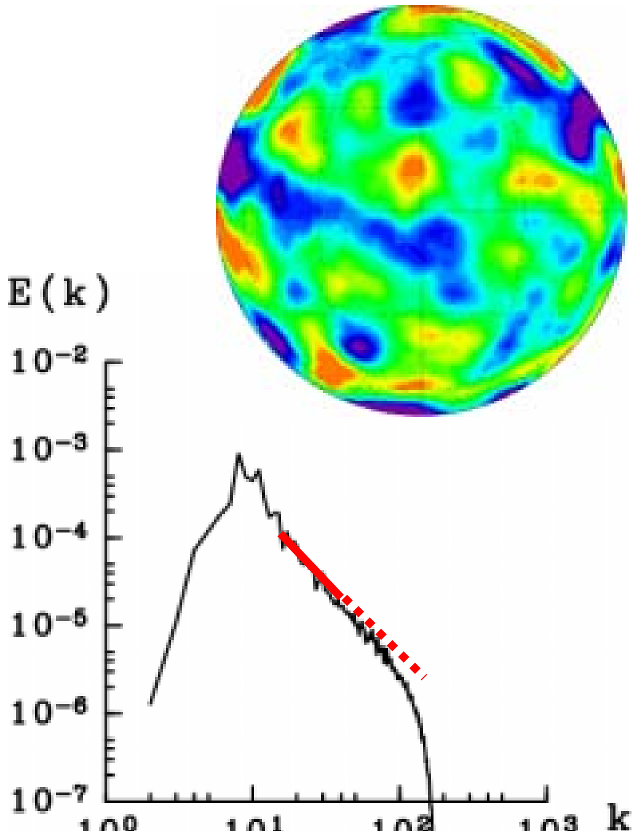
DNS
SH spectral

2048×1024 (T682)

LES
Double Fourier

512×256

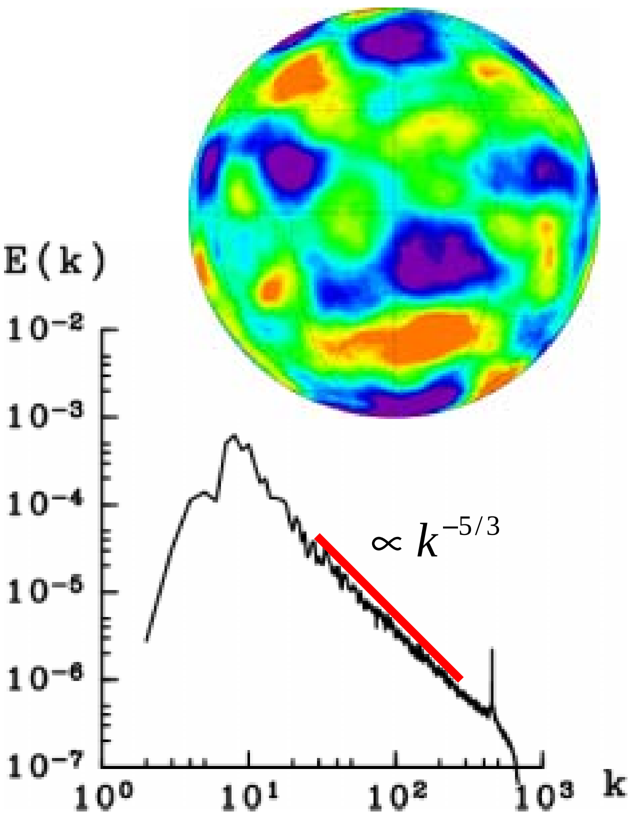




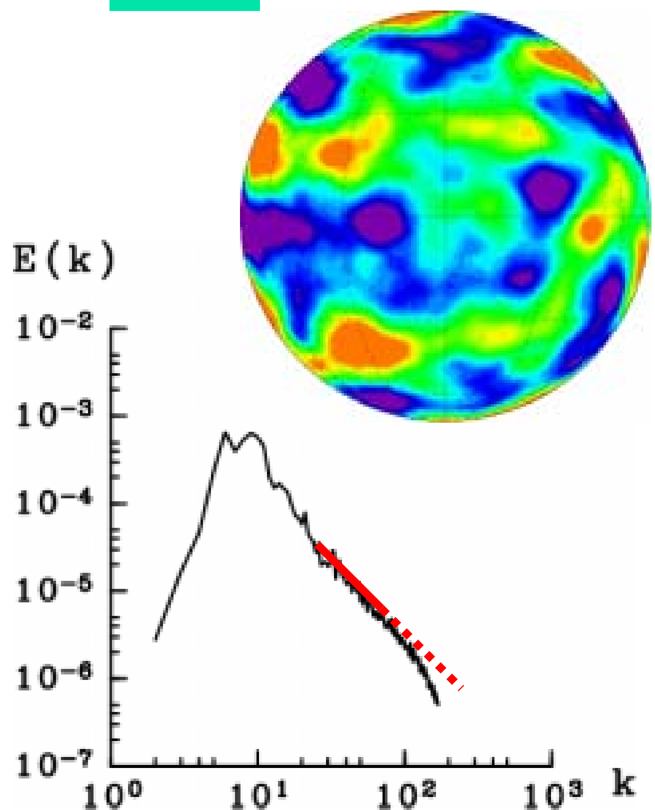
LES applied to FD schemes (2)

DNS 2048×1024 (T682)

SH spectral



LES CCD 512×256



Application to stratified turbulence

- Assume k_c is in the inertial subrange.
  - In SGS,

    u(x) -- quasi isotropic turbulence,

    Density fluctuation field -- almost passive scalar.

Yoshida, Ishihara & Kaneda (2002)

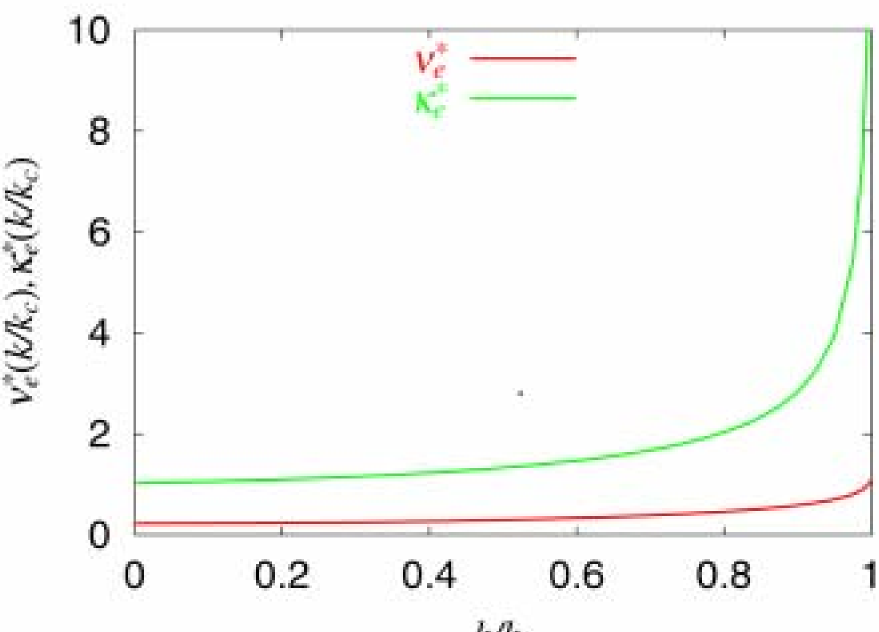
Eddy viscosity and Eddy diffusivity

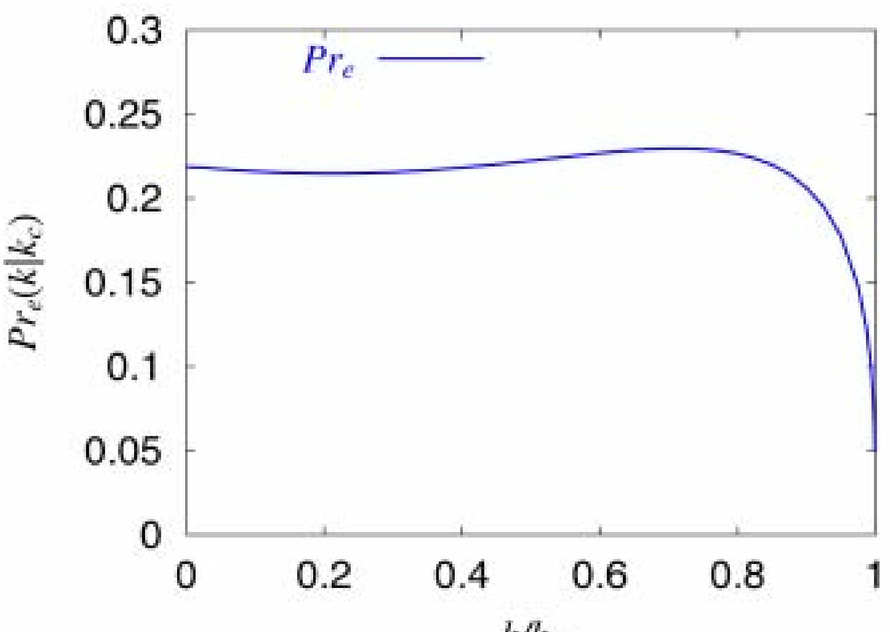
$$\nu_e(k \mid k_c) = \varepsilon^{1/3} k_c^{-4/3} \nu_e^* \left(\frac{k}{k_c} \right)$$

$$\kappa_e(k \mid k_c) = \varepsilon^{1/3} k_c^{-4/3} \kappa_e^* \left(\frac{k}{k_c} \right)$$

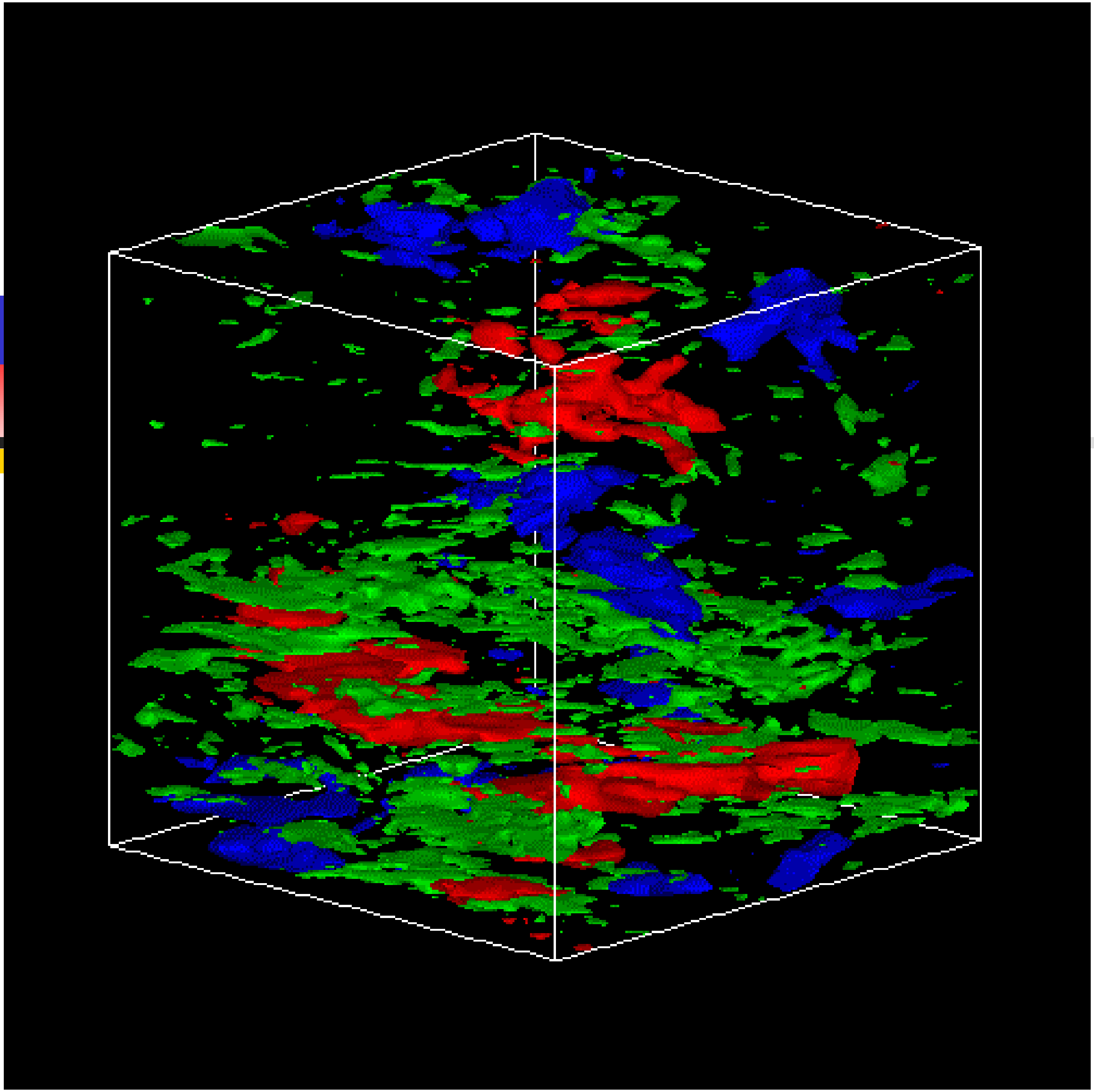
Eddy Prandtl number

$$\mathrm{Pr}_e(k \mid k_c) = \frac{\nu_e(k \mid k_c)}{\kappa_e(k \mid k_c)}$$





LES of stratified turbulence



$N = 3\pi$

$k_b = 231$

Computed resolution 512^3

Visualized resolution 64^3

Isosurface of

$\rho = +2\sigma_\rho$ (red and blue)

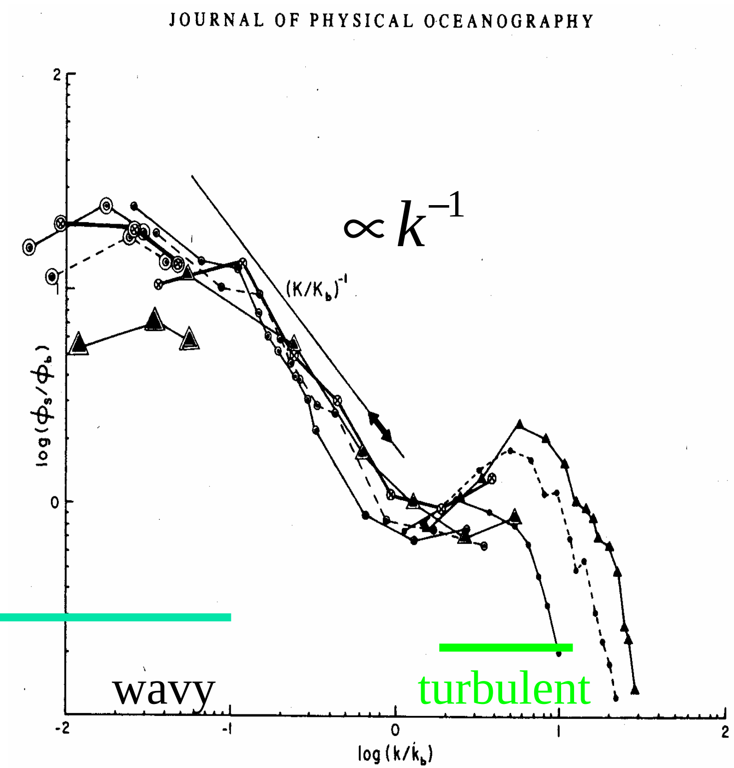
$\rho = -2\sigma_\rho$

$|\omega| = 2\sigma_\omega$ (green)

Vertical shear spectrum

$$S(r) = \sum_{i=1,2} \left\langle \frac{\partial u_i}{\partial x_3}(\mathbf{x}) \frac{\partial u_i}{\partial x_3}(\mathbf{x} + r\mathbf{e}_3) \right\rangle \longrightarrow S(k)$$

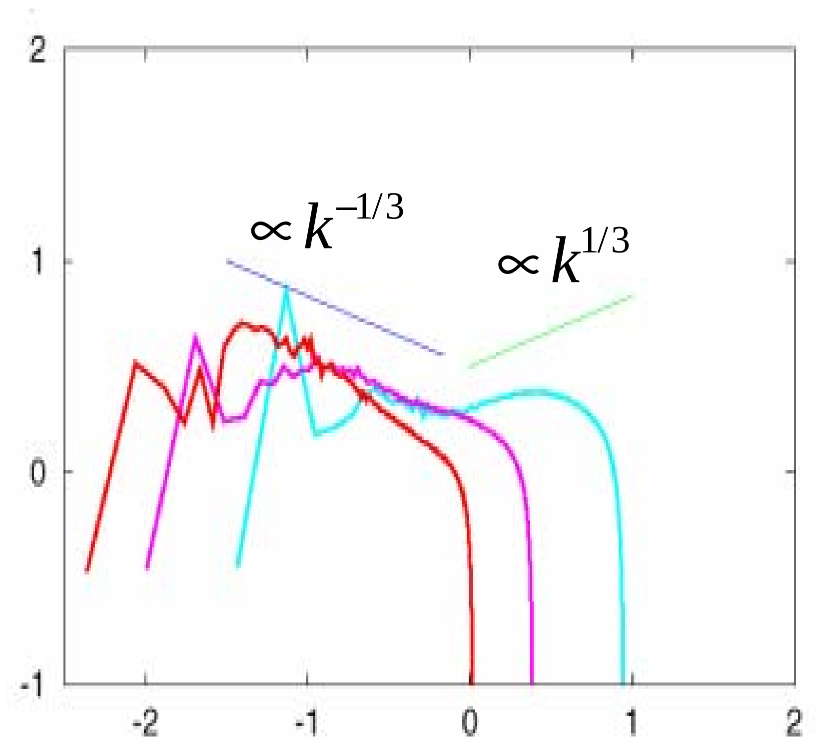
$\log_{10}[S(k)/S_b]$



$\log_{10}(k/k_b)$

(Gargett *et al.*, 1984)

$\log_{10}[S(k)/S_b]$



$\log_{10}(k/k_b)$

An application of DNS-data analysis

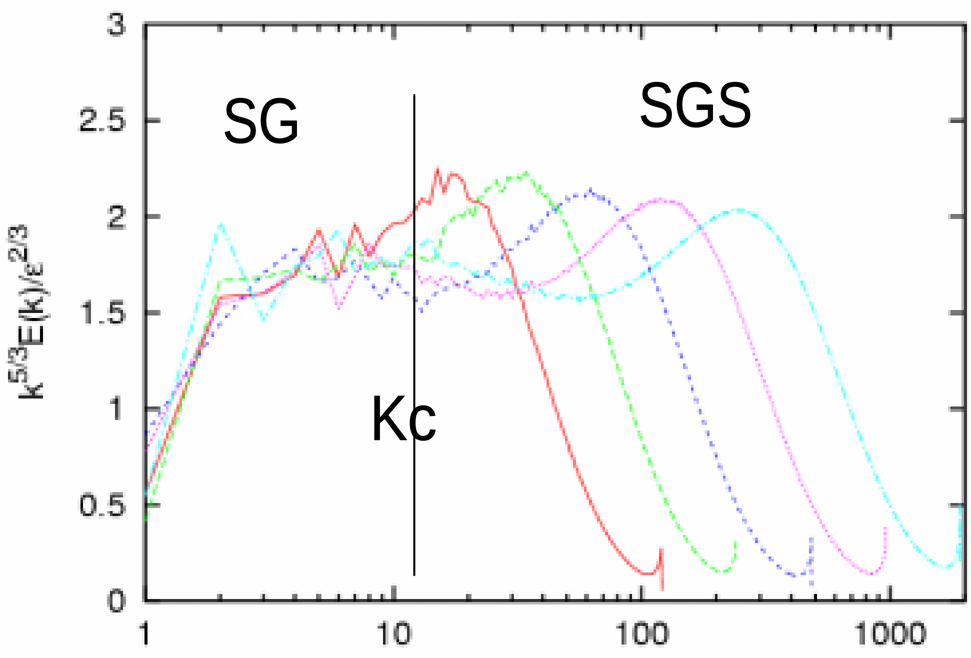
- Examination of Model:

  Eddy viscosity

  Comparison with DNS and theory

Compensated energy spectrum

Series 1



II) LES Modeling (spectral approach)

b) Predictability & Stochastic LES

LES so far

Good for energy

but ....

LES & Predictability

- From the view point of the reduction of Information;

$$? = \langle ? \mid A \rangle + (\text{Res.})$$

Projection to a space A

Residue

- the Dim of Res. is huge

  Dim (Res.) >> Dim (A)

  (in fact, the correlation between model and DNS is poor)

- Difference of u_1, u_2

Impossibility to identify small scale conditions/noise

→ inevitable uncertainty, unpredictability

Error growth due to uncertainty in SGS

$u^{(1)}, u^{(2)}$:Two velocity field with different initial conditions in large wavenumber modes ($k > k_c$).

Difference between two fields

$$\delta \boldsymbol{u} = \boldsymbol{u}^{(1)} - \boldsymbol{u}^{(2)}$$

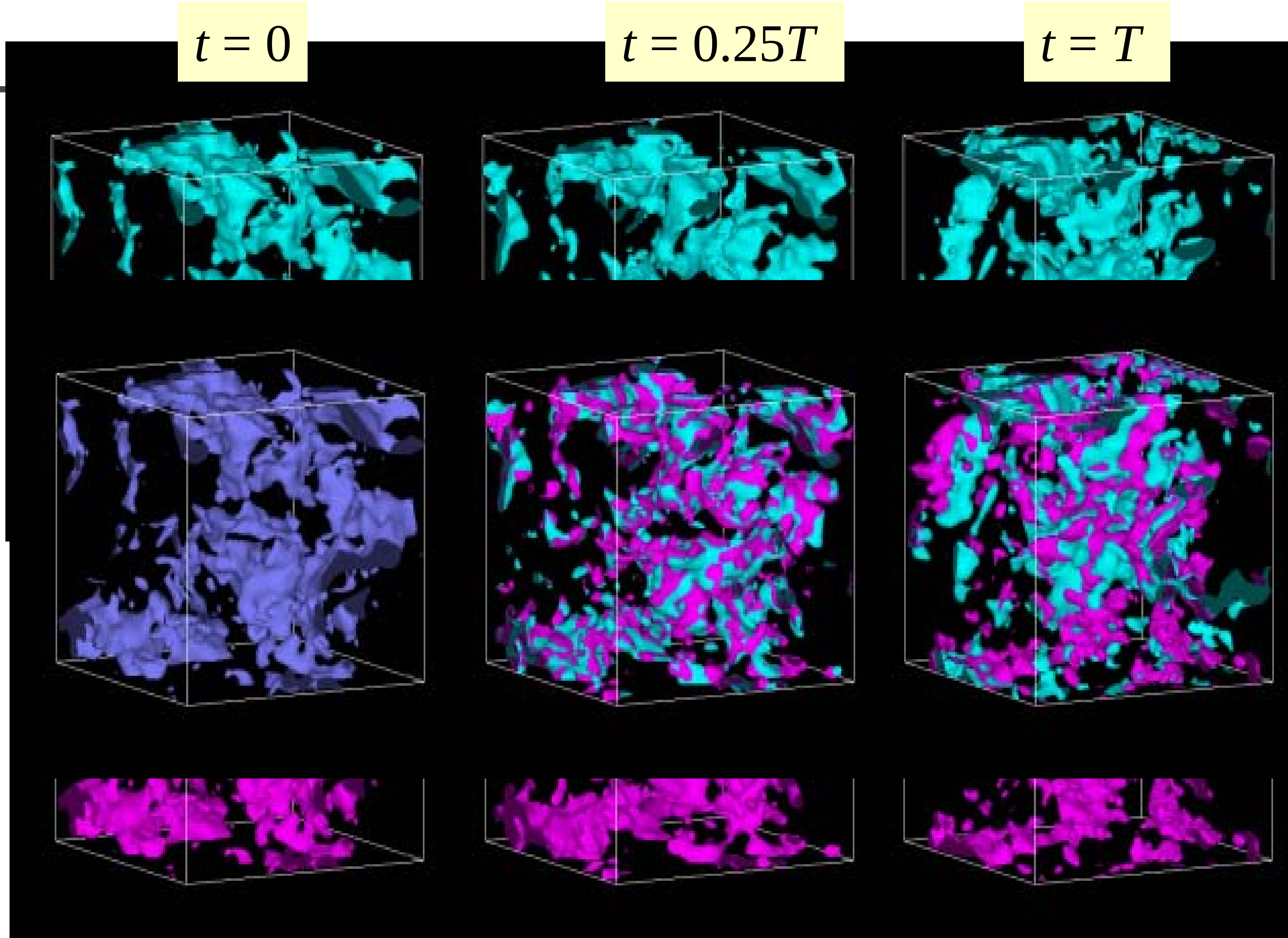
becomes non-zero in small wavenumber modes ($k \leq k_c$) for t>0.

Uncertainty due to SGS uncertainty; Predictability

Case 1

Kinetic
nergy
ontour

Case 2



Probabilistic LES (PLES) model

- Estimate the prediction error due to the uncertainty in SGS.
- Introduce **random external forcing.**

cf. Kraichnan, Bertoglio, Chasnov

$$\left(\frac{\partial}{\partial t} + [\nu + \underline{\mu_e(k|k_c)}]k^2\right)\tilde{u}_i^{(\alpha)}(\boldsymbol{k}) = M_{imn}(\boldsymbol{k}) \sum_{\boldsymbol{k}=\boldsymbol{p}+\boldsymbol{q}} \tilde{u}_m^{(\alpha)}(\boldsymbol{p})\tilde{u}_n^{(\alpha)}(\boldsymbol{q})$$

eddy viscosity

$$+f_i(\boldsymbol{k},t) + \underline{(f_e^{(\alpha)})_i(\boldsymbol{k}|k_c,t)}, \quad \alpha = 1,2$$

Forcing spectrum

$$F(k|k_c,t) = 4\pi k^2 \int_{-\infty}^{t} ds \langle \boldsymbol{f}_e^{(\alpha)}(\boldsymbol{k}|k_c,t) \cdot \boldsymbol{f}_e^{(\alpha)}(-\boldsymbol{k}|k_c,s)\rangle.$$

Requirement for the PLES model

- Error Spectrum $$\Delta(k) = \frac{1}{4} \sum_{k-1/2<|\boldsymbol{k}'|<k+1/2} \langle \delta\boldsymbol{u}(\boldsymbol{k}') \cdot \delta\boldsymbol{u}(-\boldsymbol{k}') \rangle.$$

Require the model to simulate E(k) and **Δ**(k)

$$\tilde{E}(k) = E(k), \quad \frac{\partial}{\partial t}\tilde{E}(k) = \frac{\partial}{\partial t}E(k),$$

$$\tilde{\Delta}(k) = \Delta(k), \quad \frac{\partial}{\partial t}\tilde{\Delta}(k) = \frac{\partial}{\partial t}\Delta(k), \quad \text{for } k < k_c.$$

DNS

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) E(k) = T(k),$$

$$\left(\frac{\partial}{\partial t} + 2\nu k^2\right) \Delta(k) = S(k),$$

SLES

$$\left(\frac{\partial}{\partial t} + 2[\nu + \mu_e(k|k_c)]k^2\right) \tilde{E}(k,t) = \tilde{T}(k) + F_e(k|k_c),$$

$$\left(\frac{\partial}{\partial t} + 2[\nu + \mu_e(k|k_c)]k^2\right) \tilde{\Delta}(k,t) = \tilde{S}(k) + F_e(k|k_c),$$

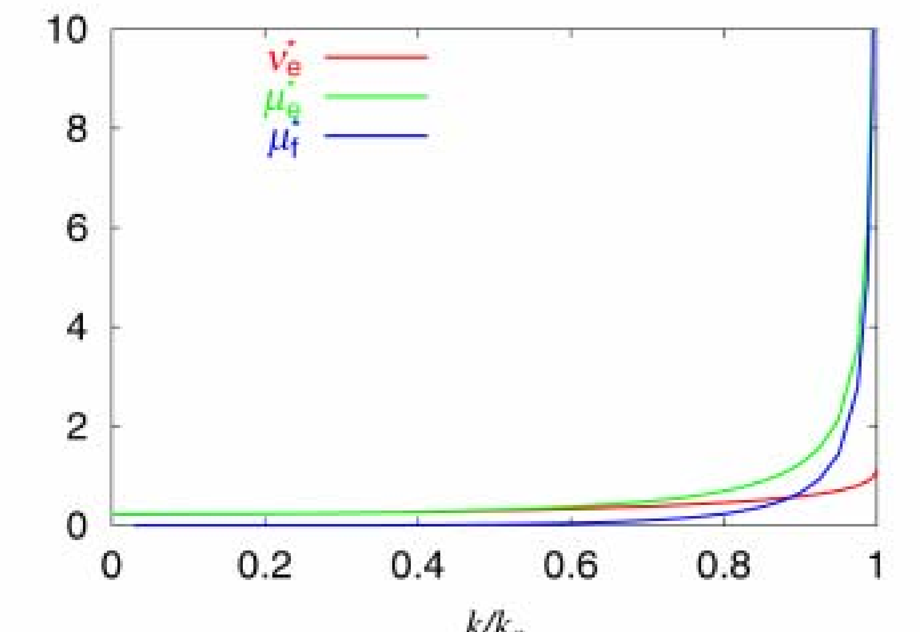
Eddy viscosity and random forcing in PLES

- After some simplifications
  - $\tilde{G}(k) = G(k)$
  - Δ(k)=E(k) for $k > k_c$

$$\mu_e(k|k_c) = \epsilon^{1/3} k_c^{4/3} \mu_e^*(k/k_c),$$

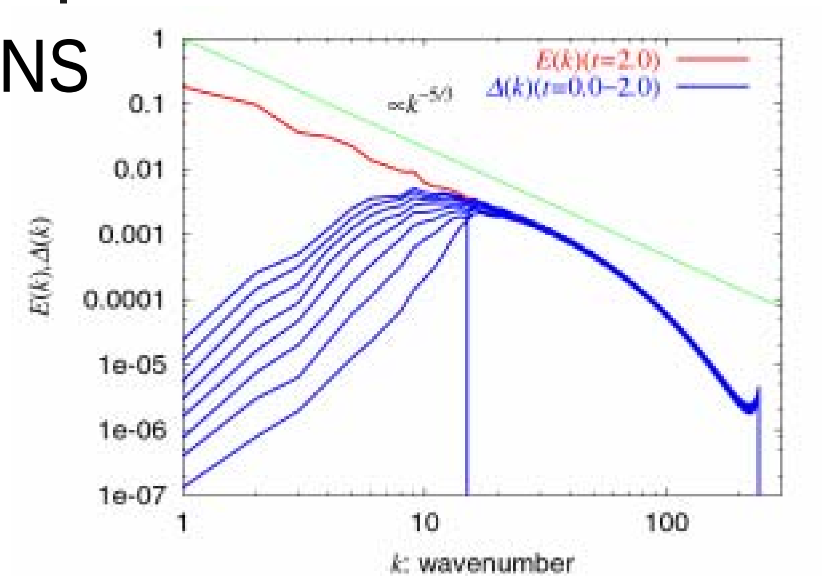
$$F_e(k|k_c) = 2K_o \epsilon k_c^{5/3} \mu_f^*(k/k_c).$$

$$\mu_e^* - \mu_f^* = \nu_e^*$$

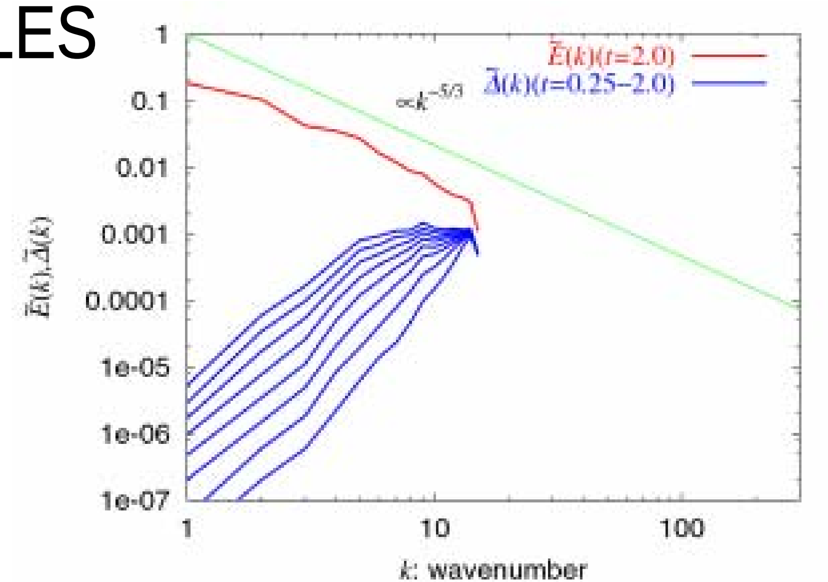


Δ (k) in DNS and PLES

DNS



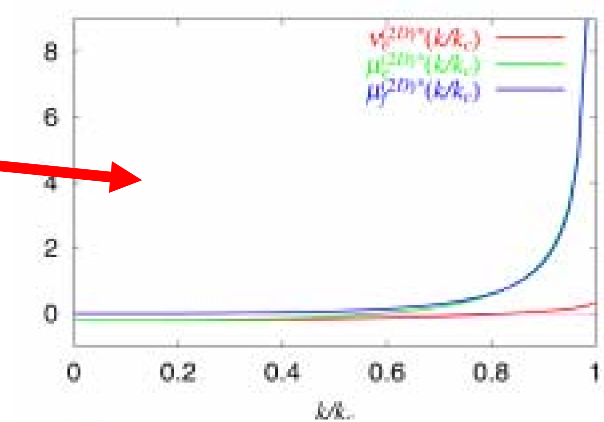
PLES



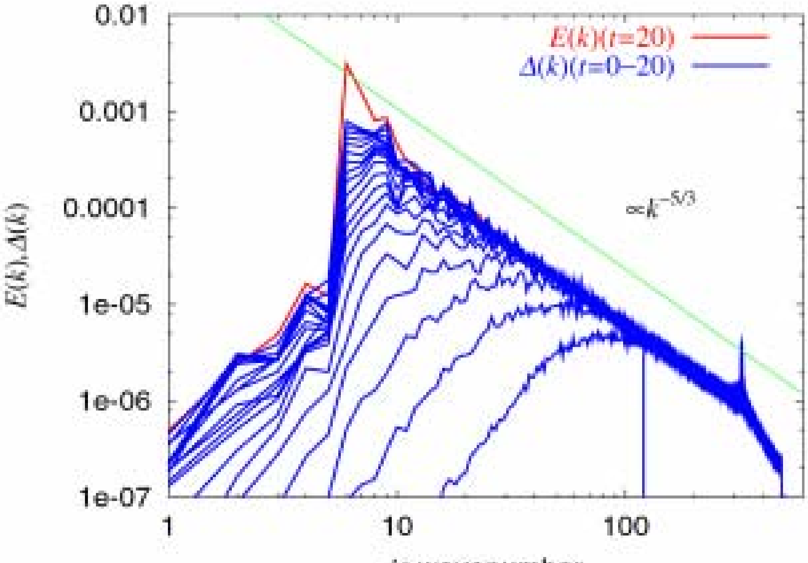
Yoshida, et al(2002)

PLES of 2D turbulence with inverse cascade range

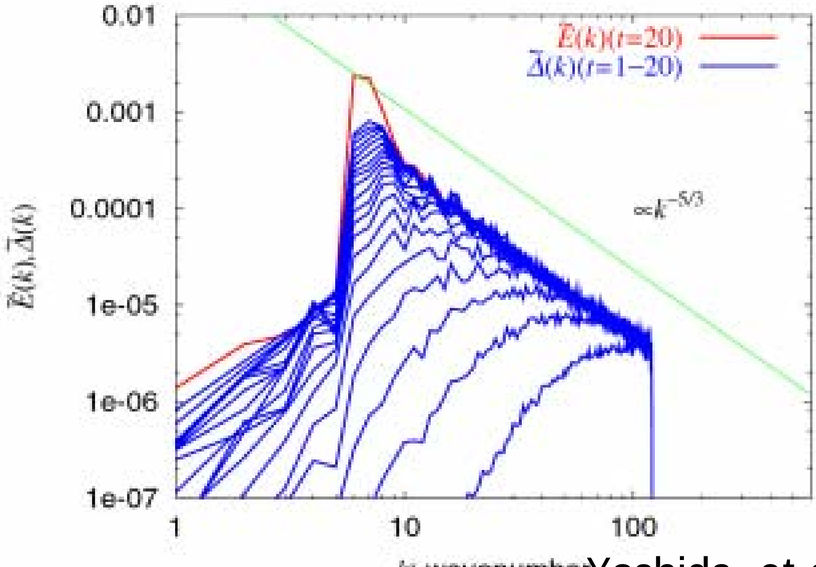
Eddy viscosity and random forcing



DNS



PLES



Yoshida, et al(200

Summary

- Spectral LES
  without ad-hoc parameter-tuning

- DNS data →
  a comparative test for the theory of
  1) eddy viscosity, 2) triad interaction, localness

- Probabilistic LES
  predictability